COMPARISON BETWEEN TURBULENT EXCHANGE COEFFICIENTS C_H and C_D IN ARPEGE AND SURFEX

Internal report

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Abstract

The aim of this document is to compare the formulations of the turbulent exchange coefficients C_H and C_D both in ARPEGE and SURFEX in the different stability cases. For that purpose, the formulations are taken from source code CY25T1_op3 of 1D ARPEGE model containing split of acdifus in order to plug surface energy budget at the level of vertical diffusion computations (work of GMGEC) and version 0.7 of SURFEX. The examined subroutines are achmtls.F90 for ARPEGE and drag.F90, surface_aero_cond.F90 and surface_cd.F90 for SURFEX. The organization of the document will separate the equations from ARPEGE and SURFEX in both stable and unstable cases for the two turbulent exchange coefficients above mentioned.
Usually, developers work consists in translating equations of a given parameterization into programming language. The current exercise does the reverse since it tries to identify the basic equations used to compute the turbulent exchange coefficients from fortran source code.

Contents

Chapter 1

C_D: turbulent exchange coefficient at surface for momentum

1.1 ARPEGE formulation (achmtls.F90)

$$PCD = \frac{(ZLOI + PSTAB \times (ZLOS - ZLOI)) \times PCDN}{ZU} \tag{1.1}$$

ZU represent the wind speed, $PCDN$ is the neutral surface exchange coefficient for wind, $ZLOS$ and $ZLOI$ are expressed as function of atmospheric stability (Richardson number) and wind speed, $PSTAB$ is an indicator of the stability: $PSTAB = 1$ means that the case is stable and $PSTAB = 0$ otherwise.

1.1.1 stable case

In the stable case, PSTAB=1 and equation 1.1 becomes:

$$PCD = \frac{ZLOS \times PCDN}{ZU} \tag{1.2}$$

Following notations are introduced:

$$ZCIS = u^2 + v^2 \tag{1.3}$$

$$ZU = \sqrt{ZCIS} \tag{1.4}$$

$$ZDS = \sqrt{ZCIS + 5|ZSTA|} \tag{1.5}$$

$$ZLOS = \frac{ZCIS \times ZDS}{ZU \times ZDS + Z2B \times |ZSTA|} \tag{1.6}$$

$ZSTA$ characterize the atmosphere stability (expression will be given later).
At this stage, it appears that the possibility to have a critical Richardson number does not exist in SURFEX. To go on with the comparison it's assumed that Richardson critical numbers are set to zero. The consistency between ARPEGE and SURFEX, as far as Richardson critical numbers are concerned will be ensured by introducing these refinements in SURFEX code.

$$USURIC = 0. \tag{1.7}$$
$$ZIXP = 1. \tag{1.8}$$
$$USURID = 0. \tag{1.9}$$

With these assumptions, expression 1.6 becomes:

$$ZLOS = \frac{ZCIS \times \sqrt{ZCIS + 5|ZSTA|}}{ZU\sqrt{ZCIS + 5|ZSTA|} + Z2B \times |ZSTA|} \tag{1.10}$$

The expression of Richardson number is given by:

$$R_i = \frac{ZSTA}{ZCIS}$$

When introducing this variable into the expression of $ZLOS$ one obtain the analytical relationship :

$$\frac{ZLOS}{ZU} = \frac{1}{1 + \frac{10R_i}{\sqrt{1+5R_i}}} \tag{1.11}$$

Neutral coefficient for momentum is given by:

$$PCDN = \frac{\kappa^2}{(\ln(1 + \frac{Z}{Z_0}))^2} \tag{1.12}$$

Where $Z = \frac{\Delta\phi}{g}$ is the height ($\Delta\phi$ is the thickness between full and half-level close to the lowest atmospheric level and g is the gravity) and Z_0 is the current roughness length that takes into account vegetation and orography. Equation 1.2 that gives expression of exchange turbulent coefficient for momentum may be expressed as follows:

$$PCD = \frac{PCDN}{1 + \frac{10R_i}{\sqrt{1+5R_i}}} \tag{1.13}$$

1.1.2 unstable case

In the unstable case, $PSATB = 0$ and equation 2.1 becomes:

$$PCD = \frac{ZLOI \times PCDN}{ZU} \tag{1.14}$$

$$ZLOI = ZU - Z2B \times ZSTA \times ZDID \tag{1.15}$$
$$ZDID = \frac{1}{ZU + ZCH \times Z3BC \times PCDN\sqrt{|ZSTA| \times ZRZD}} \tag{1.16}$$

