4. Martin GERA : "Improved representation of boundary layer"

To solve the Turbulent Kinetic Energy (TKE) equation one needs to know expressions for momentum, heat, turbulent kinetic energy fluxes, diffusion, dissipation of TKE and pressure correlation terms. Fluxes dependence of momentum and buoyancy terms (exchange coefficients) are expressed as functions of TKE :

$$K_x = cL_k E^{1/2}$$

where K_x is the exchange coefficient, c a parameterized constant, L_k the corresponding mixing length and E the TKE.

Shear-production, buoyant and dissipation terms are sources of this energy. Vertical diffusion (Di) is parameterized using the "eddy diffusivity" assumption, one can write the expression :

$$D_i \sim -c_d K_u \left(\frac{dE}{dz} \right)$$

where c_d is a parameterized constant, K_u the exchange coefficient for momentum and the last term is a vertical derivation of TKE.

To begin, we focus our attention on the pressure-correlation terms. The pressure-correlation terms include a transport term and an energy-redistribution term. This last one is important for the smoothness of energy fields. This means, that this term returns anisotropy of turbulence to the isotropic state. For finding an optimal parameterization of this term we need to know an estimation of magnitude and physical interactions for this covariance. The expression of pressure covariance has significant influence on the properties of parameterized equations. It is important to find a proper parameterization for this term.

The oldest methods of parameterization make this term proportional to the anisotropy component of momentum covariance. The newest approach, in addition tries to include the interaction between the velocity fluctuations and mean strain field. The way to find estimation is to subtract the isotropic components from the TKE equation. The anisotropy part contains a term which produces the TKE and also terms which work against the turbulence (pressure covariance, dissipation, ...). From measurements it seems that the three normal stress tensor components are comparable when mean stability in the Planetary Boundary Layer (PBL) decreases (Manton, 1979). We say that isotropy of velocity fluctuations can be assumed in this situation. Our next assumption is that the buoyant term does not contribute significantly to the anisotropy of the turbulence. We hope that this knowledge is sufficient to estimate the pressure return to the isotropy term. We did a comparison of these parameterization methods.

Nowadays, in limited-area models (LAMs), the subgrid effects are important. In models with high resolution the subgrid effects are less important. For example, for estimation of the turbulent fluxes it is sufficient assume that the mixing length equals the grid mesh. For LAMs one must find an expression for the mixing length. In addition, as the observations confirm, we must recognize the mixing and dissipative lengths (L_k and L_e respectively).

The comparison of the parameterization methods of the pressure-correlation term was done by evaluating the formulation of the ratio between mixing and dissipative lengths. This ratio has a close connection to the ratio of vertical perturbation velocity variance and TKE. We analysed properties of this ratio with dependence on the Richardson number (Ri). This evaluation was done with the aim of stationary approach in prognostic equations for heat and momentum fluxes and using parameterized formulas for fluxes. Stationary, here, means that shear, buoyant and dissipation terms are balanced in TKE equation. We started by analysing this ratio, because it contains no dependence to the dimensionless wind shear function. We suppose that L_k and L_e have same dependence to the dimensionless wind-shear function. This is an advantage of this approach.

The oldest scheme has had smaller values for stables and unstable case. The newest scheme, which is preferable in current numerical models, defines that ratio as a function of stationary TKE (E) :

$$\frac{L_k}{L_e} \sim f(Ri)$$

$$E \sim L_k L_e E^{\#} f(Ri)$$

where $E^{\#}$ is a neutral TKE.

The variations of this ratio are similar in both schemes :

- unstable case : $Ri < 0$, $L_k > L_e$
- neutral case : $Ri = 0$, $L_k = L_e$
- stable case : $Ri > 0$ $L_k < L_e$

After manipulation with parameterized equations one can refine formulas for this ratio. We can find not only dependence on the air stratification but also introduce dependence on the divergence and vertical shear of horizontal wind ($\vec{V}$) :

$$\frac{L_k}{L_e} \sim f(Ri)\left(\frac{\nabla.\vec{V}}{\left|\frac{\partial \vec{V}}{\partial z}\right|}\right)^2$$

This result is keeping the consistency for definition of momentum fluxes. We may not forget, that all results follow chosen parameterization. Phenomena are described with limits of application of our assumption.

