Current Research and Developpment in dynamics at Meteo-France

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Joint 27th ALADIN Workshop & HIRLAM All Staff Meeting 2017, 3-7/04/2017, Helsinki, Finland

Strategy for research on dynamical cores

- There are many concerns about current dynamical cores :
  - Scalability
  - Steep slopes
  - Conservation of variables...
- Scalability : still not a such a big issue “only” 15 % of forecast time spent in communication in AROME (on our current operational domain 1536x1440 grid points on 179 nodes and 7160 physical cores)
- Steep slopes : we have the impression it could be difficult to go beyond 200m of resolution.
- Conservation of variable : it is more a concern for chemistry or climate runs

Strategy for research on dynamical cores

- Still, if we want to improve the negative points listed before, one has to change the dynamical core.
- The strategy we follow is to gradually modify the causes :
- Scalability : it is link to the intense global communication induced by the fourier transforms (this is unavoidable since Fourier transform or Legendre transform in global are highly non-local operators). It is possible to compute derivatives locally, but the implicit problem is trivial in spectral space (trigonometric polynoms are eigenvalues of that operator) and not in gridpoint space.
- Steep slopes : going from a spectral model to a gridpoint model might help us, another alternative is investigated : quasi-elastic system.

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Strategy for research on dynamical cores

- For scalabilities issues the solution is to perform all the computations in gridpoint space. Consequently the pseudo-helmholtz implicit equation has to be solved in grid point space, leading us to study the performance of grid point solvers with our system of equations (part 1 of the talk).
- If we want to keep a single global and local dynamical core it is also important to be able to perform correctly computations on the sphere especially derivatives (part 2).
- To potentially improve steep slope issues a solution could be to use another set of primitive equations (part 3) or to use a different (more complex) semi-implicit operator (part 1)

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1. Using gridpoint solvers for implicit problem

Using gridpoint solvers for implicit problem

- In the current Semi-implicit algorithm after variable elimination and projection on vertical modes, 2*Nlev 2D implicit equations are solved :

$$\left(I_d - \lambda_i^2 \nabla^2\right) X\left(\vec{x}\right) = F\left(\vec{x}\right) \qquad \left(1\right)$$

- Equation (1) “Helmholtz type” is solved in spectral space where the solution is trivial.

Using gridpoint solvers for implicit problem

- The reference state for the linearization of the implicit operator does not include orography. With orography linearization leads to a more general operator depending on x, no more projection on vertical modes is possible, coefficients are not constant on x.

$$\left(I_d - B_{\vec{x}} \nabla^2\right) X\left(\vec{x}, \eta\right) = F\left(\vec{x}, \eta\right) \qquad (2)$$

- With a grid point solver, system (2) can be solved but not as easily as equation (1) of previous slide.
- It is possible that current instabilities with high slopes might be linked with the implicit system not taking account orography.

Using gridpoint solvers for implicit problem

- Direct methods are too expensive.
- Simple iterative methods such as Gauss-Seidel, Jacobi, SOR…. are not efficient enough.
- Quasi-Newton methods can be used to solve linear problem but require in general to store and approximation of the Hessian (that is to say an approximation of A), that is too large (although some memory inexpensive version exist).

Using gridpoint solvers for implicit problem

- The most successful class of methods for our problem are Krylov space methods ; the solution is seek in the successive Kn vector spaces for a Ax=b system :

$$K_n = \text{span}\{b, Ab, A^2 b \ldots A^{n-1} b\}$$

- Example : Conjugate gradient, Biconjugate gradient, Generalized Minimal Residual...
- Those methods are the most efficient for sparse matrices with a dominant diagonal.
- Among all Krylov space methods, Generalized Minimal Residual (GMRES) is the more optimal in term of number of iterations (meaning that it requires the least iterations for a given accuracy).