In these equations $Z2B = 10.$ and $Z3BC = 5. \times Z3B$ with $Z3B = 15.$ and

$$ZRZD = 1 + \frac{Z}{Z_0} \tag{1.17}$$

$$ZCD = ZCD0 \times (1 + \frac{Z}{Z_{0MR}})^{ZPD} \tag{1.18}$$

Where Z_{0MR} is the roughness length without orography. By replacing $ZRZD$, ZCD, $Z2B$ and $Z3BC$ in equation 1.14, we can express $ZLOI$ as follows:

$$ZLOI = ZU - \frac{7.5ZSTA}{ZU + 5ZCD0(1 + \frac{Z}{Z_{0MR}})^{ZPD} \times 10PCDN\sqrt{|ZSTA|(1 + \frac{Z}{Z_0})}} \tag{1.19}$$

Equation 1.14 can be rewritten as a function of Richardson number R_i, $|\overrightarrow{v}|$ the wind modulus, η_A and two other functions ψ_A and ϕ_A defined below:

$$\eta_A = (\frac{\kappa}{\ln(1 + \frac{Z}{Z_0})})^2 \tag{1.20}$$

$$\phi_A = \overrightarrow{A_\phi} \cdot \overrightarrow{X_{Z_0MR}} \tag{1.21}$$

$$\psi_A = Y_Z^{\overrightarrow{A_\psi} \cdot \overrightarrow{X_{Z_0MR}}} \sqrt{1 + \frac{Z}{Z_0}} \tag{1.22}$$

$$PCD = (|\overrightarrow{v}| - \frac{10R_i|\overrightarrow{v}|^2}{|\overrightarrow{v}| + 10\eta_A\psi_A\phi_A\sqrt{|R_i||\overrightarrow{v}|^2}}) \times \frac{\eta_A}{|\overrightarrow{v}|} \tag{1.23}$$

Eliminating $|\overrightarrow{v}|$ in the expression of PCH gives finally:

$$PCD = (1 - \frac{10R_i}{1 + 10\eta_A\psi_A\phi_A\sqrt{|R_i|}}) \times \eta_A \tag{1.24}$$

With the following notations:

$$\overrightarrow{X_{Z_0MR}} = \begin{pmatrix} 1 \\ \mu \\ \mu^2 \\ \mu^3 \end{pmatrix} \qquad \mu = \ln(\frac{Z_{0MR}}{Z_{0H}}) \qquad Y_Z = (1 + \frac{Z}{Z_{0MR}})$$

and vectors $\overrightarrow{A_\phi}$ and $\overrightarrow{A_\psi}$ are:

$$\overrightarrow{A_\phi} = \begin{pmatrix} 7.5 & 2.39 & -0.2858 & 0.01074 \end{pmatrix} \tag{1.25}$$

$$\overrightarrow{A_\psi} = \begin{pmatrix} 0. & -0.07028 & 0.01023 & -0.00067 \end{pmatrix} \tag{1.26}$$

1.2 SURFEX formulation (surface_cd.F90)

$$PCD = PCDN \times ZFM \tag{1.27}$$

ZFM is expressed differently according to stability and $PCDN$ is given by:

$$PCDN = \left(\frac{\kappa}{\ln(\frac{Z}{Z_0})}\right)^2 \tag{1.28}$$

Be carefull, meaning of $Z0$ differs from ARPEGE since it represents the vegetation roughness length without orography.

