IMPLICIT COUPLING BETWEEN SURFEX AND AN ATMOSPHERIC MODEL.

EQUATIONS AND CALCULATIONS IN THE ROUTINE
e_budget.F90

Internal report

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Abstract

The aim of this document is to describe the set of reference of the equation for the implicit coupling between SURFEX and the atmospheric model above and the calculations done in the routine e_budget to solve the surface temperature.

Contents

Chapter 1

Reference set of equations

1.1 Definition of the coefficients As and Bs

We follow here the set of equations proposed by Best et al. (2004). The surface variable evolution are done in the resolution of the vertical diffusion of the atmosphere. If X is an atmospheric variable (u, v, θ, q), at the lower atmospheric level we use the equation :

$$X_N^+ = A_{X,N}^- \times F_{X,S}^+ + B_{X,N}^- \tag{1.1}$$

We will use in this document the subscript N for the lower atmospheric level, the subscript S for the surface level. The sign - means that this value is calculated at the time t. The sign + means that the value is calculated at the time t+Δt.

- X_N^+ is the atmospheric variable at time t+Δt.
- $A_{X,N}^-$ and $B_{X,N}^-$ are the coefficient used for the vertical diffusion resolution. They are calculated by the atmospheric model. $A_{X,N}^-$ is a function of $A_{X,N-1}^-$ and the diffusion coefficients in the atmosphere between the levels N-1 and N. $B_{X,N}^-$ is a function of the diffusion coefficients in the atmosphere between the levels N-1 and N and X_{N-1}^-.
- $F_{X,S}^+$ is the surface flux of the variable X.

In our case, the variables used for the interface are : u,v, θ, q, following the variables used in the externalised surface code developped by CNRM/GMGEC (A. L. Gibelin, I. Zuurendonk). Hence Eq. 1.1 becomes :

$$u_N^+ = A_{u,N}^- \times F_{u,S}^+ + B_{u,N}^- \tag{1.2}$$

$$v_N^+ = A_{v,N}^- \times F_{v,S}^+ + B_{v,N}^- \tag{1.3}$$

$$\theta_N^+ = A_{\theta,N}^- \times F_{\theta,S}^+ + B_{\theta,N}^- \tag{1.4}$$

$$q_N^+ = A_{q,N}^- \times F_{q,S}^+ + B_{q,N}^- \tag{1.5}$$

1.2 definition of the fluxes

In the surface scheme, we need the definition of fluxes to calculate the evolution of the surface temperature, and then to calculate the fluxes at t+Δt, which will be used in the atmospheric model to solve the vertical profile of u,v, θ, q.

This fluxes are defined as follows :

$$
\begin{aligned}
F^+_{u,S} &= \rho^- C^-_D V^-_N u^+_N && (1.6)\\
F^+_{v,S} &= \rho^- C^-_D V^-_N v^+_N && (1.7)\\
F^+_{\theta,S} &= \rho^- C^-_H V^-_N (\theta^+_N - \theta^+_S) && (1.8)\\
F^+_{q,S} &= \rho^- C^-_H V^-_N (q^+_N - q^+_S) && (1.9)
\end{aligned}
$$

- Surface or atmospheric models use the latent heat fluxes (LE) or the sensible heat fluxes (H). We can easily obtain these fluxes by : $F^+_{q,S} = \frac{-LE}{Cp_S}$ and $F^+_{\theta,S} = \frac{-H}{Cp_S}$. However, it must be noted that we need coherent values of L and Cp_S for the transformation (e.g. from A and B coefficients for $F^+_{q,S}$) and the back substitution (from LE to $F^+_{q,S}$).

- the relation between θ and temperature is the classical definition : $\theta = T(\frac{P_0}{P})^\kappa$, where P_0 is a reference pressure. For convenience, we define $\beta = (\frac{P_0}{P})^\kappa$, and the Exner function π, $\pi = \frac{1}{\beta}$. Hence we have $\theta = \beta T = \frac{T}{\pi}$.

Chapter 2

Evolution of the surface temperature

In this chapter (and in the routine e_budget.F90), the aim is to calculate the new surface temperature T_S^+. For a force restore scheme, T_S^+ is given by :

$$\frac{C_s}{\Delta t}(T_S^+ - T_S^-) = R_n^+ - H^+ - LE^+ - G_1^+ \tag{2.1}$$

We will detail in the following the calculations of the fluxes. The main objective is to eliminate the air variables T_N and q_N in the equation using the As and Bs coefficients.