Using gridpoint solvers for implicit problem

- The current implicit problem leads to 2*Nlev 2D discrete implicit system

$$\left(I-\lambda_i^2\nabla^2\right)d_i^{n+1}=d^*$$
$$\text{for } i=1\ldots N_{lev}$$
then
$$\left(I-\lambda^2\nabla^2\right)D_i^{n+1}=D^*$$
$$\text{for } i=1\ldots N_{lev}$$

- Operators $\left(I-\lambda_i^2\nabla^2\right)$ are $(Nx*Ny)^2$ sparse matrices that depend on the vertical mode. In matrix writing on a given line the number of non-zero coefficient depends on the choice of the order for the derivative operator (9 non-zero coefficients for a 4^{th} order derivative).

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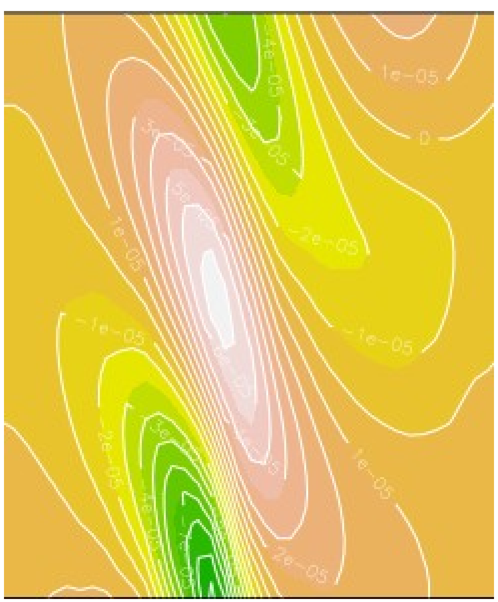
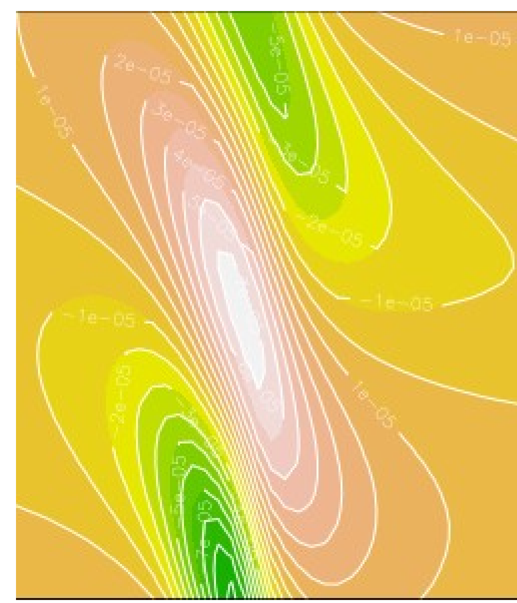
Using gridpoint solvers for implicit problem

- Test were performed with the iterative solver GMRES and the same variable elimination + projection into vertical modes
- Test with hydrostatic orography (5km length, 200m height)
- dx=2000m, dt=60s, predictor corrector. The iterative solver uses 16 iterations at each time step, that is to say one iteration every 4s



experiment



reference

Using gridpoint solvers for implicit problem

- Positive aspects :
  - The condition number of the implicit problem is ~ CFL^2 that is to say ~100. That corresponds to quite well conditioned problems.
  - In term of communications, roughly, the number of communications required for a given forecast lead time seems to be equivalent to the ones in a HEVI model.
  - If results are confirmed in 3D, that could be the first step for moving from a spectral model to a full grid-point model.
- Negative aspects :
  - The number of iterations can depend on the meteorological situation (not very convenient for operations).
  - Memory cost can be high for GMRES, other algorithms (congrad) with low recursivity require a few more iterations.

2. Circumventing Pole problem for solving PDES in spherical coordinates with local algorithms

Circumventing Pole problem for solving PDES in spherical coordinates with local algorithms

- Investigate whether spherical coordinates with unstaggered reduced lat-lon grids really makes life impossible or not, especially in view of local/scalable algorithms.

Why “attractive” ?

