First results on compactly-supported background error covariances in ALADIN

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1. Introduction

Due to biperiodisation and to the length-scales of the structure functions, some problems may occur when using observations near the border of the C+I domain (cf. previous Newsletter).

2. Principle of compactly-supported covariances

Let ENIL1 and ENIL2 be the distance for starting the modification of the covariances and the distance of effective zeroing, respectively. Let *mask* be the mask defined by :

$$mask(x) = \begin{cases} 1 & \text{if } x \le \text{ENIL1} \\ \frac{1}{2}\left(1 + \cos\left(\pi \frac{x - \text{ENIL1}}{\text{ENIL2} - \text{ENIL1}}\right)\right) & \text{if } \text{ENIL1} \le x \le \text{ENIL2} \\ 0 & \text{if } \text{ENIL2} \le x \end{cases}$$

To obtain compactly-supported ("COSU") autocorrelations, one has to apply this mask in the gridpoint space :

$$q_{\text{cosu}}(x, y) = q(x, y) \times mask(\sqrt{(x^2+y^2)}),$$

which is the exact formula if the observation is located at (0, 0). This multiplication corresponds to a convolution in the spectral space :

$$\text{F}(q_{\text{cosu}})(m, n) = \left(\text{F}(q)*\text{F}(mask)\right)(m, n)$$

According to Gaspari and Cohn (1999), this mask should be applied to the square root of the gridpoint correlations. Therefore, the autocorrelations won't be exactly zero from ENIL2, but from some distance between ENIL2 and 2×ENIL2. Here is the method used in this study, which has been proposed by Loïk Berre :

1. convert the power spectrum into modal variances
2. calculate the square roots
3. fill a 2D spectral array from the 1D square roots of the modal variances
4. convert into gridpoint structure (inverse bi-Fourier transform)
5. mask the gridpoint structure
6. convert back to spectral 2D array (direct bi-Fourier transform)
7. collect isotropically
8. square to obtain modified modal covariances
9. convert into power spectrum

3. 1D model

The 1D model used in this study is a gridpoint model, with 289 gridpoints in the C+I domain and a 11-gridpoints wide E-zone, only used to perform an analysis. It is a uni-variable (so univariate) model. Everything is done in gridpoint space. The formula used for the analysis is :

$$x_a = x_b + B\,H^{\text{T}}\,(H\,B\,H^{\text{T}} + R)^{-1}\,(y_0 - H\,x_b)$$

where x_a is the analysed state, x_b the initial state, H the observation operator, B and R the background and observation error covariances matrices. The observations are given in exact gridpoint coordinates. *H* only contains 0 and 1 (to select the gridpoints to be compared). *B* is obtained from the actual ALADIN lagged J_b power spectrum of the variable to analyse.

3.1 Modification of the length of the E-zone

The 1D model provides an opportunity to evaluate the impact of an enhancement of the length of the extension zone. In order to mimic such a modification, with constant C+I zone, we have only to modify the horizontal autocorrelations. It has been decided to extrapolate the missing values from the original gridpoint variances. The extrapolated values are all equal to each other and are continuous with the original values.

