Physics - Dynamics Interface

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Physics - dynamics interface

- general considerations
- mass conservation issues
- AROME equations
- predictor - corrector scheme

General considerations

- total tendency is a sum of the linear contribution from dynamics, the non-linear part of the dynamics and the physics part

$$\frac{d\psi}{dt} = \frac{\psi_A^+ - \psi_O^-}{2\delta t} = L(\frac{\psi_A^+ + \psi_O^-}{2}) + N(\psi_M^0) + \phi(\psi_O^-)$$

- we compute the physics tendency before the dynamical one and interpolate it to the origin point

Mass conservation

- In ALADIN/ARPEGE, two options exist:
- (1) the total mass of the atmosphere is conserved (NDPSFI=0 in NAMPHY)

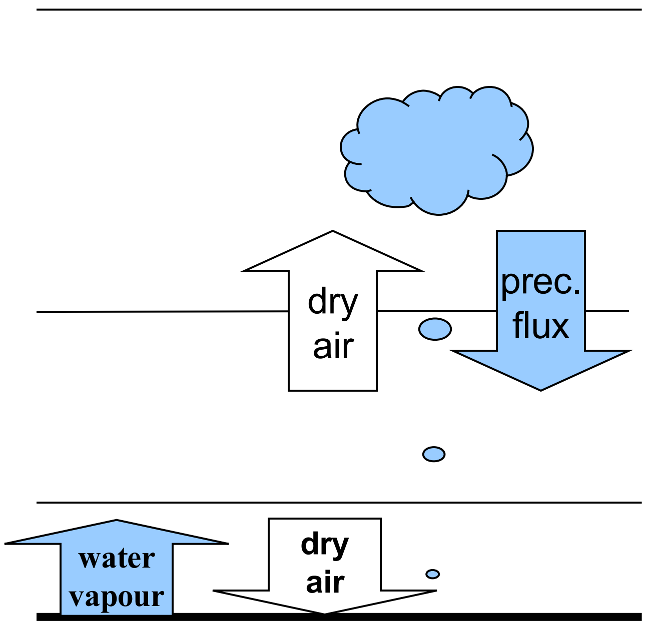
$$\rho = \rho_{dryair} + \rho_{vapour} + (\rho_{liquid} + \rho_{ice}) = cte.$$

- (2) the mass of the dry air is conserved (NDPSFI=1)

$$\rho_{dryair} = cte.$$

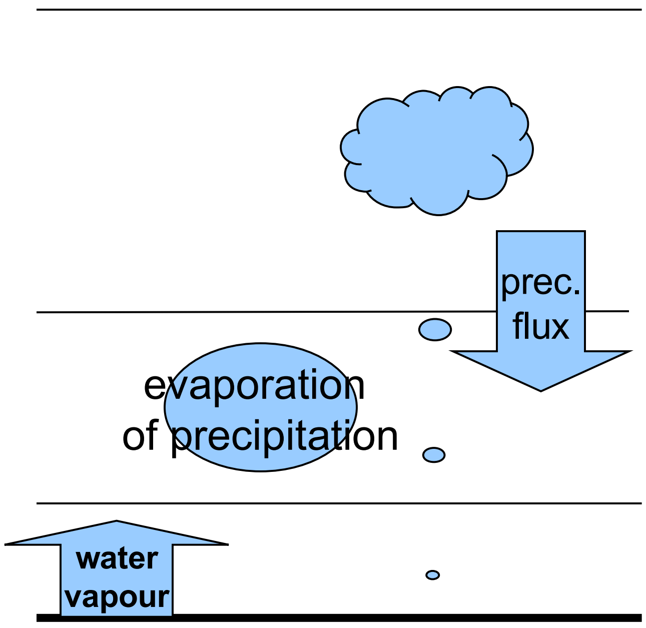
$q_v = \frac{\rho_v}{\rho_{total}}$ – is a prognostic variable