2.1 New wind speed

The calculation of the wind speed at the time $t + \Delta t$ is straightforward, because of the no-slip condition at the surface (V_S=0). By using Eq. 1.1 and 1.6, we can deduce :

$$u_N^+ = \rho^- C_D^- V_N^- u_N^+ \times A_{u,N}^- + B_{u,N}^- \tag{2.2}$$

Hence, we have for u (and in an similar way for v):

$$\begin{aligned} u_N^+ &= \frac{B_{u,N}^-}{1 - \rho^- C_D^- V_N^- \times A_{u,N}^-} && (2.3) \\ v_N^+ &= \frac{B_{v,N}^-}{1 - \rho^- C_D^- V_N^- \times A_{v,N}^-} && (2.4) \end{aligned}$$

It is then easy to calculate $F_{u,S}^+$, $F_{v,S}^+$ and $u_*^2 = \rho^- C_D^- V_N^- \sqrt{u_N^{+2} + v_N^{+2}}$

SURFEX case

In the present version of SURFEX, we have only one set of coefficient A and B for u and v. If the drag coefficient are identical for u and v, it it possible to use these coefficients. By definition, $A_{u,N} = A_{v,N}$ (both depend only on difusion terms in the atmosphere). We can define an equivalent coefficient $B^-_{V,N} = \sqrt{(B^-_{u,N})^2 + (B^-_{v,N})^2}$.

We finally obtain : $u_*^2 = \rho^- C_D^- V_N^- \times \dfrac{B^-_{V,N}}{1 - \rho^- C_D^- V_N^- \times A^-_{V,N}}$

2.2 Contribution of turbulent fluxes : elimination of T_N^+

Using the expression of the fluxes we will express T_N^+ as a function of T_S^+. We start from Eq. 1.4 and 1.8 and we obtain :

$$\theta_N^+ = A^-_{\theta,N} \times \rho^- C_H^- V_N^- \theta_N^+ - A^-_{\theta,N} \times \rho^- C_H^- V_N^- \theta_S^+ + B^-_{\theta,N} \tag{2.5}$$

And finally, using T, as in the routine e_budget.F90 :

$$T_N^+ = \frac{B^-_{\theta,N} - A^-_{\theta,N} \times \rho^- C_H^- V_N^- \beta_S T_S^+}{\beta_N (A^-_{\theta,N} \times \rho^- C_H^- V_N^-)} \tag{2.6}$$

We have here an expression of T_N^+ as a function of T_S^+. For convenience we define two new coefficients A and B for the surface. We write Eq. 2.6 as follow :

$$T_N^+ = A^-_{\theta,S} \times T_S^+ + B^-_{\theta,S} \tag{2.7}$$

with :

$$C^-_{\theta,S} = \beta_N (1 - A^-_{\theta,N} \times \rho^- C_H^- V_N^-) \tag{2.8}$$

$$A^-_{\theta,S} = \frac{-A^-_{\theta,N} \times \rho^- C_H^- V_N^- \beta_S^-}{C^-_{\theta,S}} \tag{2.9}$$

$$B^-_{\theta,S} = \frac{B^-_{\theta,N}}{C^-_{\theta,S}} \tag{2.10}$$

2.3 Contribution of the latent heat flux : elimination of q_N^+

In ISBA, the latent heat flux is defined by the following expression :

$$\begin{aligned} LE &= veg \times (1 - P_{snowV}) \times \rho^- L_v V_N^- C_H^- h_v^- \left[q_{sat}(T_S^+) - q_N^+\right] \\ &+ (1 - veg) \times (1 - P_{snowG}) \times (1 - P_{frozen}) \times \rho^- L_v V_N^- C_H^- \left[hu_g^- \times q_{sat}(T_S^+) - q_N^+\right] \\ &+ (1 - veg) \times (1 - P_{snowG}) \times P_{frozen} \times \rho^- L_s V_N^- C_H^- \left[hu_i^- \times q_{sat}(T_S^+) - q_N^+\right] \\ &+ P_{snow} \times \rho^- L_s V_N^- C_H^- \left[q_{sat}(T_S^+) - q_N^+\right] \end{aligned} \tag{2.11}$$

LE is the sum of four terms : transpiration over vegetation non covered by snow, evaporation over non frozen bare ground, sublimation over frozen bare ground, sublimation over snow.

