Big Root approximation of site-scale vegetation water uptake

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Martin Bouda
martin.bouda@ibot.cas.cz
Vegetation as a Land Surface Feature

Canyon of Vltava river at Sedlec
Vegetation as a Land Surface Feature

Canyon of Vltava river at Sedlec
Terrestrial models: soil water and root representation

\[
\Delta z_{i-1} \quad e_{i-1} & \Psi_{i-1}, \theta_{liq,i-1}, z_{i-1} \\
\Delta z_i \quad z_{h,i-1} & q_{i-1} & k[z_{h,i-1}] \\
\Delta z_{i+1} \quad z_{h,i} & k[z_{h,i}] & q_i \\
\Delta z_{i+1} \quad e_{i+1} & \Psi_{i+1}, \theta_{liq,i+1}, z_{i+1} \\
z \quad 0 & -0.5 & -1 & -1.5 & -2 & -2.5 & -3 \\
\text{C4 grass} & \text{C3 crop} & \text{Evergreen} & \text{Deciduous} \\
\text{Root Fraction} \quad 0 & 0.05 & 0.1 & 0.15 & 0.2 \\
\text{stress factor} \quad \beta = \frac{\psi - \psi_{min}}{\psi_{max} - \psi_{min}}
\]
Next generation

- Moisture gradient in soil, plant, atmosphere.
- Ohm’s law analogue.
- All root resistances assumed in parallel.
- Misrepresentation of root system architecture.
- Known problems, e.g. redistributes water too freely (Kennedy et al., 2019)

(Sperry et al., 2016, New Phytologist)
**Effects of Root System Architecture**

\[ Q_{plant}^{i \Rightarrow i-1} - Q_{plant}^{i+1 \Rightarrow i} = Q_{soil \Rightarrow plant}^i \]

\[ \frac{\Delta^2 \bar{\psi}_x}{\Delta z^2} = \frac{\bar{K}_{soil \Rightarrow plant}}{\bar{K}_z^{plant}} \left( \bar{\psi}_x - \psi_s \right) \]

- Flows accumulate upward.
- The more inflow in layer \( i \), the greater the difference in water potential across layers \( i \pm 1 \).
- RSA determines ratio between uptake and potential gradient dissipation.
Governing Equation

\[
\frac{d^2 \psi_x}{ds^2} = \frac{k_r}{K_x} (\psi_x - \psi_s)
\]

- $\psi_x$ (Pa) ‘xylem’ water potential
- $s$ (m) length along root
  (0 at tip, $S$ at base)
- $\psi_s$ (Pa) ‘soil’ water potential
- $k_r$ (m$^2$s$^{-1}$Pa$^{-1}$) soil-root ‘radial’ conductance in cross-section
- $K_x$ (m$^4$s$^{-1}$Pa$^{-1}$) xylem ‘axial’ conductance in cross-section
Segment mean water potential

Can find solutions that yield mean water potential in root segments ($\bar{\psi}_x^i$):

$$\bar{\psi}_x^i = \frac{\int_{0}^{S^i} \psi_x(s) \, ds}{S^i}$$

Layer water uptake from Darcian expression using $\bar{\psi}_x$:

$$Q^i_R = -k_r S^i (\bar{\psi}_x^i - \psi_s^i)$$
Possible equations for $\bar{\psi}_x^i$

\(\bar{\psi}_x^i = \psi_s^i + (G_1^i - G_0^i)/\beta_2^i\) \hspace{1cm} (1a)

\(\bar{\psi}_x^i = c_1^i \psi_0^i + (1 - c_1^i)\psi_s^i - c_2^i G_0^i\) \hspace{1cm} (1b)

\(\bar{\psi}_x^i = c_3^i \psi_0^i + (1 - c_3^i)\psi_s^i - c_4^i G_1^i\) \hspace{1cm} (1c)

\(\bar{\psi}_x^i = c_1^i \psi_1^i + (1 - c_1^i)\psi_s^i + c_2^i G_1^i\) \hspace{1cm} (1d)

\(\bar{\psi}_x^i = c_3^i \psi_1^i + (1 - c_3^i)\psi_s^i + c_4^i G_0^i\) \hspace{1cm} (1e)

\(\bar{\psi}_x^i = c_5^i (\psi_1^i + \psi_0^i) + (1 - 2c_5^i)\psi_s^i\) \hspace{1cm} (1f)

