Introduction of the β-plane into the horizontal balance equation of Aladin Jb

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9 avril 2004

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1 Introduction

The principal objective of this study is to code and to test the β-plane approximation into the horizontal balance equation, in the sofware of covariance calculation ("festat") and in the Aladin 3D-Var, and to compare it with the previous approach (based on the f-plane approximation).

Indeed, the adoption of the f-plane approximation in the horizontal balance equation remains reasonable on small domains far from the equator, knowing that the more one moves away from the equator, the less the variations of f are fast. The situation becomes worrying when great domains are treated, in particular those close to the tropics where the variations of the Coriolis parameter are not negligible. Model Albachir NORAF is an example.

The horizontal balance equation currently used in Aladin adopts the f-plane approximation and is close to the following equation :

$$\phi = f_0 \ \ \Delta^{-1}\zeta = f_0\psi \tag{1}$$

- ϕ : linearized geopotential.
- f_0 : mean value of the Coriolis parameter.
- Δ^{-1} : inverse of the Laplacian.
- ζ : relative vorticity.
- ψ : stream function.

The equation used in this study to express the horizontal balance in β-plane in "festat" and the Aladin 3D-Var is :

$$\phi = \Delta^{-1}(f\zeta) \tag{2}$$

We took into account in this equation the main part of the horizontal variations of f (which, implicitly, amounts writing f=f_0+β y, even if the formulation and the tested balance code do not utilize explicitly the parameter β).

2 Bi-Fourier development of the horizontal balance equation

The coding of the horizontal balance equation in β-plane in "festat" and the Aladin 3D-Var is carried out in the bi-Fourier complex space (over a half ellipsis) rather than in the space of real quadruplets (over a quarter of ellipsis). The choice was made taking into account the two following elements :
- The equations are definitely simpler in the bi-Fourier complex space than in the space of real quadruplets.
- The equations connecting these two spaces are simple. The fact of using them explicitly in the code does not generate an additional cost, and improves the legibility of the code.

The development of the equation (2) in bi-Fourier is :

$$\hat{\phi}(m,n) = (\hat{f\psi})(m,n) = \sum_l \sum_p \hat{f}(l,p)\hat{\psi}(m-l,n-p)$$

If it is supposed that f depends only on the direction y of the Aladin model in grid points (and thus of the meridional wave number n in the spectral representation), one has :

$$f(x,y) = \sum_p \hat{f}(0,p)e^{2\pi i p y} \tag{3}$$

$$\hat{\phi}(m,n) = \sum_p \hat{f}(0,p)\hat{\psi}(m,n-p) \tag{4}$$

Within this framework, Loïk Berre had made beforehand a study of truncation of f and had found that a truncated representation of the Coriolis parameter around T10 and T20 is a good approximation of f over the NORAF domain (figure 1). He also checked that truncation can be reduced even more when one increases the size of the extension zone : thus a truncation of T05 is enough when the extension zone accounts for example for 25% of the field (figure 2), instead of 6% in figure 1.

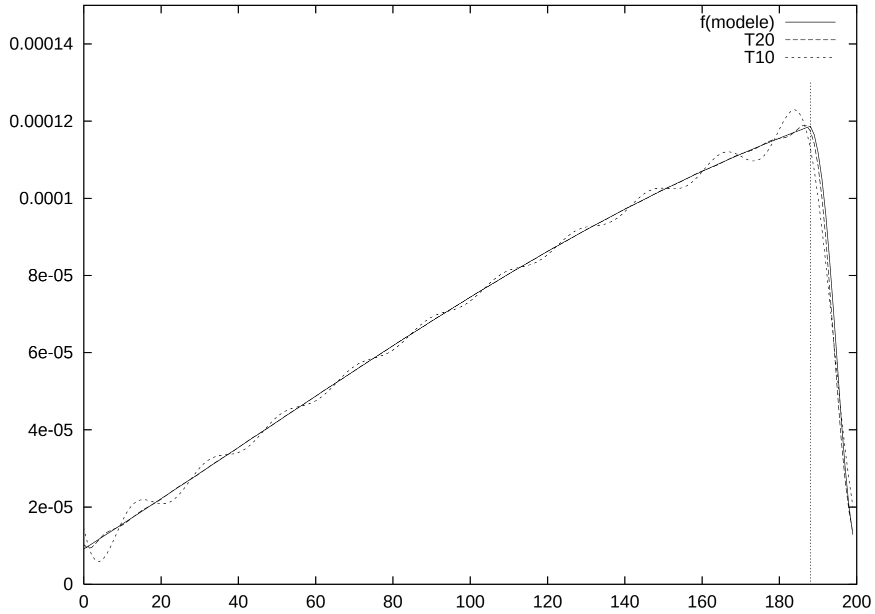

Figure 1: Representation of the Coriolis parameter with a truncation T10 and T20 and comparison with f resulting from the model.