- a global orthogonal coordinate system with simple differential operators; high-orders schemes quite easy
- reduced lat-lon grid is semi-structured – transparent use of (i, j)
- grid cells may be viewed as quadrilaterals

Inconvenience

- curvature is unbounded at poles, this generates a variety of problems

Circumventing Pole problem for solving PDES in spherical coordinates with local algorithms

- Try to keep as much as possible of properties of spectral models:
  - high-order space-accuracy
  - totally unstaggered grid
  - reduced lat-lon grid quasi uniform physical resolution
- Space-discrete algorithms as simple and efficient as possible:
  - Finite differences at least to begin (FD2, FD4, FD6, FD8).
- Prototype : Shallow Water model.

Circumventing Pole problem for solving PDES in spherical coordinates with local algorithms

- Governing equations
  - Eulerian form with (u, v) wind components
- Discretization :
  - lat-lon grid: regular $\Delta\varphi$, regular $\Delta\lambda(\varphi)$
  - unstaggered A-grid, no points at poles
  - Meridionally, tests made with FD8 (stencil width = 9)
  - Close vicinity of Pole: Fourier transform method (FFT) along λ
  - Fourier truncation variable with latitude : $M(\varphi)$
  - Away from Poles: Sine-Cosine Lagrange (SCL) representation along λ

    (similar to FD but respects the 2π periodicity of fields).

Circumventing Pole problem for solving PDES in spherical coordinates with local algorithms

- Requirements :

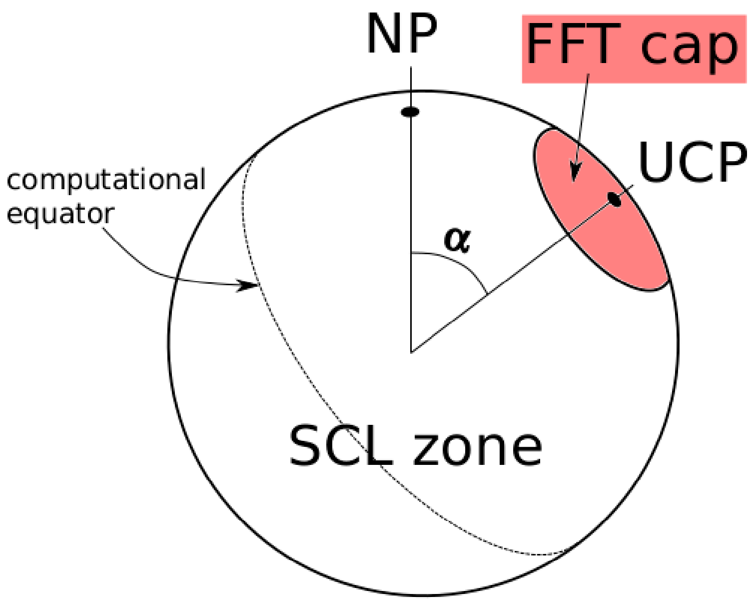
1) Always use true scalars (u cos φ, v cos φ) instead of pseudo-scalars (u, v) for evaluating Vorticity and Divergence

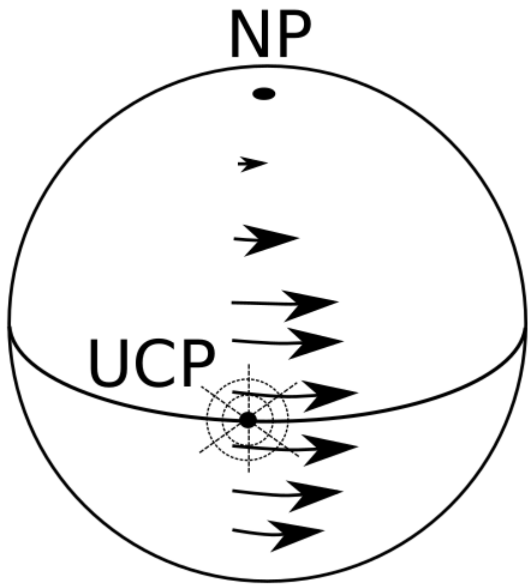
2) Increase the number of grid points on few latitude circles to avoid a pole problem similar to “Courtier and Naughton 1994” problem

3) Use different boundaries conditions at poles for zonal average of fields <ψ> than for the deviation ψ’.