1.2.1 stable case

$$ZFM = \frac{1}{1 + \frac{10R_i}{\sqrt{1+5R_i}}} \tag{1.29}$$

PCD is then expressed as:

$$PCD = \frac{PCDN}{1 + \frac{10R_i}{\sqrt{1+5R_i}}} \tag{1.30}$$

1.2.2 unstable case

In this case, ZFM is given by:

$$ZFM = 1 - \frac{10R_i}{1 + ZCM\sqrt{-R_i}} \tag{1.31}$$

$$ZCM = 10. \times ZCMSTAR \times PCDN \times \left(\frac{Z}{Z_{0eff}}\right)^{ZPM} \tag{1.32}$$

Z_{0eff} is the effective roughness length that takes into account the effect of snow and the effect of orography (possibly subscale orography if option is selected). By replacing ZFM and $PCDN$ in equation 1.27, the expression of PCD becomes:

$$PCD = \left(1 - \frac{10R_i}{1 + 10\eta_S\psi_S\phi_S\sqrt{|R_i|}}\right) \times \eta_S \tag{1.33}$$

$$\eta_S = \left(\frac{\kappa}{\ln(\frac{Z}{Z_0})}\right)^2 \tag{1.34}$$

$$\phi_S = \overrightarrow{S_\phi} \cdot \overrightarrow{X_{Z_0}} \tag{1.35}$$

$$\psi_S = Y_Z^{\overrightarrow{S_\psi} \cdot \overrightarrow{X_{Z_0}}} \tag{1.36}$$

With the following notations:

$$\overrightarrow{X_{Z_0}} = \begin{pmatrix} 1 \\ \mu \\ \mu^2 \\ \mu^3 \end{pmatrix} \qquad \mu = \ln(\frac{Z_{0eff}}{Z_{0H}}) \qquad Y_Z = \frac{Z}{Z_{0eff}}$$

and vectors $\overrightarrow{S_\phi}$ and $\overrightarrow{S_\psi}$ are:

$$\overrightarrow{S_\phi} = \begin{pmatrix} 6.8741 & 2.6933 & -0.3601 & 0.0154 \end{pmatrix} \tag{1.37}$$

$$\overrightarrow{S_\psi} = \begin{pmatrix} 0.5233 & -0.0815 & 0.0135 & -0.0010 \end{pmatrix} \tag{1.38}$$

Chapter 2

C_H: turbulent exchange coefficient at surface for heat

2.1 ARPEGE formulation (achmtls.F90)

$$PCH = \frac{(ZLOI + PSTAB \times (ZLOS - ZLOI)) \times ZCDNH}{ZU} \tag{2.1}$$

ZU represent the wind speed, $ZCDNH$ is the neutral surface thermal exchange coefficient, $ZLOS$ and $ZLOI$ are expressed as function of atmospheric stability (Richardson number) and wind speed, $PSTAB$ is an indicator of the stability: $PSTAB = 1$ means that the case is stable and $PSTAB = 0$ otherwise.

2.1.1 stable case

In the stable case, $PSTAB = 1$ and equation 2.1 becomes:

$$PCH = \frac{ZLOS \times ZCDNH}{ZU} \tag{2.2}$$

Following notations are introduced:

$$ZCIS = u^2 + v^2 \tag{2.3}$$

$$ZU = \sqrt{ZCIS} \tag{2.4}$$

$$ZHS = \sqrt{ZCIS + 5|ZSTAH|} \tag{2.5}$$

$$ZSTAH = \frac{ZSTA}{(1 + ZIXP \times ZUSURIC \times \frac{ZSTA}{ZCIS})^{\frac{1}{ZIXP}}} \tag{2.6}$$

$$ZLOS = \frac{ZCIS^2}{ZU \times ZCIS + Z3B \times |ZSTAH| \times ZHS} \tag{2.7}$$

At this stage, it appears that the possibility to have a critical Richardson numbers does not exist in SURFEX. To go on with the comparison, it's assumed that Richardson critical numbers are set to zero. The consistency between ARPEGE and SURFEX, as far as Richardson critical numbers are concerned will be ensured by introducing these refinements in SURFEX code.

