Big Root approximation of site-scale vegetation water uptake

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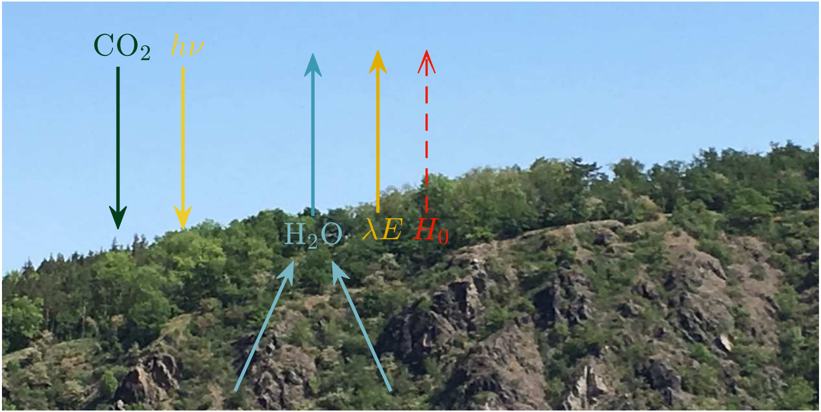
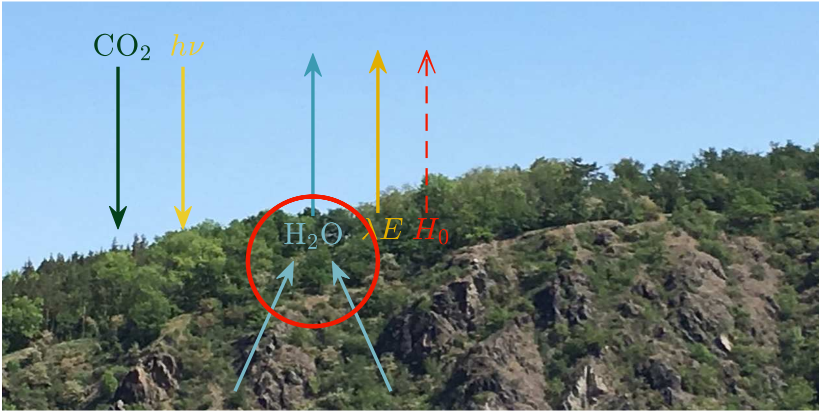
Vegetation as a Land Surface Feature



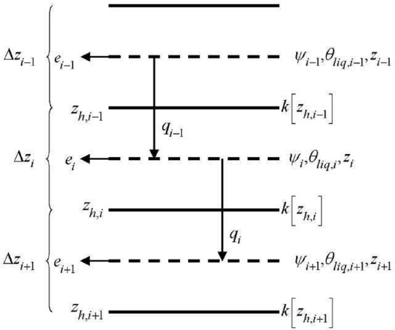
Canyon of Vltava river at Sedlec

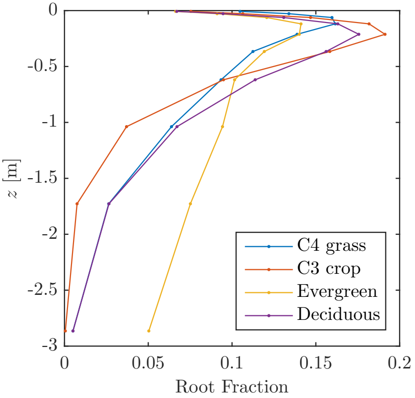
Vegetation as a Land Surface Feature



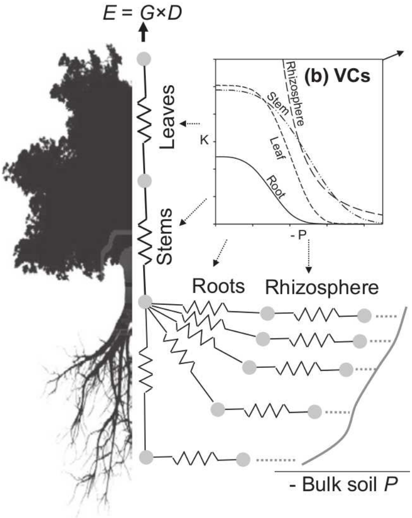
Canyon of Vltava river at Sedlec

Terrestrial models: soil water and root representation





$$\text{stress factor } \beta = \frac{\psi - \psi_{min}}{\psi_{max} - \psi_{min}}$$