Circumventing Pole problem for solving PDES in spherical coordinates with local algorithms

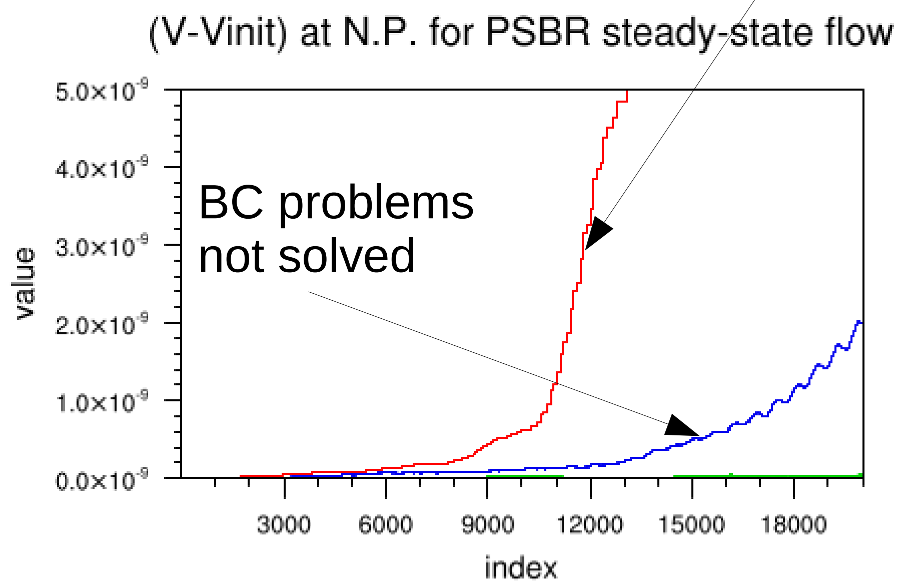
- Test flow : Polar Solid Body Rotation

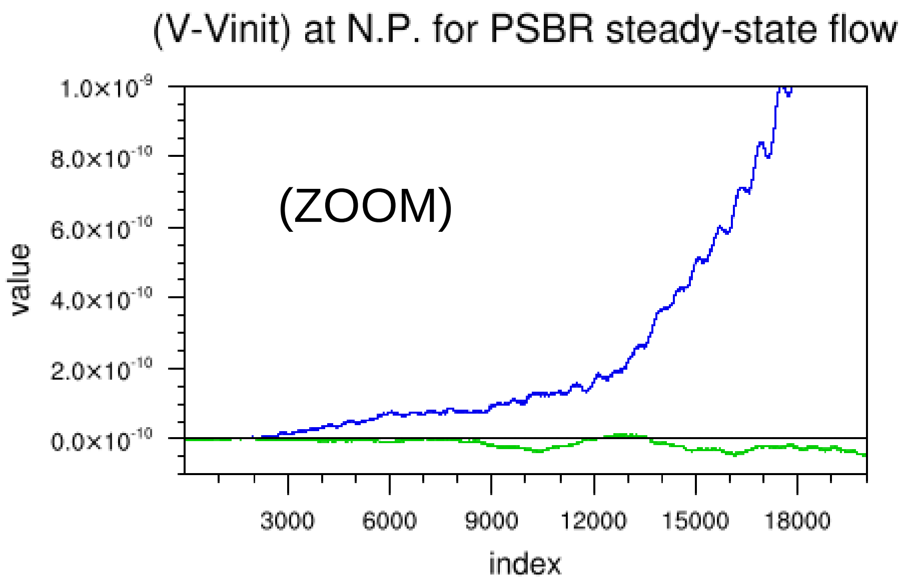




Circumventing Pole problem for solving PDES in spherical coordinates with local algorithms

CN94 and BC problems not solved





V(t) minus V(0), should remain 0 since the flow is stationnary.

3. A quasi-elastic semi-implicit dynamical system in mass-based coordinate

A quasi-elastic semi-implicit dynamical system in mass-based coordinate

- Objective : Enhance the stability of AROME non-hydrostatic (NH) model by suppressing the most restrictive fast dynamical process : « NH compressibility ».

- A possible solution : Design a set of approximate non-hydrostatic equations in mass-based coordinate, viable at small and large scales, that are free from the vertically-propagating acoustic wave.