(1) When the total mass of the atmosphere is conserved



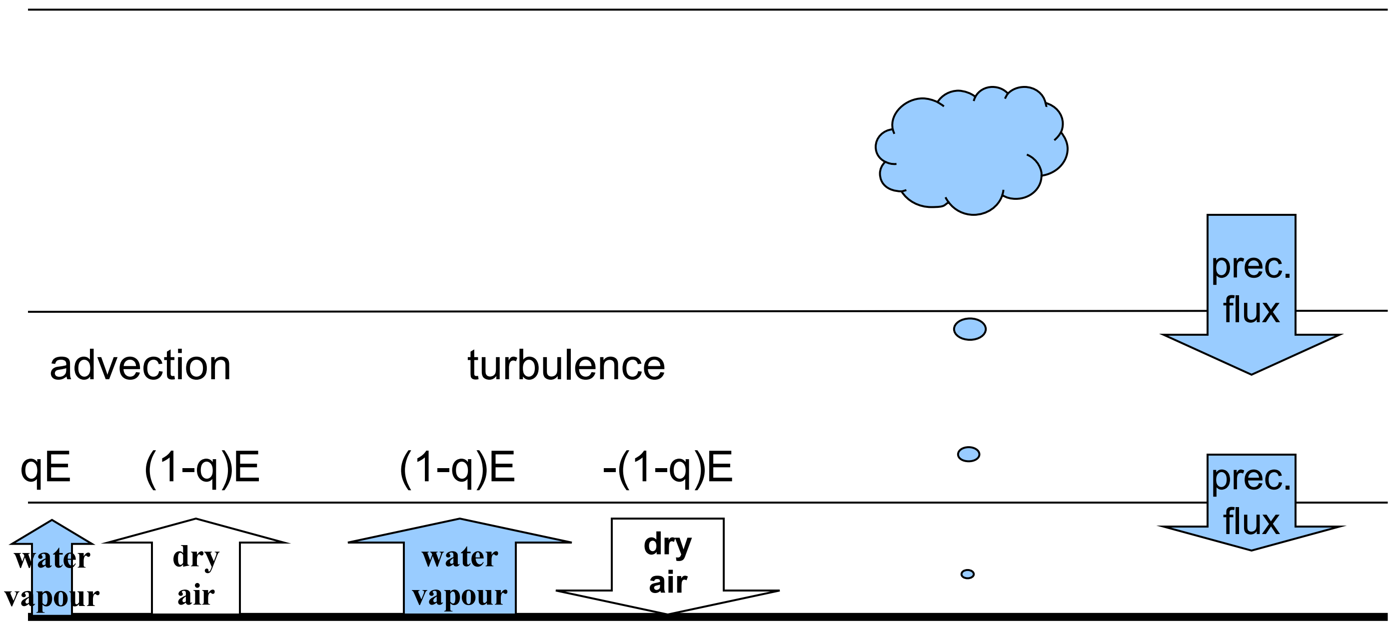
- the mass of water removed from the atmosphere is replaced by the dry air
- the mass of water vapour evaporated from the bottom surface (or falling precipitation) is compensated by a removal of the dry air

(2) When the mass of the dry air is conserved



- the total mass of the atmosphere changes due to the precipitation - evaporation budget
- condensation produces a local mass deficit
- evaporation produces a local mass increase

(2) When the mass of the dry air is conserved (2)



Equations

$$\frac{dV}{dt} = -fk \times \vec{V} - \nabla\phi - RT\nabla \ln p + \left(\frac{\partial V}{\partial t}\right)_{phy} - \delta_m g P \frac{\partial V}{\partial p}$$

horisontal wind

$$\dot{\eta}\frac{\partial p}{\partial \eta} = -B(\eta)\frac{\partial \pi_s}{\partial t} - \int_0^{\eta} \nabla_{\eta} \cdot \left(\vec{V}\frac{\partial p}{\partial \eta}\right) d\eta - \delta_m g P$$

vertical velocity

$$\frac{dT}{dt} = \frac{RT}{C_p}\frac{\omega}{p} + \left(\frac{\partial T}{\partial t}\right)_{phy} - \delta_m g P \frac{\partial \phi}{\partial p} + (1-\delta_m) g \frac{C_{pd}}{C_p} P \frac{\partial T}{\partial p}$$

temperature

$$\frac{dq}{dt} = -g\frac{\partial P}{\partial p} + \left(\frac{\partial q}{\partial t}\right)_{phy} + \delta_m g q \frac{\partial P}{\partial p}$$

moisture

- Impact of the variable mass assumption on the evolution of the model variables

Vertical co-ordinate

$$\pi = A(\eta)\pi_{top} + B(\eta)\pi_{surface}$$

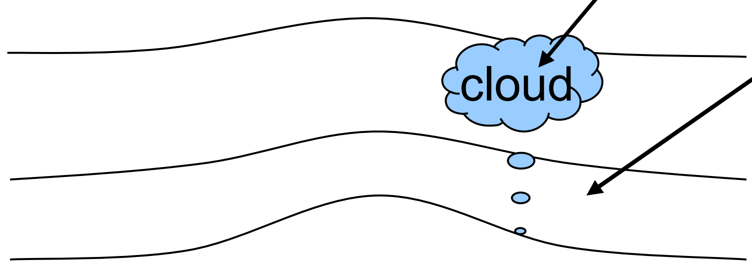
$$\frac{\partial \pi}{\partial z} = -\rho g$$