For simplifications and to have an expression as close as possible to the classical one we introduce L_{avg}, which is an area average of the different latent heat in the surface (see 2.13). LE is then given by :

$$LE = \rho^- L_{avg} V_N^- C_H^- \left[h_S^- \times q_{sat}(T_S^+) - h_N^- \times q_N^+ \right] \tag{2.12}$$

Where :

$$\begin{aligned} L_{avg} &= veg \times (1 - P_{snowV}) \times L_v \\ &+ (1 - veg) \times (1 - P_{snowG}) \times (1 - P_{frozen}) \times L_v \\ &+ (1 - veg) \times (1 - P_{snowG}) \times P_{frozen} \times L_s \\ &+ P_{snow} \times L_s \end{aligned} \tag{2.13}$$

$$\begin{aligned} h_S^- &= \frac{L_v}{L_{avg}} \times veg \times (1 - P_{snowV}) \times h_v^- \\ &+ \frac{L_v}{L_{avg}} \times (1 - veg) \times (1 - P_{snowG}) \times (1 - P_{frozen}) \times hu_g \\ &+ \frac{L_v}{L_{avg}} \times (1 - veg) \times (1 - P_{snowG}) \times P_{frozen} \times hu_i^- \\ &+ \frac{L_v}{L_{avg}} \times P_{snow} \end{aligned} \tag{2.14}$$

$$\begin{aligned} h_N^- &= \frac{L_v}{L_{avg}} \times veg \times (1 - P_{snowV}) \times h_v^- \\ &+ \frac{L_v}{L_{avg}} \times (1 - veg) \times (1 - P_{snowG}) \times (1 - P_{frozen}) \\ &+ \frac{L_v}{L_{avg}} \times (1 - veg) \times (1 - P_{snowG}) \times P_{frozen} \\ &+ \frac{L_v}{L_{avg}} \times P_{snow} \end{aligned} \tag{2.15}$$

The next step is the linearization of LE with respect to temperature. Then we use Eq. 1.5 (which is a relation between q_N^+ and $F_{q,S}^+$) and replace $F_{q,S}^+$ by its expression as a function of LE. To derive LE from $F_{q,S}^+$ we use L_{avg}. We obtain the following equations :

$$LE = \rho^- L_{avg} V_N^- C_H^- \left[h_S^- \left(q_{sat}(T_S^-) - \frac{\partial q_{sat}}{\partial t} T_S^- \right) + \frac{\partial q_{sat}}{\partial t} T_S^+ - h_N^- q_N^+ \right] \tag{2.16}$$

$$q_N^+ = -A_{q,N}^- \times \frac{LE}{L_{avg}} + B_{q,N}^- \tag{2.17}$$

We can then subtitute the expression of LE in 2.17 by the expression given in 2.16 to obtain q_N^+ as a function of T_S^+. We define two new coefficients A and B for the surface, as in the previous section.

$$q_N^+ = A_{q,S}^- \times T_S^+ + B_{q,S}^- \tag{2.18}$$

$$C = 1 - A_{q,N}^- \times \rho^- V_N^- C_H^- h_N^- \tag{2.19}$$

$$A_{q,S} = \frac{-A_{q,N} \rho^- V_N^- C_H^- \frac{\partial q_{sat}}{\partial T} h_S^-}{C} \tag{2.20}$$

$$B_{q,S} = \frac{B_{q,N}^- - A_{q,N}^- \times \rho^- V_N^- C_H^- h_S^- (q_{sat}(T_S^-) - \frac{\partial q_{sat}}{\partial T}(T_S^-))}{C} \tag{2.21}$$

2.4 Effective resolution of the surface temperature

Equation 2.1 gives the evolution of the surface temperature. We recall it below :

$$\frac{C_s}{\Delta t}(T_S^+ - T_S^-) = R_n^+ - H^+ - LE^+ - G_1^+ \tag{2.22}$$

First, we express the different terms as a function of T_N^+, T_g^+ and q_N^+ by using the expression of the fuxes given in Eq. 1.8, Eq. 1.9, the expression of the ground heat flux and the linearisation of the infrared flux :

$$\begin{aligned}
R_n^+ &= R_g(1-\alpha) + \epsilon R_{atm} + 3\epsilon\sigma {T_S^-}^4 - 4\epsilon\sigma {T_S^-}^3 T_S^+ \\
H^+ &= \rho^- C_p^- C_H^- V_N^- \left(\beta_S T_S^+ - \beta_N T_N^+\right) \\
LE &= \rho^- C_H^- V_N^- L_{avg} \left[h_S^- \left(q_{sat}(T_S^-) - \frac{\partial q_{sat}}{\partial T} T_S^- + \frac{\partial q_{sat}}{\partial T} T_S^+\right) - h_N^- q_N^+\right] \\
G_1^+ &= \frac{2\pi C_S}{\tau}\left(T_S^+ - T_2^-\right)
\end{aligned}$$