3 Coding in the software of covariance calculation ("festat") and diagnosis of the explained variance

We were interested in the study of the β-plane on the level of error statistics on the Albachir-NORAF domain.
To test the contribution of the β-plane approximation in the horizontal balance equation, one plotted α, the variance percentage of linearized geopotential ϕ, which is explained by the horizontal balance equation used in this study.

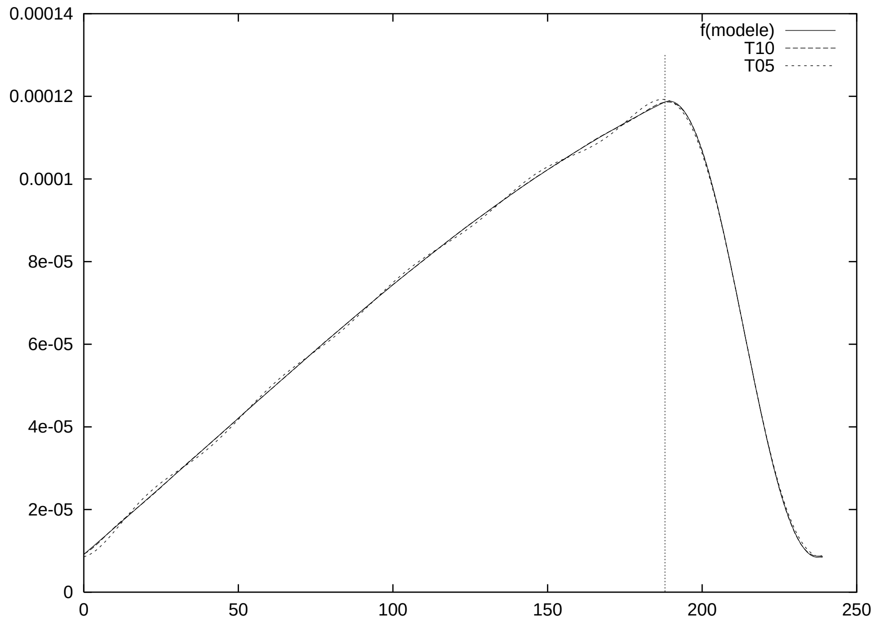

Figure 2: Truncation can be reduced even more when the size of the extension zone E increases.

We plotted the parameter α (explained variance) in the case of the f-plane and the β-plane T15 (3). We note that the explained variance clearly increased while passing from the f-plane (maximum around 43%) to the β-plane (maximum around 54%) ; this is a first validation of the approach of the β-plane in the horizontal balance equation. We were then interested in coding in the 3D-Var Aladin.

4 Experiments with an observation in two different latitudes

The goal of this experiment is to check that the variations of the temperature and wind increments are coherent with the effects of f on the guess error statistics. One varies the position of an observation, while keeping constant the innovation amplitude. The increment equations (Berre 2001), which we use to validate our results, are detailed in the following section.

4.1 Increment equations and variations awaited

Impact of the β-plane on the guess error statistics of ϕ :

$$\phi = f\psi + \phi_u \ ^1$$

$$\sigma_\phi{}^2 = (f\sigma_\psi)^2 + \sigma_{\phi_u}{}^2 \quad (cov(f\psi, \phi_u) = 0)$$

$$cov(\phi, \psi) = f\sigma_\psi{}^2$$

[1]This equation is a simplified version of the coded exact equation, which is : $\phi{=}\gamma\ \Delta^{-1}(f\zeta){+}\phi_u$