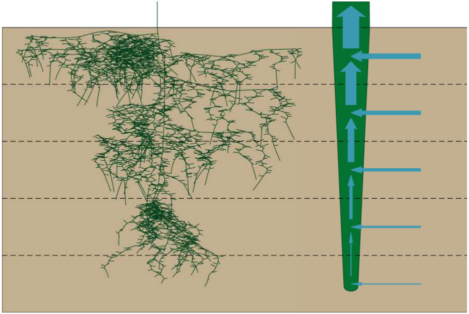


(Sperry *et al.*, 2016, New Phytologist)

Next generation

- Moisture gradient in soil, plant, atmosphere.
- Ohm's law analogue.
- All root resistances assumed in parallel.
- Misrepresentation of root system architecture.
- Known problems, e.g. redistributes water too freely (Kennedy et al., 2019)

Effects of Root System Architecture

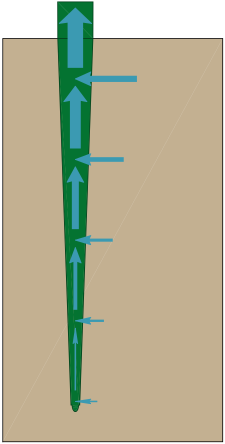


$$Q_{plant}^{i \Rightarrow i-1} - Q_{plant}^{i+1 \Rightarrow i} = Q_{soil \Rightarrow plant}^{i}$$

$$\frac{\Delta^2 \bar{\psi}_x}{\Delta z^2} = \frac{\bar{K}_r^{soil \Rightarrow plant}}{\bar{K}_z^{plant}} \left(\bar{\psi}_x - \psi_s \right)$$

- Flows accumulate upward.
- The more inflow in layer i, the greater the difference in water potential across layers $i \pm 1$.
- RSA determines ratio between uptake and potential gradient dissipation.

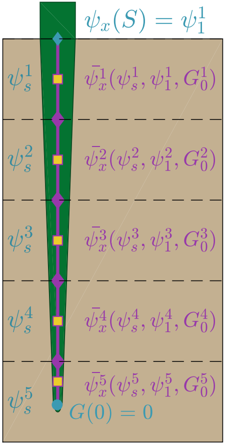
Governing Equation



$$\frac{d^2\psi_x}{ds^2} = \frac{k_r}{K_x}(\psi_x - \psi_s)$$

- ψ_x (Pa) 'xylem' water potential
- s (m) length along root (0 at tip, S at base)
- ψ_s '(Pa) soil' water potential
- k_r ($m^2\,s^{-1}\,Pa^{-1}$) soil-root 'radial' conductance in cross-seciton
- K_x ($m^4\,s^{-1}\,Pa^{-1}$) xylem 'axial' conductance in cross-section

Segment mean water potential



Can find solutions that yield mean water potential in root segments ($\bar{\psi}_x^i$):

$$\bar{\psi}_x^i = \frac{\int_0^{S^i} \psi_x(s) ds}{S^i}$$

Layer water uptake from Darcian expression using $\bar{\psi}_x$:

$$Q_R^i = -k_r^i S^i (\bar{\psi}_x^i - \psi_s^i)$$

Possible equations for $\bar{\psi}_x^i$

$$\bar{\psi}_x^i = \psi_s^i + (G_1^i - G_0^i)/\beta_2^i \tag{1a}$$

$$\bar{\psi}_x^i = c_1^i \psi_0^i + (1 - c_1^i)\psi_s^i - c_2^i G_0^i \tag{1b}$$

$$\bar{\psi}_x^i = c_3^i \psi_0^i + (1 - c_3^i)\psi_s^i - c_4^i G_1^i \tag{1c}$$

$$\bar{\psi}_x^i = c_1^i \psi_1^i + (1 - c_1^i)\psi_s^i + c_2^i G_1^i \tag{1d}$$

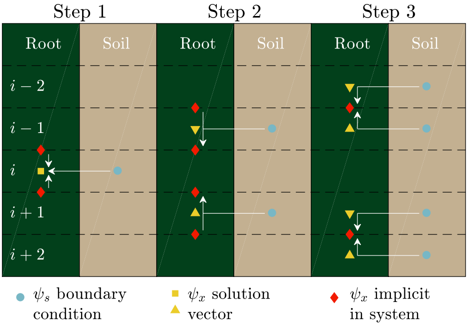
$$\bar{\psi}_x^i = c_3^i \psi_1^i + (1 - c_3^i)\psi_s^i + c_4^i G_0^i \tag{1e}$$

$$\bar{\psi}_x^i = c_5^i (\psi_1^i + \psi_0^i) + (1 - 2c_5^i)\psi_s^i \tag{1f}$$

Table: List of variables

Symbol	Expression
α^i	$\sqrt{k_r^i / K_x^i}$
β^i	$\alpha^i S^i$
β_2^i	$(\alpha^i)^2 S^i$
c_1^i	$\sinh(\beta^i)/\beta^i$
c_2^i	$(1 - \cosh(\beta^i))/\beta_2^i$
c_3^i	$\tanh(\beta^i)/\beta^i$
c_4^i	$(\text{sech}(\beta^i) - 1)/\beta_2^i$
c_5^i	$\tanh(\beta^i/2)/\beta^i$

Analytical equation for single root $\bar{\psi}_x^i$

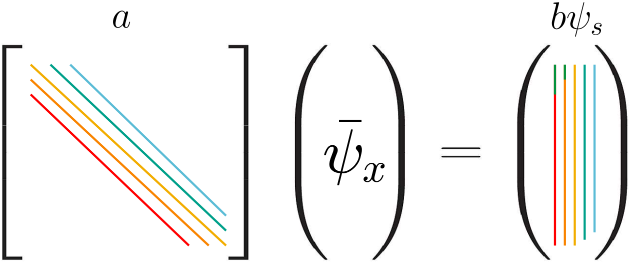


Derivation for layer i:

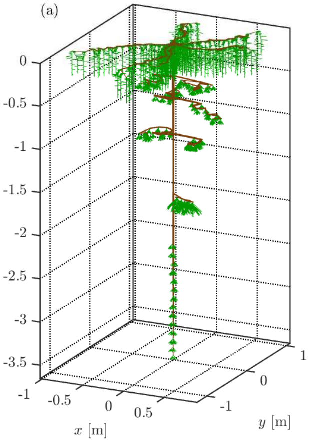
1. start from eq. 1f,
2. substitute for $\psi_{0/1}^i$ from eq. 1f,
3. substitute for $\psi_{0/1}^{i\pm1}$ from combined eqs. 1b and 1d.

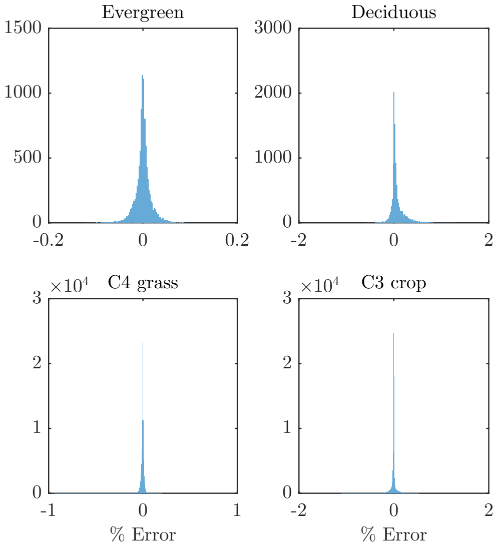
$$\bar{\psi}_x^i = \sum_{j=i-2}^{i+2} a^j \bar{\psi}_x^j + \sum_{k=i-2}^{i+2} b^k \psi_s^k, (j \neq i)$$