$$USURIC = 0. \tag{2.8}$$
$$USURID = 0. \tag{2.9}$$
$$ZIXP = 1. \tag{2.10}$$

With these assumptions,

$$ZSTAH = ZSTA \tag{2.11}$$
$$ZHS = \sqrt{ZCIS + 5|ZSTA|} \tag{2.12}$$

And expression 2.7 is simplified into:

$$ZLOS = \frac{ZCIS^2}{ZU \times ZCIS + Z3B \times |ZSTA| \times ZHS} \tag{2.13}$$

$\Leftrightarrow$

$$ZLOS = \frac{ZCIS^2}{ZU \times ZCIS + Z3B \times |ZSTA|\sqrt{ZCIS + 5|ZSTA|}} \tag{2.14}$$

The expression of Richardson number is given by:

$$R_i = \frac{ZSTA}{ZCIS}$$

When introducing this variable into the expression of $ZLOS$ one obtain the analytical relationship :

$$\frac{ZLOS}{ZU} = \frac{1}{1 + 15R_i\sqrt{1 + 5R_i}} \tag{2.15}$$

As established in previous chapter,

$$ZCDNH = \frac{\kappa^2}{\ln(1 + \frac{Z}{Z_{0H}})\ln(1 + \frac{Z}{Z_{0MR}})} \tag{2.16}$$

Equation 2.2 that gives expression of exchange turbulent coefficient for heat may be expressed as follows:

$$PCH = \frac{ZCDNH}{1 + 15R_i\sqrt{1 + 5R_i}} \tag{2.17}$$
$$ZCDNH = \frac{\kappa^2}{\ln(1 + \frac{Z}{Z_{0H}})\ln(1 + \frac{Z}{Z_{0MR}})} \tag{2.18}$$

2.1.2 unstable case

In the unstable case, $PSATB = 0$ and equation 2.1 becomes:

$$PCH = \frac{ZLOI \times ZCDNH}{ZU} \tag{2.19}$$

$$ZLOI = ZU - Z3B \times ZSTA \times ZDIH \tag{2.20}$$

$$ZDIH = \frac{1}{ZU + ZCH \times Z3BC \times ZCDNH\sqrt{|ZSTA| \times ZRZH}} \tag{2.21}$$

In these equations $Z3B = 15.$ and $Z3BC = 5 \times Z3B$ and

$$ZRZH = 1 + \frac{Z}{Z_{0H}} \tag{2.22}$$

$$ZCH = ZCH0 \times (1 + \frac{Z}{Z_{0H}})^{ZPH} \tag{2.23}$$

By replacing $ZRZH$, ZCH and $Z3BC$ in equation 2.21, we can express $ZLOI$ as follows:

$$ZLOI = ZU - \frac{15ZSTA}{ZU + 5ZCH0(1 + \frac{Z}{Z_{0H}})^{ZPH} \times 15ZCDNH\sqrt{|ZSTA| \times (1 + \frac{Z}{Z_{0H}})}} \tag{2.24}$$

$\Leftrightarrow$

$$ZLOI = ZU - \frac{15ZSTA}{ZU + 5ZCH0(1 + \frac{Z}{Z_{0H}})^{ZPH+\frac{1}{2}} \times 15ZCDNH\sqrt{|ZSTA|}} \tag{2.25}$$

Equation 2.18 gives the expression of ZCDNH the neutral surface thermal exchange coefficient noted η_A, thus one can write PCH as a function of Richardson number R_i, $|\overrightarrow{v}|$ the wind modulus, η_A and two other functions ψ_A and ϕ_A defined below:

$$
\begin{aligned}
\eta_A &= \frac{\kappa^2}{\ln(1+\frac{Z}{Z_{0H}})\ln(1+\frac{Z}{Z_{0MR}})} && (2.26)\\
\phi_A &= \overrightarrow{A_\phi}\cdot\overrightarrow{X_{Z_0}} && (2.27)\\
\psi_A &= Y_Z^{\overrightarrow{A_\psi}.\overrightarrow{X_{Z_0}}} && (2.28)
\end{aligned}
$$

$$
PCH = (|\overrightarrow{v}| - \frac{15R_i|\overrightarrow{v}|^2}{|\overrightarrow{v}| + 15\eta_A\psi_A\phi_A\sqrt{|R_i||\overrightarrow{v}|^2}}) \times \frac{\eta_A}{|\overrightarrow{v}|} \qquad (2.29)
$$