- Unified NH system of Arakawa and Konor (2009) in z-coordinate.

- Guidelines : Exploit Arakawa and Konor'idea together with Laprise (1992) formalism.

A quasi-elastic semi-implicit dynamical system in mass-based coordinate

- Quasi-elastic set of prognostic equations in mass-based η-coordinate :

$$\frac{\mathrm{d}V}{\mathrm{d}t} = -e^{\kappa q}\left[\left(1+\frac{\hat{\pi}}{m}\frac{\partial q}{\partial \eta}\right)\nabla\phi + R\widetilde{T}\left(\frac{\nabla\pi}{\pi}+\nabla q\right)\right] + \mathcal{V}$$

$$\frac{\mathrm{d}\,d}{\mathrm{d}t} = -\frac{g^2}{\kappa R\widetilde{T}}\left(\frac{\pi}{m}\frac{\partial}{\partial\eta}\right)\left[\frac{\pi}{m}\frac{\partial\left(e^{\kappa q}-1\right)}{\partial\eta} + \kappa\left(e^{\kappa q}-1\right)\right] + d\left(D-\mathbb{D}_3\right) + \frac{g}{R\widetilde{T}}\left(\frac{\pi}{m}\frac{\partial V}{\partial\eta}\right).\nabla w - \frac{g}{R\widetilde{T}}\left(\frac{\pi}{m}\frac{\partial \mathcal{W}}{\partial\eta}\right)$$

$$\frac{\mathrm{d}\widetilde{T}}{\mathrm{d}t} = \frac{R\widetilde{T}}{C_p}\frac{\dot{\pi}}{\pi} + \frac{Q}{C_p}e^{-\kappa q},$$

$$\frac{\partial\pi_s}{\partial t} = -\int_0^{\eta}\nabla(mV)d\eta'$$

NB : Close link with already existing HPE and EE mass-based Systems.

- Additional soundproof divergence constraint

$$\mathbb{D}_3 + \frac{C_v}{C_p}\frac{\dot{\pi}}{\pi} = \frac{Q}{C_pT}$$

- Useful diagnostic relationships

$$m = \partial\pi/\partial\eta$$

$$\mathbb{D}_3 = \nabla.V + d + \frac{g\nabla\phi}{R\widetilde{T}}.\left(\frac{\pi}{m}\frac{\partial V}{\partial\eta}\right)$$

$$\nabla\phi = \nabla\Phi_s + \int_{\eta}^{1}\nabla\left(\frac{mR\widetilde{T}}{\pi}\right)d\eta',$$

$$\frac{\dot{\pi}}{\pi} = V.\frac{\nabla\pi}{\pi} - \frac{1}{\pi}\int_0^{\eta}\nabla(mV)\mathrm{d}\eta'$$

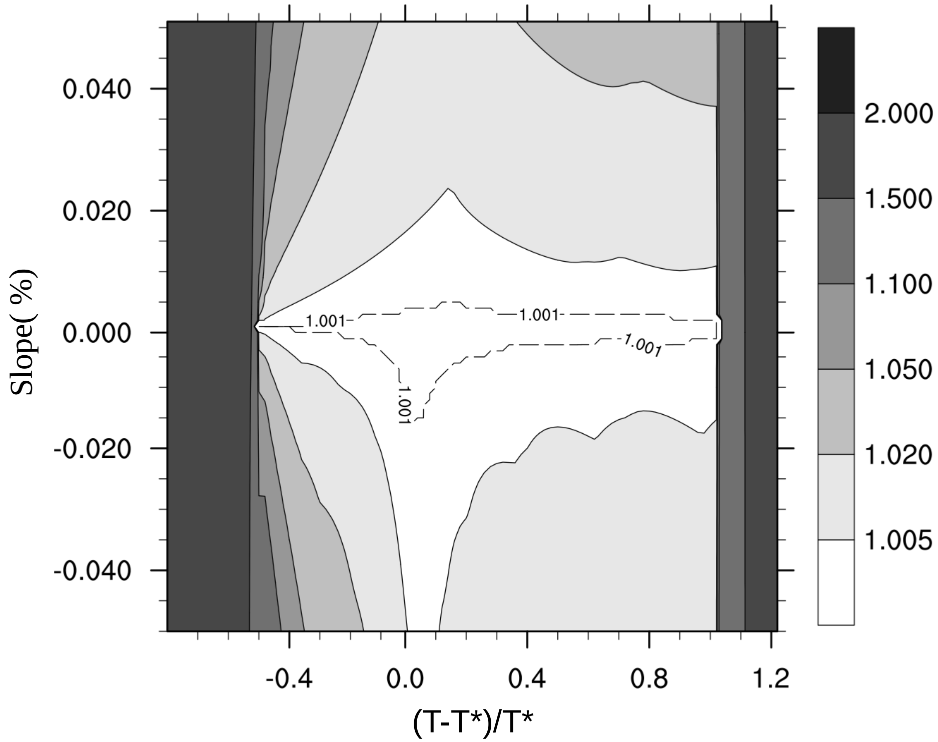
$$m\dot{\eta} = B(\eta)\int_0^1\nabla(mV)\mathrm{d}\eta' - \int_0^{\eta}\nabla(mV)\mathrm{d}\eta'$$