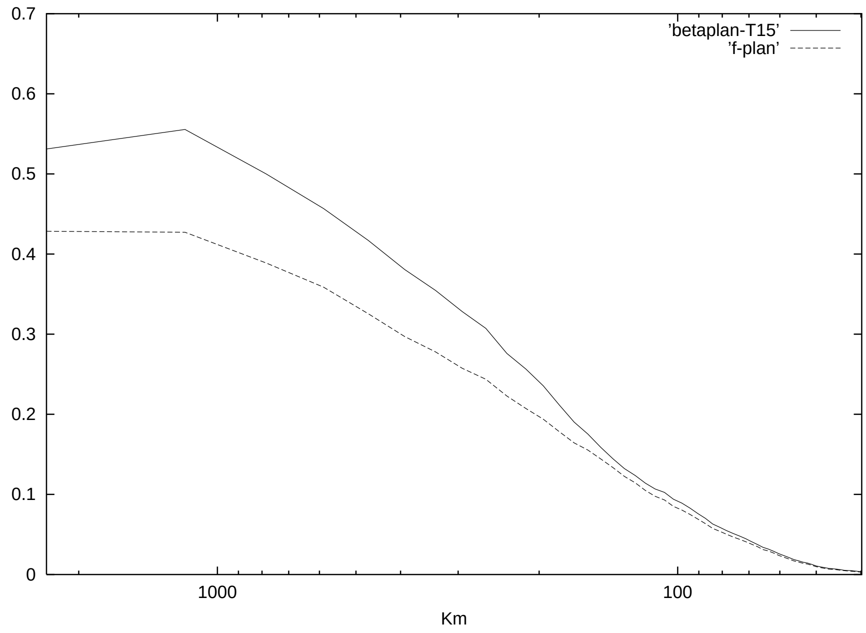

Figure 3: Percentage of variance of the geopotential one which is explained by the horizontal balance, in the case of the f-plane and the β-plane T15.

$$cor(\phi,\psi) = \frac{1}{1+(\frac{\sigma_{\phi u}}{f\sigma_\psi})^2} \tag{5}$$

σ_ϕ, $cov(\phi,\psi)$ and $cor(\phi,\psi)$ are respectively the standard deviation of ϕ, the cross-covariance between ϕ and ψ, and the cross-correlation between ϕ and ψ. These quantities decrease when the latitude gets lower. [2]

Impact of an observation of ϕ over ϕ :

$$d\phi = \frac{1}{1+(\frac{\sigma^o_\phi}{\sigma^b_\phi})^2}\delta\phi \tag{6}$$

$$= \frac{1}{1+\frac{(\sigma^o_\phi)^2}{(\mathrm{f}\sigma_\psi)^2+{\sigma_{\phi u}}^2}}\delta\phi \tag{7}$$

- $d\phi$: increment value ;
- $\delta\phi$: innovation value ;
- σ^o_ϕ : standard deviation of the observation error ;
- σ^b_ϕ : standard deviation of the guess error.

[2] σ_ψ and σ_{ϕ_u} are constants (assumption of horizontal homogeneity).

Impact of an observation of ϕ over ψ :

$$d\psi = \frac{cov(\phi,\psi)}{{\sigma_\phi}^2}\, d\phi \tag{8}$$

$$= \frac{f{\sigma_\psi}^2}{f^2{\sigma_\psi}^2 + {\sigma_{\phi_u}}^2} d\phi \tag{9}$$

$$= \underbrace{\frac{1}{1+(\frac{\sigma_{\phi_u}}{f\sigma_\psi})^2}}_{1\,\searrow}\ \underbrace{\frac{1}{f}}_{2\,\nearrow}\ \underbrace{d\phi}_{3\,\searrow} \tag{10}$$

The product of terms 1 and 2 increases slightly when the latitude drops (this result is valid in the mid-latitudes, whereas it is the reverse in the tropics) (Berre 2001).
Term 3 ($d\phi$) decreases when the latitude drops.
The product of terms 1, 2 and 3 should decrease when the latitude drops, because the variations of term 3 have the largest effect in the mid-latitudes.

Impact of an observation of ψ over ψ

$$d\psi = \frac{1}{1+(\frac{\sigma^o_\psi}{\sigma_\psi})^2}\delta\psi \tag{11}$$

As σ^o_ψ and σ_ψ are constants, $d\psi$ should remain constant.

Impact of an observation of ψ over ϕ

$$d\phi = \frac{cov(\phi,\psi)}{{\sigma_\psi}^2}\, d\psi \tag{12}$$

$$= \frac{f{\sigma_\psi}^2}{{\sigma_\psi}^2} d\psi \tag{13}$$

$$= f\ d\psi \tag{14}$$

As $d\psi$ remains constant, $d\phi$ should decrease when f decreases.

The two selected positions are :
- in the Channel (Lat = 50 ° , Lon = 0 °).
- in Spain (Lat = 40 ° , Lon = 0 °).

4.2 Observation of temperature

For the validation of our results using the preceding equations of increments, one considers that the increments of temperature T vary in a way similar to those of geopotential ϕ (T and ϕ are connected by the hydrostatic relation), and the variations of the increments of the wind $\vec{u}$ are close to those of the incremens of ψ (knowing that $\vec{u} \approx \vec{k} \times \vec{\nabla}\ \psi$ in the mid-troposphere of the mid-latitudes).

We will show here the results obtained when using a single observation of temperature. Similar consistent results were obtained when using a single observation of wind, in accordance with the increment equations of the previous subsection.

