# Limitations of projected limited-area spectral models for large-domains

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# 1. Abstract

A particular attention is given to operational limited-area models (LAM) in order to make detailed weather forecast (high resolution) over areas of interest. High resolution is required to adequately simulate mesoscale processes which affect the atmosphere's evolution. To this purpose, Météo-France has developed, in cooperation with other European partners, a LAM named ALADIN using classical conformal projections (Lambert, Mercator) which enable to represent any part of the sphere on a plane. Furthermore, the space-discretization of ALADIN is performed by a spectral bi-Fourier decomposition over the horizontal, and finite differences over the vertical. The time discretization is semi-implicit (SI), which implies the resolution of an Helmholtz equation at each time-step.

Nevertheless one of the main issue concerning LAM forecast reliability is the negative impact of the boundary-induced error. Previous researches have shown that the computational error generated by the over-specification of the lateral boundary conditions (LBC) can propagate inward into the domain and contaminate at the centre the forecast over the area of interest. In order to reduce the shortcomings of forcing by LBC, a pragmatical way would be to extend the geographical domain of the LAM, with an increase of computational power. But even if this latter approach seems to be intuitively easy to understand, in practice there is no guarantee that it could be done for projected spectral regional model like ALADIN. The feasibility (or not) of the extension of ALADIN domains has to be demonstrated.

# 2. Problematics

It is obvious that the limitations of ALADIN strategy for large domains involve the link between the spectral method, the semi-implicit time-stepping and the horizontal projection used in the model. As mentioned above, ALADIN runs with projection on a map. Any of the projections used is characterized by its map factor, denoted $m$ and defined as the ratio between the distance on the map and the associated distance on the sphere at a considered location. The map factor varies over the horizontal.

As the spectral part of the model computations handles map variables and horizontal derivatives, the map factor has to be taken into account in the SI scheme. In order to keep a diagonal Helmholtz operator in the spectral space, and ease the resolution of the implicit problem, the map-factor $m$ is replaced in the SI formulation by its maximum value over the domain (denoted $m^*$).

This restrictive simplification appears to be legitimate for usual projected LAMs, for which $m$ remains close to one. But the larger the domain is, the larger the map factor values becomes. Therefore, it is highly likely that this helpful simplification shall not any longer be reliable for very large domains. For the global stretched ARPEGE configuration where the map factor reaches large values, Yessad and Bénard (1996) have shown that this simplification cannot be applied since it leads to dramatic computational instabilities, especially for long time-steps within the semi-Lagrangian (SL) advection scheme. Therefore, the question to be answered is if such instabilities can occur when the ALADIN domain is extended. What we formally know is that the incorrect treatment of the map factor in the SI scheme introduces additional nonlinear (NL) residual terms explicitly treated which could jeopardize the stability of the scheme when the LAM domain is extended.

The aim of this study is to examine more in detail the limitations of projected limited area spectral models for large domains. Thanks to the ARPEGE/ALADIN system (hydrostatic primitive equation (HPE) and Euler equations (EE) non-hydrostatic versions), we have investigated the response of the three-time-levels (3TL) SISL scheme with respect to the domain extension; more precisely we focus on the impact of an increase of $m^*$ in the SI scheme.

# 3. Hydrostatic Primitive Equation (HPE) case : ALADIN-H

In the case of the HPE system, a preliminary experimental study has shown that an unstable

behaviour of the SI scheme occurs with respect to the increase of $m^*$, exclusively when the orographic forcing interacts with the nonlinear residual terms associated with the map factor. At first sight, the physical processes don't seem to be of primary importance in such an observed behaviour. A more detailed stability analysis, in presence of a simple orography consisting in a "uniform slope mountain", has shown that the increase of the map factor reinforces the destabilizing effects of the leading nonlinear orographic source on the stability of the SI scheme. Even if for $m < 2$ the resulting instabilities appear to be too weak to endanger the current NWP application, our results indicate that some care has to be taken for very large domains, especially over mountainous regions.

### 4. Euler Equation (EE) case: ALADIN-NH

Conversely to what occurs with the HPE system, the EE system shows a larger sensitivity of the 3TL SISL scheme stability to an increase of the map factor (to $m^*$), and in consequence, to the domain extension. In ALADIN-NH, orographic forcing is no more strictly required to trigger the instability. The nature of this instability is only intuitively understood, but thanks to a numerical stability analysis together with the use of an alternative prognostic variable for vertical divergence, can give a better understanding of this phenomenon. This new variable is denoted $d'$ and is defined as the ratio between the actual vertical divergence $d$ and the square of the map-factor : $d' = d \, / \, m^2$ .

The use of $d'$ instead of $d$ in the spectral part of the computations leads to a substantial stabilisation of the SI scheme. This stabilisation seems to be good enough to be able to run ALADIN-NH over larger domains than at present.

### 5. A new strategy for projected limited area spectral model

As regards the previous results, the approach which consists in replacing the local map factor $m$ by its maximum value $m^*$ in the SI scheme is irrelevant for large-domains in HPE and especially EE models. As a consequence some improvements have to be made to better take into account the map factor in the SI. To that effect, we have shown that the rotated Mercator projection offers some possibilities to improve the treatment of the map factor in the SI, and that the rotated Mercator projection allows the feasibility of the extension of ALADIN domains (with a very good degree of confidence), even for the Euler equations system.

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