Eliminating $|\overrightarrow{v}|$ in the expression of PCH gives finally:

$$
PCH = (1 - \frac{15R_i}{1 + 15\eta_A\psi_A\phi_A\sqrt{|R_i|}}) \times \eta_A \qquad (2.30)
$$

With the following notations:

$$
\overrightarrow{X_{Z_0}} = \begin{pmatrix} 1 \\ \mu \\ \mu^2 \\ \mu^3 \end{pmatrix} \qquad \mu = \ln(\frac{Z_{0MR}}{Z_{0H}}) \qquad Y_Z = (1+\frac{Z}{Z_{0H}})
$$

and vectors $\overrightarrow{A_\phi}$ and $\overrightarrow{A_\psi}$ are:

$$
\begin{aligned}
\overrightarrow{A_\phi} &= \begin{pmatrix} 5. & 4.513 & 0.3401 & -0.0533 \end{pmatrix} && (2.31)\\
\overrightarrow{A_\psi} &= \begin{pmatrix} 0.5 & -0.09421 & 0.01463 & -0.00099 \end{pmatrix} && (2.32)
\end{aligned}
$$

2.2 SURFEX formulation (drag.F90, surface_aero_cond.F90)

$$
PCH = \frac{PAC}{PVMOD} \qquad (2.33)
$$

$PVMOD$ represents the wind speed and PAC is the aerodynamical conductance.

2.2.1 stable case

Aerodynamical conductance is given by:

$$
PAC = \frac{ZCDN \times PVMOD}{1 + \frac{15 \times ZSTA \times ZDI}{PVMOD^3}} \times \frac{\ln(\frac{Z}{Z_0})}{\ln(\frac{Z}{Z_{0H}})} \qquad (2.34)
$$

where

$$ZCDN = \frac{\kappa^2}{(\ln(\frac{Z}{Z_0}))^2} \quad (2.35)$$

$$ZDI = \sqrt{PVMOD^2 + 5ZSTA} \quad (2.36)$$

$$ZSTA = R_i \times PVMOD^2 \quad (2.37)$$

At this stage we can express PCH given in equation 2.33 as a function of R_i:

$$PCH = \frac{ZCDNH}{1 + 15R_i\sqrt{1 + 5R_i}} \quad (2.38)$$

$$ZCDNH = \frac{\kappa^2}{\ln(\frac{Z}{Z_{0H}})\ln(\frac{Z}{Z_0})} \quad (2.39)$$

2.2.2 unstable case

In this case, aerodynamical conductance is given by:

$$PAC = ZCDN \times (PVMOD - 15ZSTA \times ZDI) \quad (2.40)$$

where

$$ZDI = \frac{1}{PVMOD + 15 \times ZCHSTAR \times ZCDN(\frac{Z}{Z_{0H}})^{ZPH} \times ZFH\sqrt{-ZSTA}} \quad (2.41)$$

With the same approach as it was done for the Arpege case, we introduce the following quantities: $|\overrightarrow{v}|$ for wind speed (the equivalent of PVMOD), η_S for $ZCDNH$ (cf equation 2.39) and two other functions ψ_S and ϕ_S defined below. We can express PCH as follows:

$$\eta_S = \frac{\kappa^2}{\ln(\frac{Z}{Z_{0H}})\ln(\frac{Z}{Z_0})} \quad (2.42)$$

$$\phi_S = \overrightarrow{S_\phi} \cdot \overrightarrow{X_{Z_0}} \quad (2.43)$$

$$\psi_S = Y_Z^{\overrightarrow{S_\psi} \cdot \overrightarrow{X_{Z_0}}} \quad (2.44)$$

$$PCH = (|\overrightarrow{v}| - \frac{15R_i|\overrightarrow{v}|^2}{|\overrightarrow{v}| + 15\eta_S\psi_S\phi_S\sqrt{|R_i||\overrightarrow{v}|^2}}) \times \frac{\eta_S}{|\overrightarrow{v}|} \quad (2.45)$$