4.2.1 Temperature increment

As the equation (7) suggests, one notices that the amplitude of the increment of the temperature drops towards the low latitudes : the increment in the Channel has a larger amplitude from approximately 10% than the increment in Spain (figures 5 and 4).

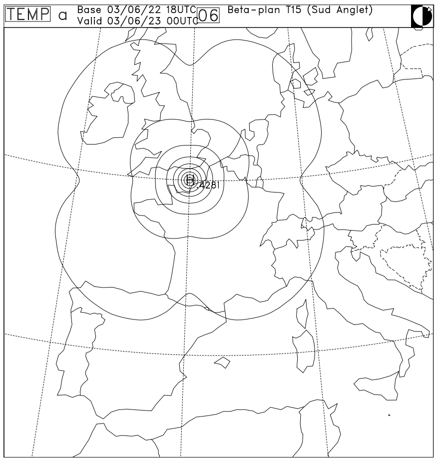

Figure 4: Temperature increment : Channel

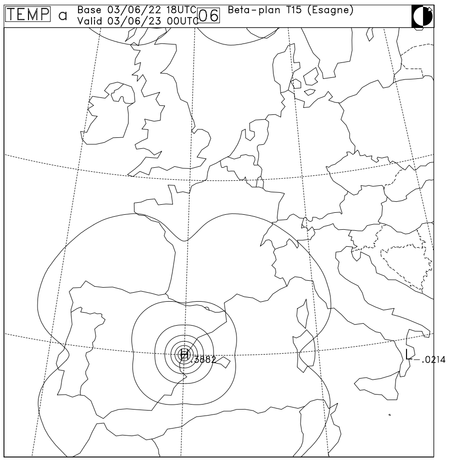

Figure 5: Temperature increment : Spain

4.2.2 Wind increment

As the equation (10) suggests, the amplitude of the meridional wind increment drops towards the low latitudes. The variation is approximately 10% between the two positions of observations.

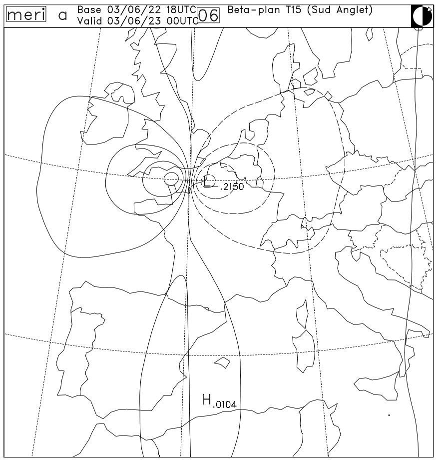

Figure 6: Meridional wind Increment : Channel

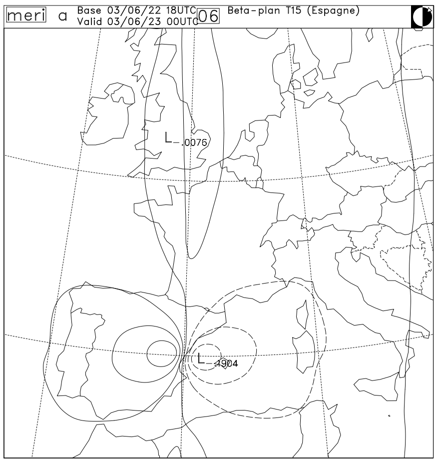

Figure 7: Meridional wind increment : Espagne

5 Conclusions and perspectives

A beta-plane balance has been developed and coded for the Jb term of the Aladin 3D-Var and for the associated software of error covariance calculations.

The formulation is based on a truncated spectral expansion of the meridional variations of the Coriolis parameter. It can be seen as a multi-diagonal approach, in contrast with the purely diagonal approach of the f-plane balance.

This approach was first validated by examining, over the Aladin NORAF domain, the increase of explained variance by the beta-plane balanced geopotential, compared with the f-plane balanced geopotential.

The formulation was then coded in the Aladin 3D-Var, and it was validated by using in particular some single observation experiments.

A natural prolongation of this work would be to develop and test other new formulations such as the non linear and omega balances (Fisher 2003).

6 References

Berre, L., 2001 : Représentation des covariances spatiales des erreurs de prévision pour une assimilation variationnelle dans un modèle atmosphérique à aire limitée. PhD Thesis of Paul Sabatier University, Toulouse, France, 250 pp.

Fisher, M., 2003 : Background error covariance modelling. ECMWF seminar on recent developments in data assimilation for atmosphere and ocean, 8-12 September 2003, 45-63.