for the $\delta_m = 1$ option ρ changes

— $\frac{\partial \pi}{\partial z}$ changes

— η changes

- in the case of condensation, and precipitation we have a removal of mass here
- but this precipitation may evaporate on the way to the ground so we have extra mass here
- we get vertical velocity due to a mass flux due to precipitation-evaporation budget



Arome

Respective conservation of dry air an total water is assumed. The equation for the total density of the mixture is

$$\frac{\partial \rho}{\partial t} = -\vec{\nabla} \cdot (\rho \vec{u})$$

Where ρ is the total density of the mixture

$$\rho = \sum_{k=0}^{N} \rho_k$$

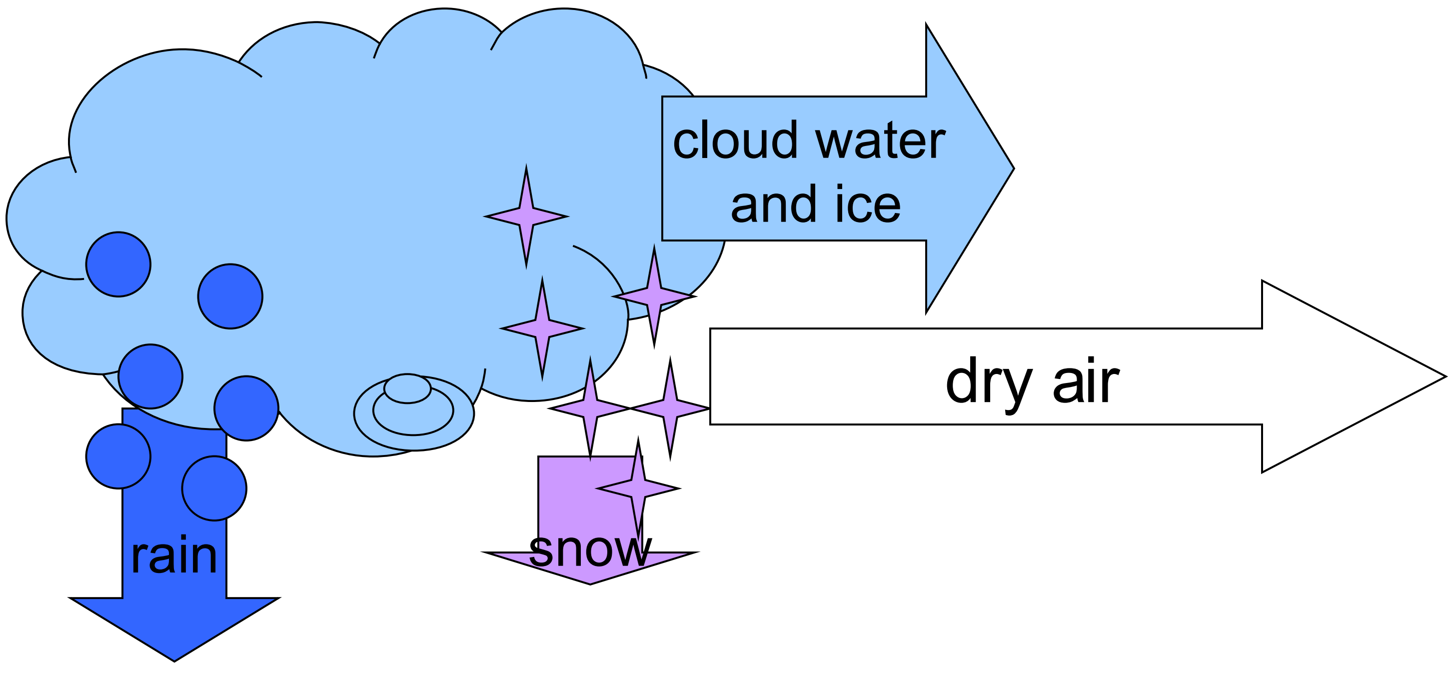
And $\vec{u}$ is the barycentric velocity is defined as

$$\vec{u} = \frac{1}{\rho} \sum_{k=0}^{N} \rho_k \vec{u}_k$$

It is allowed for different species in the atmosphere to have different velocities.