Eliminating $|\overrightarrow{v}|$ in the expression of PCH gives finally:

$$PCH = (1 - \frac{15R_i}{1 + 15\eta_S\psi_S\phi_S\sqrt{|R_i|}}) \times \eta_S \quad (2.46)$$

With the following notations:

$$\overrightarrow{X_{Z_0}} = \begin{pmatrix} 1 \\ \mu \\ \mu^2 \\ \mu^3 \end{pmatrix} \qquad \mu = \ln(\frac{Z_0}{Z_{0H}}) \qquad Y_Z = \frac{Z}{Z_{0H}}$$

and vectors $\overrightarrow{S_\phi}$ and $\overrightarrow{S_\psi}$ are:

$$\overrightarrow{S_\phi} = \begin{pmatrix} 3.2165 & 4.3431 & 0.5360 & -0.0781 \end{pmatrix} \tag{2.47}$$

$$\overrightarrow{S_\psi} = \begin{pmatrix} 0.5802 & -0.1571 & 0.0327 & -0.0026 \end{pmatrix} \tag{2.48}$$

Chapter 3

Convergence between SURFEX and ARPEGE codes

In this chapter, we're going to compare the formulations of C_D and C_H that have been established in the previous chapters. For that purpose, we need the same naming convention, especially for roughness length: in ARPEGE, Z_0 is the current roughness length: it takes into account the effects of vegetation, snow and orography while Z_{0MR} is the roughness length without the effect of the orography. In SURFEX, Z_0 is the the vegetation roughness length (no orography) and Z_{0eff} is the roughness length of vegetation plus snow and orography. In this part of the document, the SURFEX notation for roughness length is used. A second difference of convention concerns the height at which computations are done: from ground in SURFEX, above Z_0 in ARPEGE. We call Z^* the reference height, it goes from 0. to Z in SURFEX and from Z_0 to Z in ARPEGE. Considering that Z is sufficiently large compared to Z_0 or Z_{0H} or Z_{0MR}, we can assume that $1 + \frac{Z}{Z_0}$, $1 + \frac{Z}{Z_{0H}}$ and $1 + \frac{Z}{Z_{0MR}}$ are respectively close to $\frac{Z}{Z_0}$, $\frac{Z}{Z_{0H}}$ and $\frac{Z}{Z_{0MR}}$.

parameters that account in roughness length definition	ARPEGE notation	SURFEX notation
vegetation	$Z_0 - Z_{0MR}$	Z_0
vegetation + orography	Z_0	Z_{0eff}

3.1 C_D

3.1.1 stable case

ARPEGE and SURFEX formulations are the same in the stable case:

$$C_D = \frac{1}{1 + \frac{10R_i}{\sqrt{1+5R_i}}} \times \frac{\kappa^2}{(\ln(\frac{Z^*}{Z_{0eff}}))^2} \tag{3.1}$$

3.1.2 unstable case

ARPEGE and SURFEX formulations are different. If we remind the expression of C_D found in unstable case for ARPEGE and SURFEX, we have the same formula:

$$PCD = (1 - \frac{10R_i}{1 + 10\eta\psi\phi\sqrt{|R_i|}}) \times \eta \tag{3.2}$$