$$g\nabla w = g\nabla w_s + \int_{\eta}^{1}\left(\frac{mR\widetilde{T}}{\pi}\right)\nabla d\,\mathrm{d}\eta' + \int_{\eta}^{1}d\,\nabla\left(\frac{mR\widetilde{T}}{\pi}\right)\mathrm{d}\eta'$$

A quasi-elastic semi-implicit dynamical system in mass-based coordinate

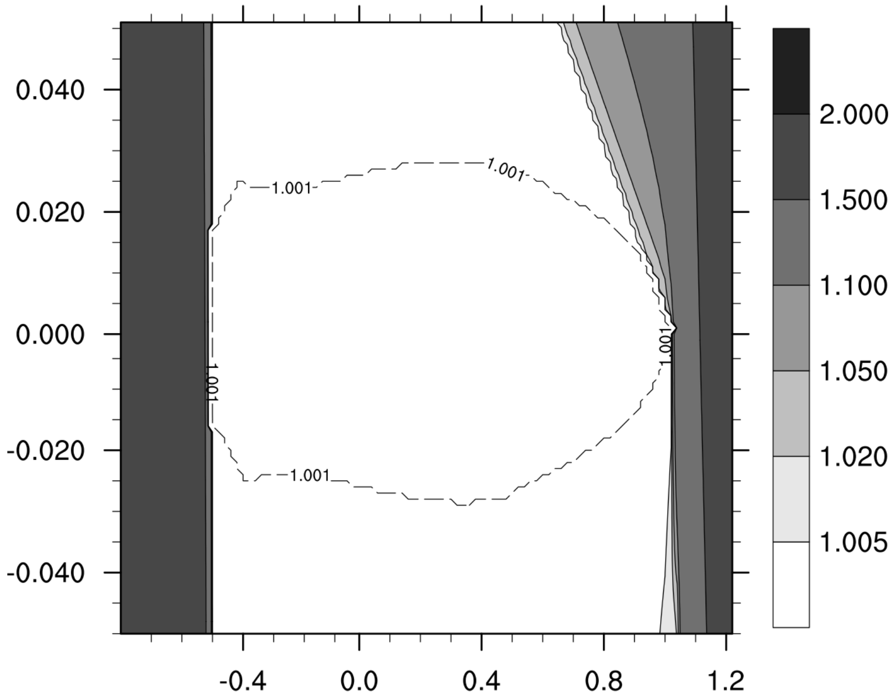
- The Quasi-elastic system is integrated with a newly derived semi-implicit time scheme, suggested by Voitus et al. (2017).

Stability diagram EE-3TL-SI with d4,
(courtesy of Bénard *et al.*, 2005)



- White colored regions indicate ranges of apparent stability.