Barycentric velocity

- velocities of the different atmospheric constituents



Arome - Conservation of species

- The conservation equation for the species k is

$$\frac{dq_k}{dt} = -\frac{1}{\rho}\vec{\nabla} \cdot (\rho q_k(\vec{u}_k - \vec{u})) + \frac{\dot{\rho}_k}{\rho}$$

$q_k = \frac{\rho_k}{\rho}$ is the specific mass content in an unit volume of the species k.

- species: dry air, water vapour, liquid water, cloud ice, rain, snow and graupel.

Arome - Velocity equation

The evolution equation for the barycentric velocity

$$\frac{d\vec{u}}{dt} = -\frac{1}{\rho}\vec{\nabla} p + \frac{1}{\rho}\frac{\partial \sigma_{\alpha\beta}}{\partial x_{\beta}} + \rho\vec{g}$$

Where $\sigma_{\alpha\beta}$ is viscous part of the stress.

Arome - Enthalpy equation

- The evolution equation for enthalpy is

$$\frac{dh}{dt} = \frac{1}{\rho}\frac{dp}{dt} + \dot{Q}_i + \frac{1}{\rho}\sigma_{\alpha\beta}\frac{\partial u_\beta}{\partial x_\alpha} - \frac{1}{\rho}\nabla \cdot \sum_{k=0}^{N} h_k \rho_k (\vec{u}_k - \vec{u})$$

where $h = \Sigma_{k=0}^{N} h_k q_k = e_{int} + \frac{p}{\rho}$ is the enthalpy.

Arome - Temperature equation

- The evolution equation for temperature is

$$C_p \frac{dT}{dt} = \frac{1}{\rho}\frac{dp}{dt} + \dot{Q}_i + \frac{1}{\rho}\sigma_{\alpha\beta}\frac{\partial u_\beta}{\partial x_\alpha} - \frac{1}{\rho}\sum_{k=0}^{N}\rho_k(\vec{u}_k - \vec{u})\vec{\nabla}\cdot h_k - \frac{1}{\rho}\sum_{k=0}^{N} h_k\dot{\rho}_k$$

where $C_p = \Sigma_{k=0}^{N} q_k C_{pk}$ and $h_k = C_{pk}T$.

Aladin

- all the non-precipitating species move with the same speed
- mass and volume of precipitation is neglected
- velocity of precipitation is infinite
- we define the product of mass to the velocity of precipitation as

$$\rho_{prec}\vec{u}_{prec} = -P\vec{k}$$

- where P is a precipitation flux

Aladin - barycentric velocity

- with the Aladin assumptions, the barycentric velocity is

$$\vec{u} = \frac{1}{\rho} \sum_{k=0}^{N} \rho_k \vec{u}_k = \vec{u}_a - \frac{P}{\rho} \vec{k}$$

Instead of introducing artificial sources and sinks of dry air to compensate for the loss of mass in the case $\delta_m = 0$ we can introduce a mass flux of dry air $\rho_{air} \vec{u}_{air} = P\vec{k}$. The barycentric velocity becomes:

$$\vec{u} = \vec{u}_a - \frac{P}{\rho} \vec{k} + (1 - \delta_m) \frac{P}{\rho} \vec{k} = \vec{u}_a - \delta_m \frac{P}{\rho} \vec{k}$$

Aladin - barycentric velocity (2)

The only purpose of the additional term $\frac{P}{\rho}\vec{k}$ is to compensate for the loss of mass, it does not advect the air properties. In the $\delta_m = 0$ case, velocity of the dry air is not equal to the velocity of the water species. All the properties of the air mixture are advected with the barycentric velocity (kinetic energy, momentum, enthalpy), but the species k in the mixture is advected with the velocity $\vec{u}_k$.

Aladin - barycentric velocity (3)

The Lagrangian derivative becomes

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{u}_a \cdot \nabla - \delta_m \frac{P}{\rho} \frac{\partial}{\partial z}$$

the last part represents the vertical pseudo-advection because of the precipitation flux. We have the advection due to dynamics (forces acting on the mixture) with $\vec{u}_a \cdot \nabla$ and advection term due to physics $\delta_m \frac{P}{\rho} \frac{\partial}{\partial z}$ that may or may not have an influence, depending on δ_m.