But, if we can assume that η function is the same for ARPEGE and SURFEX ($\ln(\frac{Z^*}{Z_{0eff}})$, this is not the case for ψ and ϕ. First main difference concerns definition of: $\overrightarrow{X_{Z_0MR}(\mu)}$ since in ARPEGE $\mu = \ln(\frac{Z_{0MR}}{Z_{0H}})$ while in SURFEX for definition of C_D the expression is $\mu = \ln(\frac{Z_{0eff}}{Z_{0H}})$. It means that in one case, the orography roughness length is taken into account but not in the other case. If we look to the other terms: ϕ and ψ we have:

$$\phi_A = \overrightarrow{A_\phi} \cdot \overrightarrow{X_{Z_0MR}} \tag{3.3}$$
$$\phi_S = \overrightarrow{S_\phi} \cdot \overrightarrow{X_{Z_0}} \tag{3.4}$$
$$\psi_A = Y_Z^{\overrightarrow{A_\psi} \cdot \overrightarrow{X_{Z_0MR}}} \sqrt{1 + \frac{Z}{Z_0}} \tag{3.5}$$
$$\psi_S = Y_Z^{\overrightarrow{S_\psi} \cdot \overrightarrow{X_{Z_0}}} \tag{3.6}$$

Even if we can adjust the coefficients of A_ϕ and S_ϕ or A_ψ and S_ψ, since they're not applied on the same variable, there's no possible convergence. Situation is even more difficult for ψ function since Y_Z is different and there's multiplication by $\sqrt{1 + \frac{Z}{Z_0}}$ that appears in the ARPEGE formulation.

Let's come back to the operational formulation of C_D as computed in achmt.F90 and not achmtls.F90. According to the code and with the same approach that was made in previous chapters, it can be expressed as:

$$PCD = (1 - \frac{10R_i}{1 + 10\eta_A\psi_A\phi_A\sqrt{|R_i|}}) \times \eta_A \tag{3.7}$$

$$\eta_A = (\frac{\kappa}{\ln(1 + \frac{Z}{Z_0})})^2 \tag{3.8}$$
$$\phi_A = \overrightarrow{A_\phi} \cdot \overrightarrow{X_{Z_0}} \tag{3.9}$$
$$\psi_A = Y_Z^{\overrightarrow{A_\psi} \cdot \overrightarrow{X_{Z_0}}} \tag{3.10}$$

With the following notations:

$$\overrightarrow{X_{Z_0}} = \begin{pmatrix} 1 \\ \mu \\ \mu^2 \\ \mu^3 \end{pmatrix} \qquad \mu = \ln(\frac{Z_0}{Z_{0H}}) \qquad Y_Z = 1 + \frac{Z}{Z_0}$$

and vectors $\overrightarrow{A_\phi}$ and $\overrightarrow{A_\psi}$ are:

$$\overrightarrow{A_\phi} = \begin{pmatrix} 7.5 & 2.39 & 0.2858 & 0.01074 \end{pmatrix} \quad (3.11)$$

$$\overrightarrow{A_\psi} = \begin{pmatrix} 0.5 & -0.07028 & 0.01023 & -0.00067 \end{pmatrix} \quad (3.12)$$

This set of equations is comparable to the one obtained in SURFEX unstable case. The adjustment needed can be made at the level of coefficients of $A_{\phi|\psi}$ and $S_{\phi|\psi}$ functions (note that coefficients differ slightly).

The question here concerns the formulation of C_D in achmtls.F90 which appears to be different as the one written in achmt.F90 and its counterpart from SURFEX. Note that we obtain the same formulations for achmtls and achmt if the orography roughness length is set to zero.

3.2 C_H

3.2.1 stable case

ARPEGE and SURFEX formulations are the same in the stable case:

$$C_H = \frac{1}{1 + \frac{15R_i}{\sqrt{1+5R_i}}} \times \frac{\kappa^2}{\ln(\frac{Z^*}{Z_{0H}})\ln(\frac{Z^*}{Z_0})} \quad (3.13)$$

3.2.2 unstable case

For C_H coefficient, the problem found for the expression of C_D does not exist and C_H coefficient may be written like this:

$$PCH = (1 - \frac{15R_i}{1 + 15\eta\psi\phi\sqrt{|R_i|}}) \times \eta \quad (3.14)$$