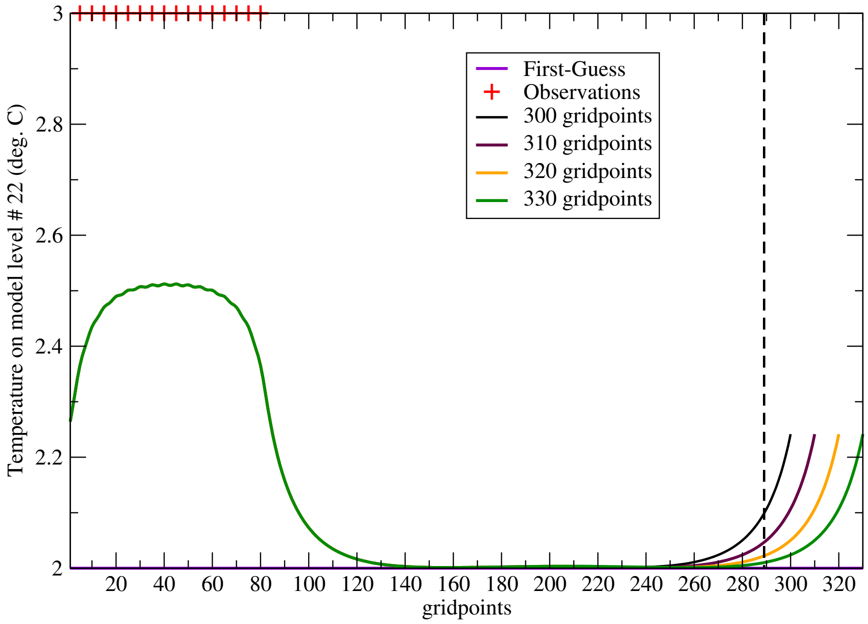


Figure 1 : Observations, first-guess and analysis for various lengths of the E-zone, for temperature on model level # 22.

Figure 1 shows that there is no modification in the neighbourhood of the observations. But the value of the analysis (and the value of the analysis increment) at the border of C+I and E-zones is not the same. The analysis increment at the border can be reduced to 22.5 % of its initial value when using a three times bigger E-zone. Even if it was quite obvious, this is a really positive cure to the "wrap-around" effect due to the biperiodisation.

3.2 Compact support

From now, "no COSU" refers to the original power spectrum, and "COSU xx-yy" refers to compactly-supported autocorrelations with ENIL1=xx and ENIL2=yy (xx and yy are gridpoint values; remember that ENIL2 is the distance of zeroing for the square root of the autocorrelations).

The modified power spectra are obtained following the above-mentioned method. The impact on the power spectrum, for various values of ENIL1 and ENIL2 (and various combinations), is shown on Fig. 2a. In a global overview, since the autocorrelations are compactly supported, the values of the power spectrum for the 3 first total wavenumbers are decreased. There is hardly no change of the power spectrum for total wavenumbers ranging from 40 to 140. Some oscillations appear when ENIL1 and ENIL2-ENIL1 are too small. The tuning "COSU 10-20" is to be rejected, for instance.

To observe the real impact on the autocorrelations in gridpoint space, the power spectra previously generated are converted into gridpoint structures. Figure 2b shows these gridpoint structures for the reference ("no COSU") and various tunings of compact support. First, with a zoom (not shown), one could notice that compactly-supported autocorrelations are not exactly zero. It is due to not totally symmetric steps (direct and inverse Fourier transforms, and fill-in of the ellipse and collect).

But the values for the COSU autocorrelations are quite satisfying : for a distance greater than 50 gridpoints, values are less than 2.10^{-4}, that is to say less than $1/5000^{th}$ of the maximum value. The general impact is as expected.