Then we substitute in 2.1 the expression of the fluxes. We obtain then an expression of T_S^+ as a function of T_N^+, T_g^+ and q_N^+ :

$$T_S^+ = Z_0 + Z_T T_N^+ + Z_{Tg}^+ T_2^- + Z_q q_N^+ \tag{2.23}$$

with :

$$\begin{aligned}
Z_c &= \frac{C_S}{\Delta t} + 4\epsilon\sigma {T_S^-}^3 + \rho^- C_p^- C_H^- V_N^- \beta_S + \rho^- C_H^- V_N^- L_{avg} h_S^- \frac{\partial q_{sat}}{\partial T} + \frac{2\pi C_S}{\tau} \\
Z_0 &= \frac{\frac{C_S}{\Delta t} T_S^- + R_g(1-\alpha) + \epsilon\left(R_{atm} + 3\epsilon\sigma {T_S^-}^4\right) - \rho^- C_H^- V_N^- L_{avg} h_S^- \left(q_{sat}(T_S^-) - \frac{\partial q_{sat}}{\partial T} T_S^-\right) + \frac{2\pi C_S}{\tau}}{Z_c} \\
Z_T &= \frac{\rho^- C_p^- C_H^- V_N^- \beta_N}{Z_c} \\
Z_q &= \frac{\rho^- C_H^- V_N^- L_{avg} h_N^-}{Z_c} \\
Z_{Tg} &= \frac{\frac{2\pi}{\tau}}{Z_c}
\end{aligned} \tag{2.24}$$

After that, we use the coefficients As and Bs for the surface obtained in 2.7 and 2.18 to replaceT_N^+, and q_N^+ in 2.23. We recall here 2.7 and 2.18 :

$$\begin{cases} T_N^+ = A_{\theta,S}^- \times T_S^+ + B_{\theta,S}^- \\ q_N^+ = A_{q,S}^- \times T_S^+ + B_{q,S}^- \end{cases}$$

And finally deduce the expression of T_S^+ :

$$T_S^+ = \frac{Z_0 + B_{\theta,S}^- Z_T + Z_{Tg} T_2^- + B_{q,S}^- Z_q}{1 - A_{\theta,S}^- Z_T - A_{q,S}^- Z_q} \tag{2.25}$$

Correspondence with the code in e_budget.F90

In the code, we have slightly different coefficients, which can be easily derived from equation 2.25. The coefficient ZA, ZB and ZC are introduced. We also introduce $C_T = \frac{1}{C_s}$. Eq. 2.25 is written as :

$$ZA \times T_S^+ = ZB \times T_S^- + ZC$$

With :

$$\begin{aligned} ZA &= \frac{1}{\Delta t} + C_T \left[4\epsilon\sigma {T_S^-}^3 + \rho^- C_H^- V_N^- L_{avg} h_S^- \frac{\partial q_{sat}}{\partial T} - A_{q,S} \times \rho^- C_H^- V_N^- L_{avg} h_N \right. \\ &\quad \left. + \rho^- C_p^- C_H^- V_N^- \left(\beta_S - A_{\theta,S}^- \times \beta_N \right) \right] + \frac{2\pi}{\tau} && (2.26) \\ ZB &= \frac{1}{\Delta t} + C_T \left[3\epsilon\sigma {T_S^-}^4 + \rho^- C_H^- V_N^- L_{avg} h_S^- \frac{\partial q_{sat}}{\partial T} \right] && (2.27) \\ ZC &= \frac{2\pi}{\tau} T_2^- + C_T \left[B_{\theta,S}^- \times \rho^- C_p^- C_H^- V_N^- \beta_N + R_g(1-\alpha) + \epsilon R_{atm} \right. \\ &\quad \left. - \rho^- C_H^- V_N^- L_{avg} h_S^- q_{sat}(T_S^-) + B_{q,S} \times \rho^- C_H^- V_N^- L_{avg} h_N \right] && (2.28) \end{aligned}$$