Aladin - Continuity equation

- The continuity equation becomes

$$\frac{\partial \rho}{\partial t} = -\vec{\nabla} \cdot (\rho \vec{u})$$

$$\frac{\partial \rho}{\partial t} = -\vec{\nabla} \cdot (\rho \vec{u}_a) + \delta_m \frac{\partial P}{\partial z}$$

or

$$\frac{d\rho}{dt} = -\rho \vec{\nabla} \cdot \vec{u}$$

$$\frac{d\rho}{dt} = -\rho \vec{\nabla} \cdot \vec{u}_a + \delta_m \rho \frac{\partial}{\partial z} \frac{P}{\rho}$$

Aladin - Conservation of water species

$$\frac{\partial q_k}{\partial t} + \vec{u}_a \cdot \nabla q_k = \frac{\dot{\rho}_k}{\rho} - \delta_m \frac{q_k}{\rho} \frac{\partial P}{\partial z}$$

The last term on the right side is the consequence of the total mass not being conserved. Because $q_k = \frac{\rho_k}{\rho}$, q_k changes if ρ changes.

Aladin - Velocity equation

$$\frac{d\vec{u}}{dt} = -\frac{1}{\rho}\vec{\nabla}p + \frac{1}{\rho}\frac{\partial\sigma_{\alpha\beta}}{\partial x_\beta} + \rho\vec{g}$$

introduce the Lagrangian derivative

$$\frac{\partial\vec{u}}{\partial t} + \vec{u}_a \cdot \nabla\vec{u} - \delta_m \frac{P}{\rho}\frac{\partial\vec{u}}{\partial z} = -\frac{1}{\rho}\vec{\nabla}p + \frac{1}{\rho}\frac{\partial\sigma_{\alpha\beta}}{\partial x_\beta} + \vec{g}$$

$$\frac{\partial\vec{u}}{\partial t} + \vec{u}_a \cdot \nabla\vec{u} = -\frac{1}{\rho}\vec{\nabla}p + \frac{1}{\rho}\frac{\partial\sigma_{\alpha\beta}}{\partial x_\beta} + \vec{g} + \delta_m \frac{P}{\rho}\frac{\partial\vec{u}}{\partial z}$$

Aladin - Temperature equation

start from the Arome temperature equation

$$C_p \frac{dT}{dt} = \frac{1}{\rho}\frac{dp}{dt} + \dot{Q}_i + \frac{1}{\rho}\sigma_{\alpha\beta}\frac{\partial u_\beta}{\partial x_\alpha} - \frac{1}{\rho}\sum_{k=0}^{N} C_{pk}\rho_k(\vec{u}_k - \vec{u})\vec{\nabla}\cdot h_k - \frac{1}{\rho}(L_{21}(\dot{\rho}_2 + \dot{\rho}_3) + L_{41}(\dot{\rho}_4 + \dot{\rho}_5 + \dot{\rho}_6))$$

where $h_k = CpkT$. Introduce

$$\frac{dT}{dt} = \frac{\partial T}{\partial t} + \vec{u}_a \cdot \nabla T - \delta_m \frac{P}{\rho}\frac{\partial T}{\partial z}$$

$$\frac{dp}{dt} = \frac{\partial p}{\partial t} + \vec{u}_a \cdot \nabla p - \delta_m \frac{P}{\rho}\frac{\partial p}{\partial z}$$

Aladin - Temperature equation (2)

For the summation term, we have

$$C_{pd}\rho_0(\vec{u}_0 - \vec{u}) \cdot \nabla T = C_{pd}(\rho_0 \delta_m \frac{P}{\rho}\frac{\partial T}{\partial z} + (1-\delta_m)P\frac{\partial T}{\partial z})$$

for dry air

$$C_{pk}\rho_k(\vec{u}_k - \vec{u}) \cdot \nabla T = C_{pk}\rho_k \delta_m \frac{P}{\rho}\frac{\partial T}{\partial z}$$

for the non-precipitable species of water

$$C_{pk}\rho_k(\vec{u}_k - \vec{u}) \cdot \nabla T = -C_{pk}P\frac{\partial T}{\partial z}$$

for precipitation species.