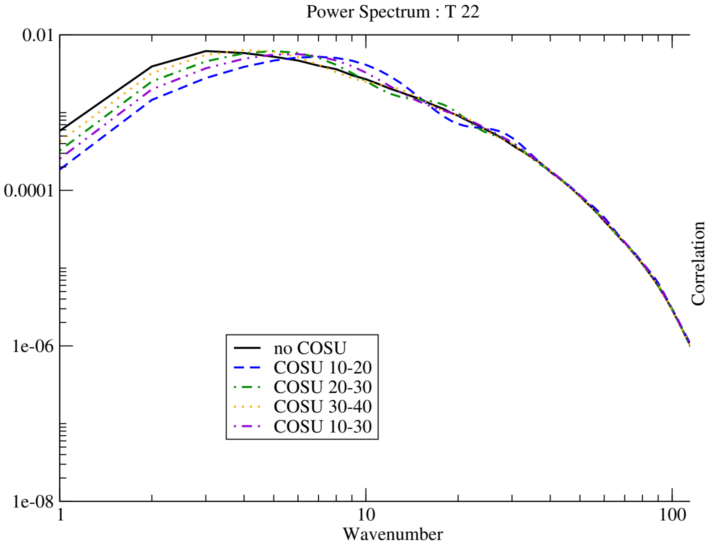


Figure 2a : COSU power spectra for temperature on model level # 22

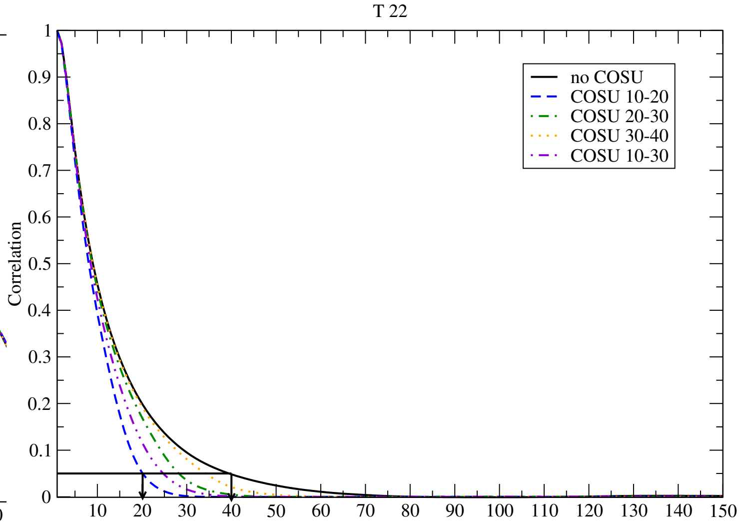


Figure 2b : COSU gridpoint autocorrelations for temperature on model level # 22.

The decrease of the length-scale obtained for the gridpoint autocorrelations is confirmed in the analysis of 15 observations (cf. Fig. 3). The shape of the analysis increment is slightly modified. The values of the analysis increment in the area containing no observations, and far enough from the observations, are efficiently reduced thanks to the compact support. This method also offers a cure to the "wrap-around" effect. The value of the analysis increment at the border between C+I and E zones is 4.5 times smaller in the COSU experiment than in the reference (which is equivalent to the results obtained with the modification of the E-zone length).

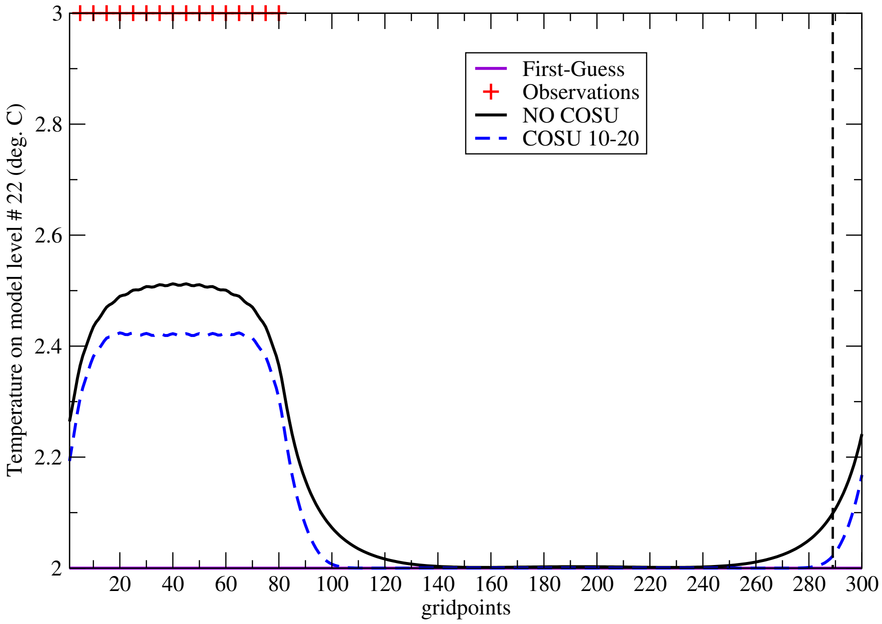


Figure 3 : Observations, first-guess and analysis for COSU and non COSU covariances, for temperature on model level # 22.