There is close relation between TKE (E) and friction velocity (u^*). In simplest case, when homogeneous turbulence and neutral air stratification are assumed, we can write that E is proportional to the square of friction velocity , E ~ a u^{*2}.

Generally, dependence is more complicated especially in unstable case. TKE is represented with diagonal part of Reynolds stress-tensor (isotropic part) and anisotropy of turbulence is usually more significant in vertical direction. Cascading of eddies to the viscous dissipation is different in horizontal and vertical directions. One can write

$$E \sim \alpha S(Ri) u^{*2}\left(\frac{L_e}{L_k} f(Ri)\right)^{1/2}$$

where $S(Ri)$ is a stability function.

Analysing the ratio of mixing to dissipative lengths shows that this parameterization presents the simplest relation which is close to the homogeneous and the neutral air stratify turbulence. For neutral and stable case expression ($E \sim a\ u^{*2}$) gives a good approximation of the relationship between energy and surface velocity. It is no surprise when we remember the manner of derivation of the covariances. From this it seems that stable situations are described more correctly than unstable cases with the present parameterization. For unstable cases is necessary to introduce some stability function $S(Ri)$ to have a correct result or introduce this contribution to the ratio (L_k / L_e).

One possible way is to estimate dissipation of the energy *(DE)* in the same manner as TKE. We obtain similar results as for TKE, but with different coefficients (we start from stationary equilibrium for the dissipation term).

$$DE = DE^{\#} g(Ri)$$

For this approach one has:

$$S(Ri) \sim \left(\frac{f(Ri)}{g(Ri)}\right)^{3/2}.$$

Other possibility to attenuate this disadvantage is to import features for the mixing and dissipative lengths from observations. However measurements have limitation too. Problems are to determine the mixing length during convective situations with weak vertical gradient ... On the next, the results show weak dependence of presented scheme to the convective situation. On the other hand, there is no reason to describe convective processes in turbulent scheme. This assertion supports, for example, Redelsperger (2001) results.

Before solving the prognostic TKE we did a deep analysis of the proposed parameterization. Again we use the support of stationary theory. This approach allows us to estimate the magnitude of fluxes and the suitability of coefficients settings. We find formulas for surface velocity, scaled potential temperature, eventually expression for vertical heat-flux. From these expressions we can get an estimation of system properties. It allow us to obtain a critical Richardson flux number (Ri_{fc}.), defined as:

$$Ri_{fc} = Ri\left(\frac{K_h}{K_m}\right)$$

where K_h and K_m are the exchange coefficients for heat and momentum respectively.

From analysing the series properties for $Ri \to \infty$ one gets

$$Ri_{fc} = c_2 Ri\varphi(Ri) = 1\text{-}f(Ri) = \left(\frac{c_2}{c_2 + c_3}\right)$$

where c_2 , c_3 are parameterized constants and $\varphi(Ri)$ is a stationary approximation of stability function from heat flux. Important is to say that this stability function has a little bit different expression when we compare results with other articles. Our results is the same as in Sommeria (1976), but differ by factor 2 in denominator with Cuxart (2000) and others.

This has an influence on the setting of parameterized coefficients. Coefficients setting can be done with help of measurements of fluxes. Concerning the setting of $R=K_m/K_h$, R=0.74, (0.95) in neutral stratification of air, keeping similarity with Monin-Obukhov theory, one obtains a condition for parameterized coefficients c_m, c from $1/c^2=R$, because $c^2=5c_m/(2c)$. Analogically, we can set a condition for c_0 and c_{ep} . We try to estimate this value from formula for Ri_{fc}. From measurement we expected, that the critical Richardson flux number Ri_{fc}. should be about 0.14. On the next, relation between TKE and momentum flux in neutral conditions is prescribed from measurements. It tell us that E/u^{*2} ranges from 3.75 to 5.47. These coefficient relations are coming from the choice of flux parameterizations. From these conditions we have still one parameter free. We set parameter c_m to c_m=3.115, (4.13). Constants c, c_m come from the pressure-correlation term, c=5.768, (7.648). Constants c_{ep}, c_0 come from dissipation parameterization, c_{ep}=0.831, (1.101) and c_0=0.271, (0.359). The setting of coefficients is very important for system "behaviour". The estimated values of constant correspond to the observations. In this way we adjust the shape of stratify functions $f(Ri)$ and $\varphi(Ri)$.