Pentadiagonal system ('RSA Stencil') for $\bar{\psi}_x$



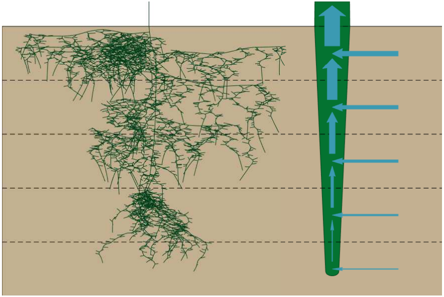
RSA Stencil: accurate $\bar{\psi}_x$ & Q_R predictions





Bouda and Saiers (2017) *Advances in Water Resources*

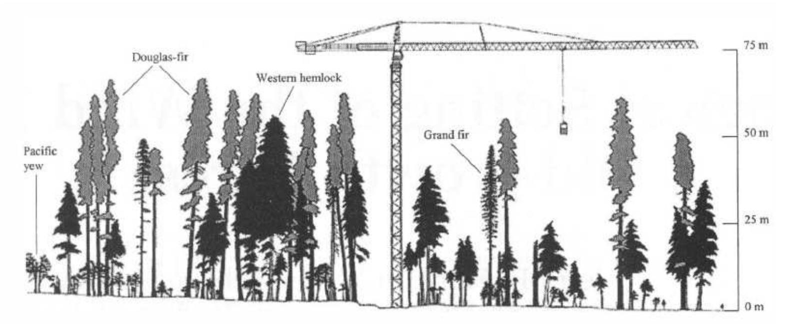
Big Root Model:



Use single-root equations to represent non-trivial RSA

- Lowers model skill as compared to 'unconstrained' RSA-Stencil fit.
- Provides a clear physical basis:
  - helps inverse model convergence
  - more easily interpretable parameters ($k_r S$, K_x/S)
  - greater consistency in prediction beyond calibration data

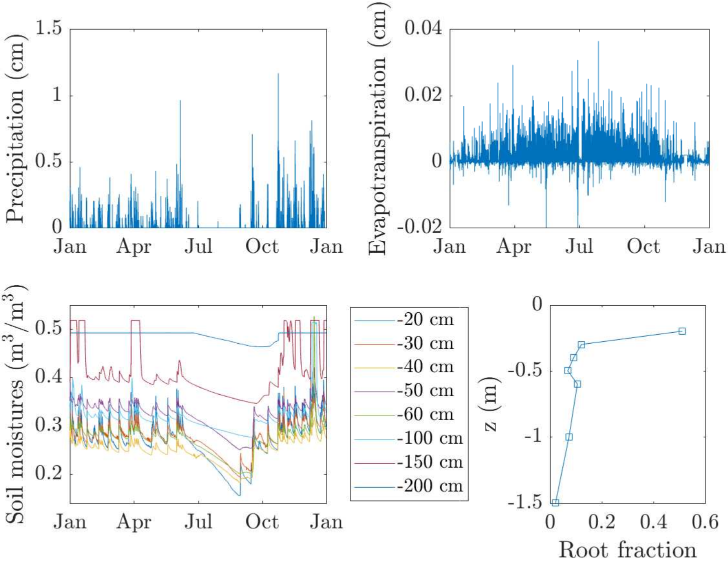
Big Root case study: prediction of field data from Wind River Crane



Shaw et al. (2004) *Ecosystems*

(Extensively studied old growth douglas-fir / hemlock site)