4. Implementation in ALADIN

Following both the implementation of compactly-supported horizontal correlations in ARPEGE (cf. routine SUJBCOSU written by François Bouttier) and the first results of COSU horizontal autocorrelations, the SUEJBCOSU routine has been implemented in ALADIN, with the great help of Claude Fischer. Its purpose is to compactly support the horizontal correlations and the vertical cross-correlations (and also the horizontal balance).

4.1 Univariate approach

The univariate case is the closest to what was done in the 1D model. The compact support has only to be applied to horizontal autocorrelations. COSU horizontal autocorrelations imply a damping of residual noise farther than a given distance (between ENIL2 and 2×ENIL2). Some geometric noise still remains (cross centred on the observation, plus a rhombus). But the results are quite the same as in the 1D model and encourage us to run a multivariate 3D-VAR with COSU horizontal autocorrelations.

4.2 Multivariate approach

The multivariate formulation used in this study is based on the work of Loïk Berre (2000). In this section, single observation (of temperature at 500 hPa) experiments are performed and compared through their temperature analysis increment on model level # 15.

<u>Naive first experiments</u>

In this paragraph, only the horizontal autocorrelations are compactly supported. Neither vertical cross-correlations nor balance operators are modified. Some astonishing results are obtained. Though quite few benefits (even neutral results) were expected, "worse" patterns are generated. These results remain unchanged whatever the values of ENIL1 and ENIL2. An explanation could be that the main part of the temperature increment is balanced, while only the vorticity (ζ) horizontal correlations are compactly supported, but not $H_b\zeta$, where H_b is the horizontal balance operator.

<u>Digging a bit deeper</u>

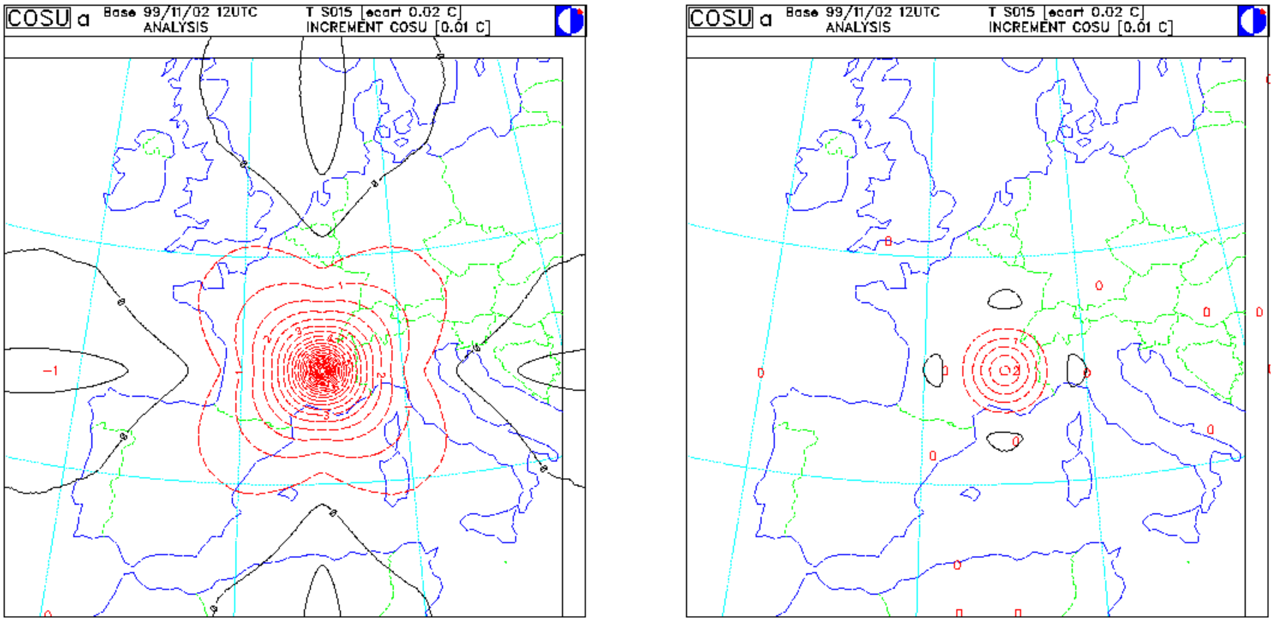
If we consider that the statistical inverse Laplacian H_b and the analytical inverse Laplacian Δ^{-1} are equivalent, $H_b\zeta$ is equivalent to $\Delta^{-1}\zeta$, that is to say the streamfunction. The power spectrum of the vorticity can be modified to obtain COSU horizontal correlations for the streamfunction. Compactly supporting the streamfunction gives neutral results, but it allows to eliminate the "worse and weird" increments. Moreover, using COSU vertical cross-correlations additionally leads to the same results (that is a mostly neutral impact).