New formulation of fluxes (dependence to TKE) in different atmospheric stability conditions yields the following exchange-coefficient property for big Ri.

$$K_m \sim L_k{}^2\left(\frac{L_e}{L_k}\right)^{1/2} f^{1/2}(Ri) \sim L_k{}^2\left(\frac{L_e}{L_k}\right)^{1/2}\left(\text{constant} + Ri^{-1/2}\right)$$

$$K_h \sim L_k{}^2\left(\frac{L_e}{L_k}\right)^{1/2} c_2 f^{1/2}(Ri)\varphi(Ri) \sim K_m \frac{c_2}{(c_3 + c_2)Ri} \sim L_k{}^2\left(\frac{L_e}{L_k}\right)^{1/2} Ri^{-3/2}$$

These features allow us to express an estimation of the time-scale of eddies overturning

$$\tau = \frac{E}{DE} = S(Ri)\alpha\left(\frac{L_e}{L_k f(Ri)}\right)^{1/2}\left|\frac{\partial \vec{V}}{\partial z}\right|^{-1}.$$

We know that for closing the non-stationary system of parameterized equations we need to set a dependence of mixing and dissipative lengths on the air stratification. For computation of fluxes it

is necessary to add two parameters from outside in our case. With help of stationary equilibrium we eliminate this task to set only one parameter, for example mixing length from measurement. During the parameterized system analysis we eliminate this one unknown by analysing the ratio L_k / L_e.
We need to underline that this stationary simplification is interested for estimation of magnitude of parameterized terms and not for integration, because increment TKE in stationary case is zero.
In first step to do integration of TKE for simplicity we take the expression for L_k from the current ALADIN model and we apply ratio result $L_k=L_e.f(Ri)$. On the next we will improve ratio result by function $S(Ri)$ or we will take Redelsperger results (Redelsperger, 2001) for L_k and L_e, where blocked energy nearest the surface will be considered.

To describe the" history" of turbulence it is necessary to solve prognostic TKE equation. We had the idea to benefit knowledge from stationary balance for finding a more stable scheme for integration of TKE equation. The subtracting balanced energy from unbalanced system from equation terms trigger new difficulties. TKE as nonlinear equation contains new interaction terms with unbalanced and balanced energy, what in final point of view complicated solution.
For this reason we decided to solve TKE equation directly for full energy without subtraction (exactly say for square root of TKE). During the solution we can now get a better estimated range of processes for the knowledge acquired from stationary balance. In TKE equation Redelsperger term (*RE*) in stability function $\varphi(RE)$ is simplified by using result from stationary approach, Redelsperger term is substituted with Richardson number in this case one has function $\varphi(Ri)$. Integration of nonlinear TKE (second order differential equation in space) is done by the Powell hybrid method (compute the QR factorization of the Jacobian). Initial vector of solution has been set by balanced TKE. Boundary values of TKE on the ground and at the top are set to the zero.

References :

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AOB : Workshop on boundary-layer clouds and consultation about progress on ALATNET project "Improved representation of boundary layer of model ALADIN".

The meeting was considered as productive. The most important problems of the current TKE were discussed. The main discussed topics wer e:

- current situation in models ALADIN, ARPEGE, ARPEGE-Climate and their 1D versions with TKE and implementation
- verification of current scheme and new scheme with TKE, at beginning with 1D ALADIN model on sensitive chosen situations, for stable, unstable profiles, for clouds developing in PBL and

the situations with wind shears; comparison of results on domains with cyclogenesis and inverse - stratified layer, concentration on antifibrillation scheme behaviour, (for example, test on EUROS frames data - shallow convection)

- advantages and disadvantages of TKE computation in half and full model levels

- surface or boundary value problem, anisotropy of turbulence

- integration of TKE equation, implicit or semi implicit scheme..., consistency, TKE as a new variable with other parts of model (dynamics, physics).

In addition I attended the Workshop on boundary-layer clouds, which was accessible at that time.