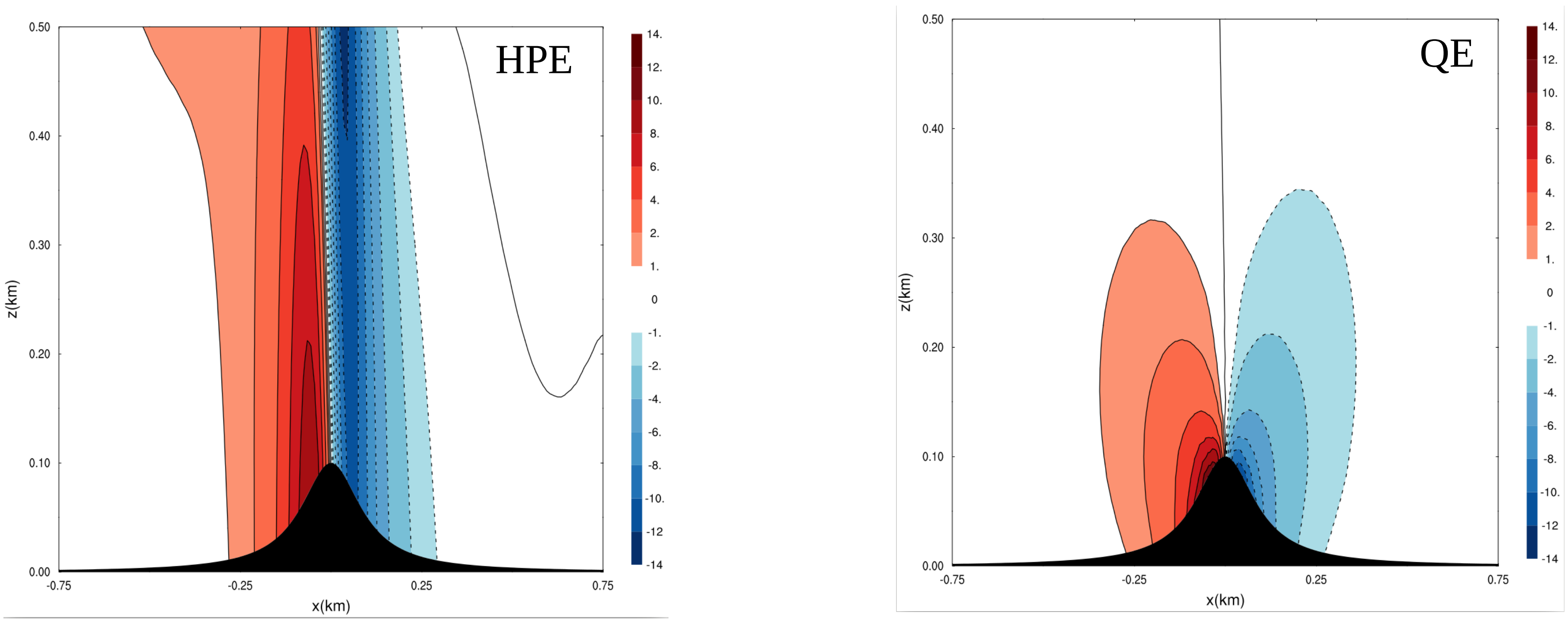
Stability diagram QE-3TL-SI with d3,
(Voitus *et al.*, 2017)



- The stability property of QE 3TL-SI in presence orography is close to the one of HPE 3TL-SI.

A quasi-elastic semi-implicit dynamical system in mass-based coordinate

- **Potential flow test-case** : The basic-state is defined by : U=15 m/s, N=0.02 s, a=100 m, h=100 m
  Settings of the experiment : Δx=20 m, Δt=0.4 s, Δη is chosen so that Δz≈20 m.



- QE system better captures the small-sclale features of the flow than HPE one (not a scoop, but nevertheless expected).

Conclusions

- Grid point solvers might be efficient enough for grid-point computations, replacing global communications by eulerian local ones with an efficiency close to the HEVI models.
- The global model might follow the same path if we carefully compute the derivatives on the reduced gaussian grid.
- Steep slopes issues might be more problematic than scalability, if we are not able go beyond 500m resolution, is the quasi-elastic system solving the issues we are facing today ?

THANK YOU FOR YOUR ATTENTION