<u>Drastic measures</u>

Another point of view, a bit more drastic, is to consider the horizontal balance as an operator which can be compactly supported. The compact support is first applied with "short" ENIL1 and ENIL2 (Fig. 4b, to be compared to the reference, Fig. 4a). This method is really efficient in controlling the length-scale of the increment. Note that COSU horizontal balance is an "antidote" to COSU vorticity horizontal correlations. Other values for ENIL1 and ENIL2 have been tested (not shown). It seems that the length-scale of the horizontal balance is the leading one, as the shape of the increment seems to depend only on what is applied to the horizontal balance. Some experiments using different distances of zeroing for correlations and horizontal balance have been performed. They reinforce the feeling that the horizontal balance is the most important element to be modified to obtain COSU analysis increments.

4.3 Towards operational applications

All these single observation experiments are only a step towards the use of the SUEJBCOSU routine with real observations on real cases. That is why preliminary tests of compactly-supported structure functions are performed (not shown). First, a band of observations (all observation types) over a southern third of the domain is considered. There are only few changes in comparison to the reference, but the "wrap-around" effect is a bit reduced. As a second step, a 3D-VAR analysis is performed with all observations (i.e. as "usual"). There is *no* modification, despite very short distances of zeroing. This has to be further investigated.



(a) (b)

Figure 4 : Temperature on model level # 15, analysis increment of a single temperature observation at 500 hPa (units : 0.01 K).
(a) reference, with original B; (b) all COSU 10-30 (horizontal correlations, horizontal balance operator).

5. Conclusion

Having a wide enough E-zone is important if all observations inside the C+I domain are used : it prevents the analysis increment from "wrapping around". But, one should be aware of the over-costs generated by a drastic enhancement of the E-zone. In the case of ALADIN with a 289-gridpoints wide square C+I domain, a 320-gridpoints (or more) wide square C+I+E domain is recommended.

To control the length-scale of the increment, compactly-supported horizontal correlations can be used (background statistics). In the univariate case, this is sufficient to have a real control. But in the multivariate case, it appears that, to obtain similar results, the horizontal balance operator has to be compactly supported too. As the distance of zeroing is tunable, one can experiment different values to reach a "realistic" limit. To keep in mind the theoretical benefit of COSU structure functions : for temperature on model level # 22, the distance from which the horizontal correlation is less than 0.05 is 400 km for the original B and *only* 250 km for the COSU 10-30 experiment.

References

Berre, L. (2000). Estimation of synoptic and mesoscale forecast error covariances in a limited-area model. *Mon. Wea. Rev.*, **128**, 644-667.

Gaspari, G. and S. Cohn (1999). Construction of correlation functions in two and three dimensions. *Quart. J. Roy. Meteor. Soc.*, **125**, 723-757.