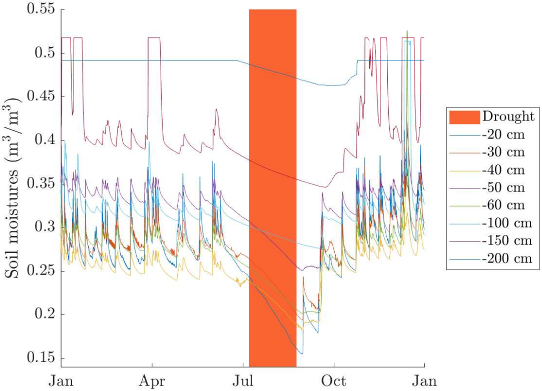
2010 Data Overview



Sonia Wharton AmeriFlux US-Wrc Wind River Crane Site, doi:10.17190/AMF/1246114

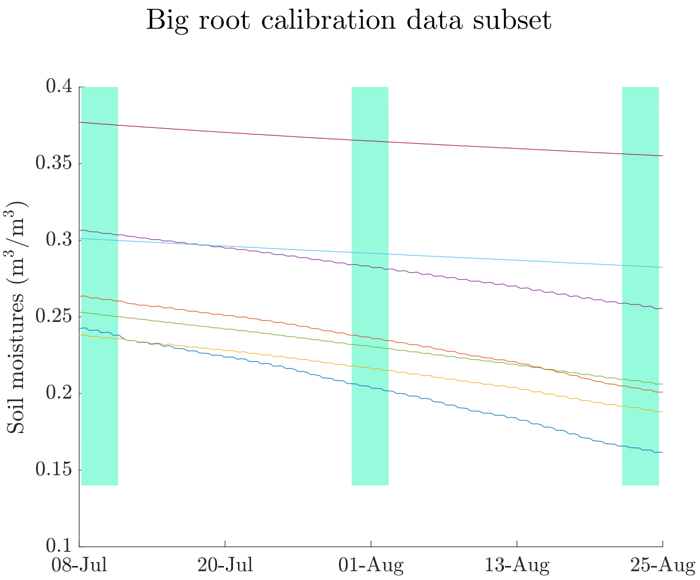
Warren et al. (2005) *Agr. & For. Meteorol.*

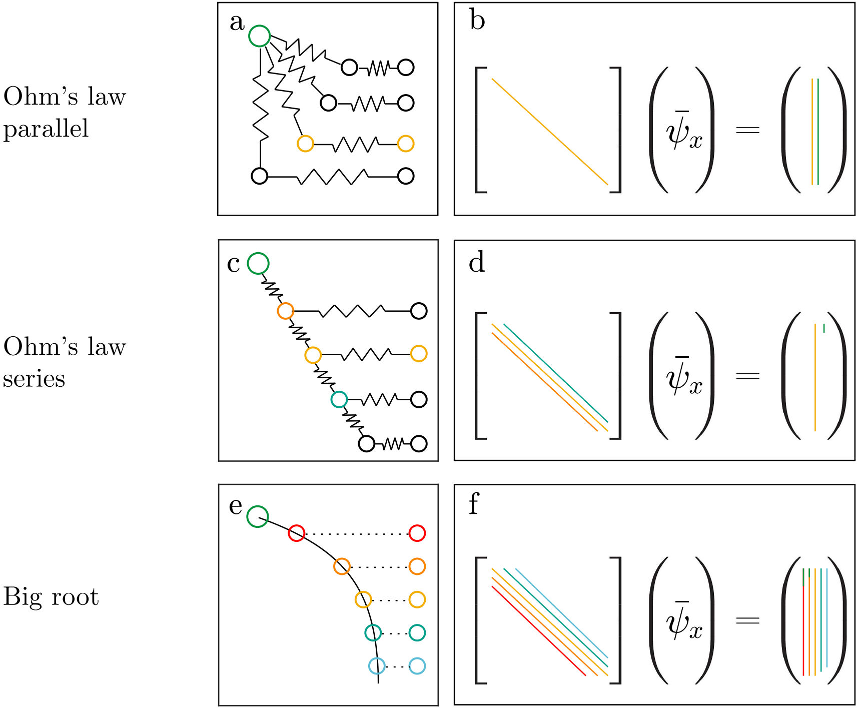
2010 Drought



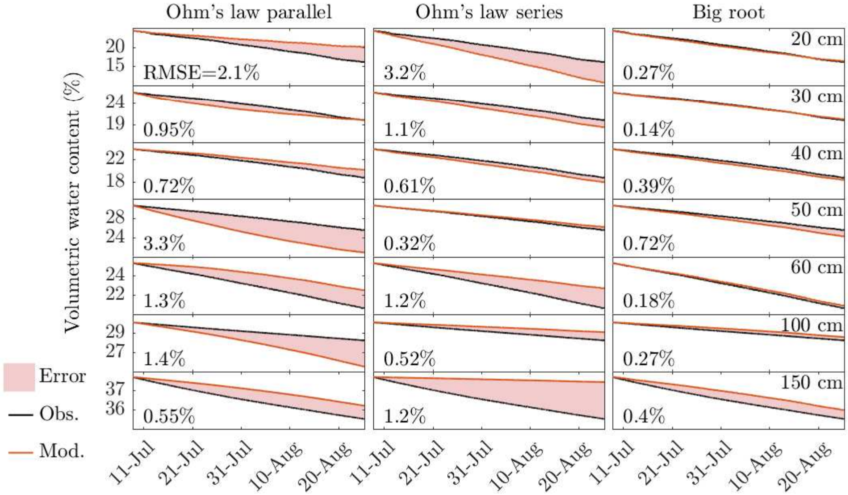
Assumptions:

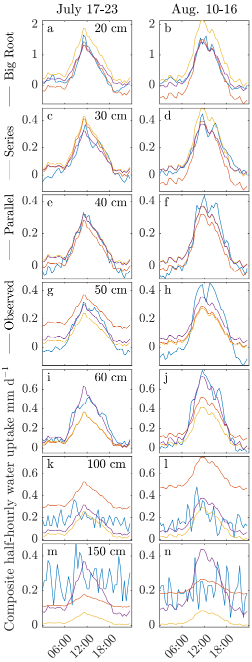
- all water movement is through plant
- all ET is transpiration
- published ψ_s–θ relations
- published basal area

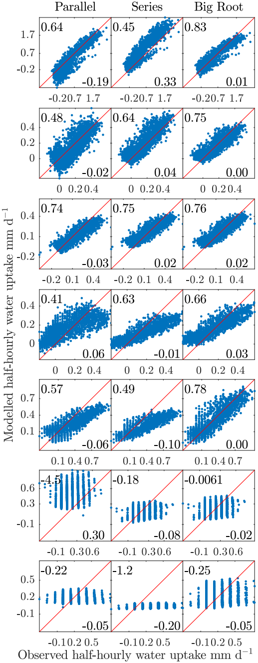

Big root calibration data subset
Soil moistures (m^3/m^3)
0.4
0.35
0.3
0.25
0.2
0.15
0.1
08-Jul
20-Jul
01-Aug
13-Aug
25-Aug


Ohm's law parallel
Ohm's law series
Big root
a
b
c
d
e
f
$\bar{\psi}_x$
=

Soil Moisture Predictions



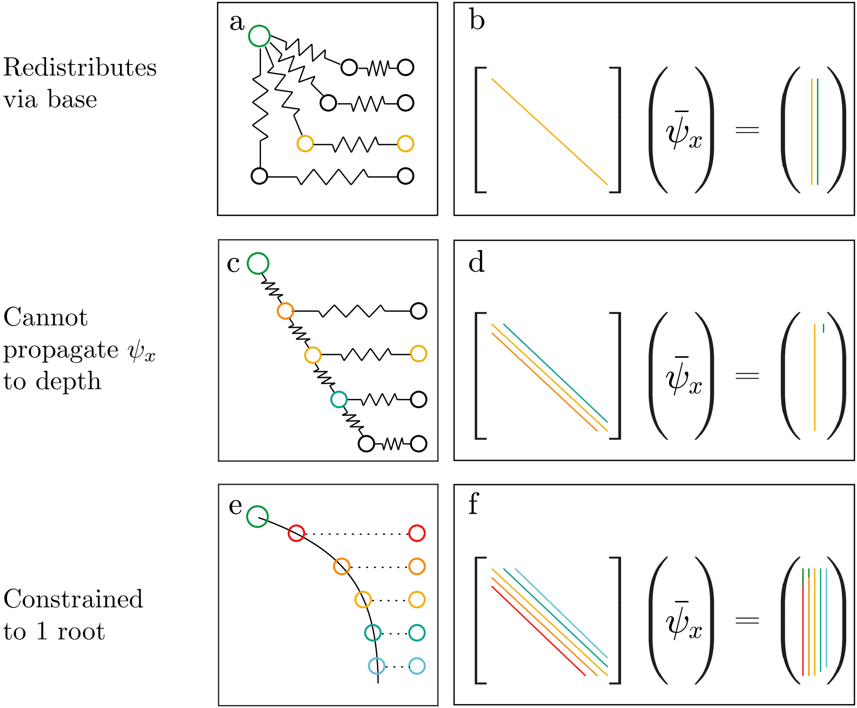




Water Uptake Predictions

- Ohm's law parallel:
  overly redistributes
  dry bias in wet
  wet bias in dry
- Ohm's law series:
  shallow dry bias
  deep wet bias
- Big root:
  more flexible RSA
  no systematic bias