With the following notations:

$$\eta_A = \frac{\kappa^2}{\ln(1 + \frac{Z}{Z_{0H}})\ln(1 + \frac{Z}{Z_{0MR}})} \quad (3.15)$$

$$\phi_A = \overrightarrow{A_\phi} \cdot \overrightarrow{X_{Z_0}} \quad (3.16)$$

$$\psi_A = Y_Z^{\overrightarrow{A_\psi} \cdot \overrightarrow{X_{Z_0}}} \quad (3.17)$$

$$\eta_S = \frac{\kappa^2}{\ln(\frac{Z}{Z_{0H}})\ln(\frac{Z}{Z_0})} \quad (3.18)$$

$$\phi_S = \overrightarrow{S_\phi} \cdot \overrightarrow{X_{Z_0}} \quad (3.19)$$

$$\psi_S = Y_Z^{\overrightarrow{S_\psi} \cdot \overrightarrow{X_{Z_0}}} \quad (3.20)$$

If we apply the assumptions for Z^*, this system becomes:

$$\eta_A = \eta_S = \frac{\kappa^2}{\ln(\frac{Z^*}{Z_{0H}})\ln(\frac{Z^*}{Z_0})} \tag{3.21}$$

$$\phi_A = \overrightarrow{A_\phi} \cdot \overrightarrow{X_{Z_0}} \tag{3.22}$$

$$\phi_S = \overrightarrow{S_\phi} \cdot \overrightarrow{X_{Z_0}} \tag{3.23}$$

$$\psi_A = Y_Z^{\overrightarrow{A_\psi} \cdot \overrightarrow{X_{Z_0}}} \tag{3.24}$$

$$\psi_S = Y_Z^{\overrightarrow{S_\psi} \cdot \overrightarrow{X_{Z_0}}} \tag{3.25}$$

X_{Z_0} and Y_Z have the same definition in the two systems:

$$\overrightarrow{X_{Z_0}} = \begin{pmatrix} 1 \\ \mu \\ \mu^2 \\ \mu^3 \end{pmatrix} \qquad \mu = \ln(\frac{Z_0}{Z_{0H}}) \qquad Y_Z = \frac{Z}{Z_{0H}}$$

The adjustment of coefficients of functions A_ϕ, S_ϕ in one hand and A_ψ, S_ψ on the other hand for ARPEGE and SURFEX will return the same value of C_H coefficient.

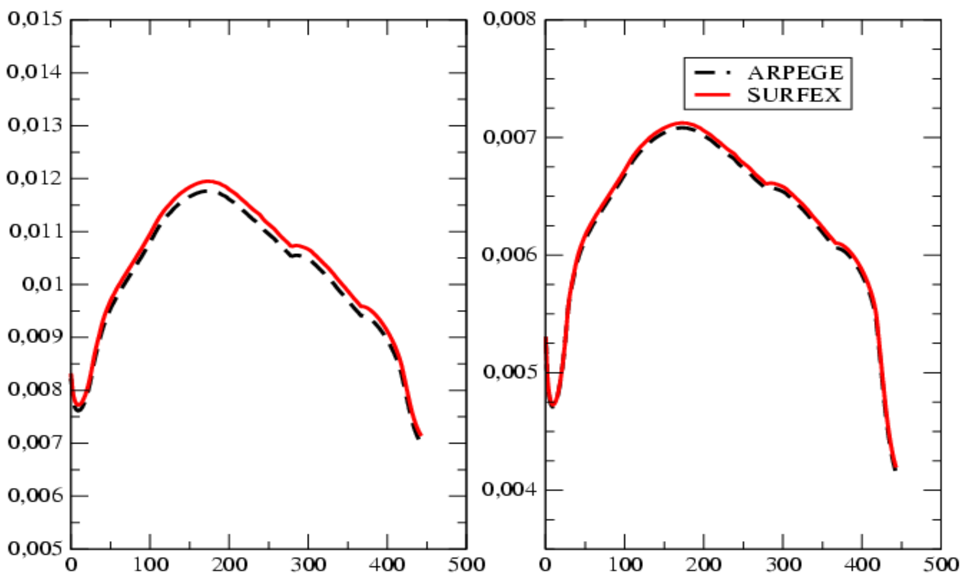

Figure 3.1: *Comparison between ARPEGE and SURFEX of C_D (left picture) and C_H (right picture) coefficients*