Aladin - Temperature equation (3)

$$\frac{\partial T}{\partial t} + \vec{u}_a \cdot \nabla T = \frac{1}{C_p \rho}(\frac{\partial p}{\partial t} + \vec{u}_a \cdot \nabla p) + \frac{1}{C_p}\dot{Q}_i + \frac{1}{C_p \rho}\sigma_{\alpha\beta}\frac{\partial u_\beta}{\partial x_\alpha}$$
$$-\delta_m \frac{P}{C_p \rho^2}\frac{\partial p}{\partial z} - \frac{1}{C_p}((1-\delta_m)C_{pd}\frac{P}{\rho}\frac{\partial T}{\partial z} - C_{prec}\frac{P}{\rho}\frac{\partial T}{\partial z})$$
$$-\frac{1}{\rho}(L_{21}(\dot{\rho}_2 + \dot{\rho}_3) + L_{41}(\dot{\rho}_4 + \dot{\rho}_5 + \dot{\rho}_6))$$

where C_{prec} is specific heat of precipitation (any species). The second line shows the effect of different assumptions for the conservation law. There is a term contributing to the temperature tendency due to the difference of specific heats of dry air and precipitaton in the case $\delta_m = 0$.

PC scheme

The way it is coded now

Predictor

$$\frac{X_{Apred}^{t+\delta t} - X_O^t}{\delta t} = L(\frac{X_{Apred}^{t+\delta t}) + X_O^t}{2}) + N(X_M^{t+\frac{\delta t}{2}}) + \phi(X_O^t)$$

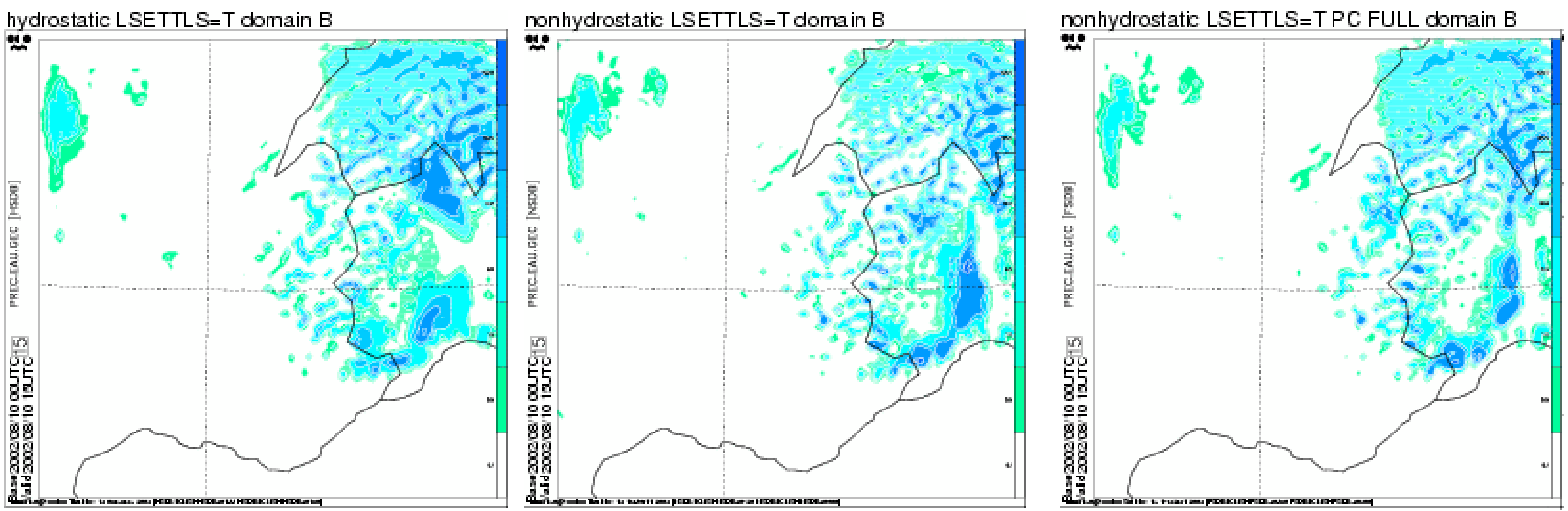
Corrector

$$\frac{X_{Acorr}^{t+\delta t} - X_O^t}{\delta t} = L(\frac{X_{Acorr}^{t+\delta t} + X_O^t}{2}) + \frac{N(X_{Fpred}^{t+\delta t}) + N(X_O^t)}{2} + \phi(X_O^t)$$

but, we can use $\phi(X_O^t)$, $\phi(X_A^t)$ or $\frac{\phi(X_O^t)+\phi(X_A^t)}{2}$

Under LPC_FULL, the position of the O points is recomputed and the values are re-interpolated.

And a few figures



- Stratiform precipitation woth 5 km resolution over Alps, August 10th 2002.

Conclusion

- both options for the mass conservation assumptions may be kept