bioRXiv
https://doi.org/10.1101/559237



Redistributes via base
a
b
$\bar{\psi}_x$
=
Cannot propagate ψ_x to depth
c
d
$\bar{\psi}_x$
=
Constrained to 1 root
e
f
$\bar{\psi}_x$
=

Conclusions

- RSA imposed by Ohm's law analogue models leads to biases.
- Big root model includes interactions between layers, is more flexible at represening RSA.
- Systematic biases eliminated; errors due to assumption of single effective root; but this also puts predictions on a physical basis, increasing robustness.
- Next steps (in cooperation with CNRM, Météo France):
  - implement big root model in SurfEx,
  - show impact of root scheme on soil moistures and surface fluxes at scale.

Acknowledgements

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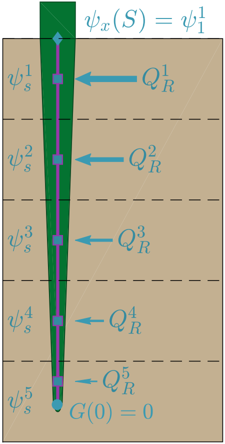
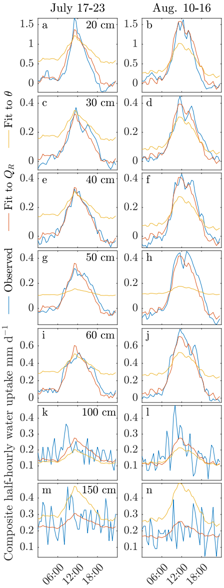
Model inversion

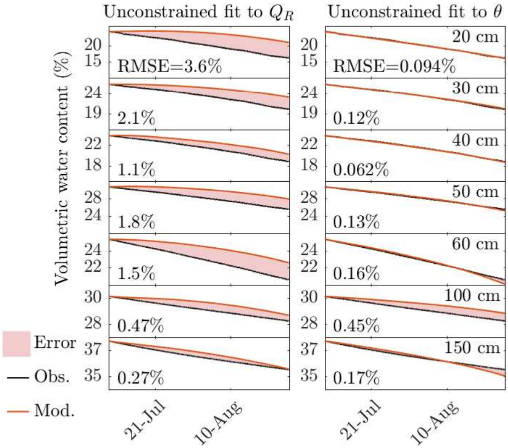


- Cannot uniquely resolve k_r^i, K_x^i, S^i without further constraint.
- Inversion yields $k_r^i S^i$ and K_x^i / S^i or just stencil a and b, predicting flows (Q_R):

Data	Boundary conditions	Inversion yields:
$\bar{\psi}_x$, Q_R	ψ_1^1, $G_0^n = 0$	$k_r S$, K_x/S
$\bar{\psi}_x$, Q_R	G_1^1, $G_0^n = 0$	$k_r S$, K_x/S
Q_R	ψ_1^1, $G_0^n = 0$	$k_r S$, K_x/S *
Q_R	G_1^1, $G_0^n = 0$	stencil a, b

* except layer n


July 17-23
Aug. 10-16
a
20 cm
b
c
30 cm
d
e
40 cm
f
g
50 cm
h
i
60 cm
j
k
100 cm
l
m
150 cm
n
Fit to θ
Fit to Q_R
Observed
Composite half-hourly water uptake mm d^{-1}
06:00
12:00
18:00


Unconstrained fit to Q_R
Unconstrained fit to θ
Volumetric water content (%)
RMSE=3.6%
2.1%
1.1%
1.8%
1.5%
0.47%
0.27%
RMSE=0.094%
0.12%
0.062%
0.13%
0.16%
0.45%
0.17%
20 cm
30 cm
40 cm
50 cm
60 cm
100 cm
150 cm
Error
Obs.
Mod.
21-Jul
10-Aug