**Table: List of variables**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha^i$</td>
<td>$\sqrt{k_r^i/K_x^i}$</td>
</tr>
<tr>
<td>$\beta^i$</td>
<td>$\alpha^i S^i$</td>
</tr>
<tr>
<td>$\beta_2^i$</td>
<td>$(\alpha^i)^2 S^i$</td>
</tr>
<tr>
<td>$c_1^i$</td>
<td>$\sinh(\beta^i)/\beta^i$</td>
</tr>
<tr>
<td>$c_2^i$</td>
<td>$(1 - \cosh(\beta^i))/\beta_2^i$</td>
</tr>
<tr>
<td>$c_3^i$</td>
<td>$\tanh(\beta^i)/\beta^i$</td>
</tr>
<tr>
<td>$c_4^i$</td>
<td>$(\text{sech}(\beta^i) - 1)/\beta_2^i$</td>
</tr>
<tr>
<td>$c_5^i$</td>
<td>$\tanh(\beta^i/2)/\beta^i$</td>
</tr>
</tbody>
</table>
Analytical equation for single root $\bar{\psi}_x^i$

Derivation for layer $i$:

1. start from eq. 1f,
2. substitute for $\psi_0^{i/1}$ from eq. 1f,
3. substitute for $\psi_0^{i\pm 1}$ from combined eqs. 1b and 1d.

\[
\bar{\psi}_x^i = \sum_{j=i-2}^{i+2} a^j \bar{\psi}_x^j + \sum_{k=i-2}^{i+2} b^k \psi_s^k, (j \neq i)
\]
Pentadiagonal system (‘RSA Stencil’) for $\bar{\psi}_x$
RSA Stencil: accurate $\bar{\psi}_x$ & $Q_R$ predictions

Bouda and Saiers (2017) Advances in Water Resources
Big Root Model:

- Use single-root equations to represent non-trivial RSA

- Lowers model skill as compared to ‘unconstrained’ RSA-Stencil fit.
- Provides a clear physical basis:
  - helps inverse model convergence
  - more easily interpretable parameters ($k_rS$, $K_x/S$)
  - greater consistency in prediction beyond calibration data
Big Root case study:
prediction of field data from Wind River Crane

Shaw et al. (2004) *Ecosystems*

(Extensively studied old growth douglas-fir / hemlock site)
2010 Data Overview

Sonia Wharton AmeriFlux US-Wrc Wind River Crane Site, doi:10.17190/AMF/1246114
2010 Drought

Assumptions:

- all water movement is through plant
- all ET is transpiration
- published $\psi - \theta$ relations
- published basal area
Big root calibration data subset

Soil moistures (m$^3$/m$^3$)

08-Jul  20-Jul  01-Aug  13-Aug  25-Aug
Soil Moisture Predictions

<table>
<thead>
<tr>
<th>Date</th>
<th>Ohm’s law parallel</th>
<th>Ohm’s law series</th>
<th>Big root</th>
</tr>
</thead>
<tbody>
<tr>
<td>11-Jul</td>
<td>RMSE=2.1%</td>
<td>3.2%</td>
<td>0.27%</td>
</tr>
<tr>
<td>21-Jul</td>
<td>0.95%</td>
<td>1.1%</td>
<td>0.14%</td>
</tr>
<tr>
<td>31-Jul</td>
<td>0.72%</td>
<td>0.61%</td>
<td>0.39%</td>
</tr>
<tr>
<td>10-Aug</td>
<td>3.3%</td>
<td>0.32%</td>
<td>0.39%</td>
</tr>
<tr>
<td>21-Aug</td>
<td>1.3%</td>
<td>1.2%</td>
<td>0.72%</td>
</tr>
<tr>
<td>31-Jul</td>
<td>1.4%</td>
<td>0.52%</td>
<td>0.18%</td>
</tr>
<tr>
<td>11-Jul</td>
<td>0.55%</td>
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Volumetric water content (%)
Water Uptake Predictions

- Ohm’s law parallel: overly redistributes dry bias in wet wet bias in dry
- Ohm’s law series: shallow dry bias deep wet bias
- Big root: more flexible RSA no systematic bias

bioRXiv
https://doi.org/10.1101/559237
Redistributes via base

Cannot propagate $\psi_x$ to depth

Constrained to 1 root
Conclusions

- RSA imposed by Ohm’s law analogue models leads to biases.
- Big root model includes interactions between layers, is more flexible at representing RSA.
- Systematic biases eliminated; errors due to assumption of single effective root; but this also puts predictions on a physical basis, increasing robustness.
- Next steps (in cooperation with CNRM, Météo France):
  - implement big root model in SurfEx,
  - show impact of root scheme on soil moistures and surface fluxes at scale.
Acknowledgements

- Many thanks to Sonia Wharton and the Wind River Crane team!
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Model inversion

- Cannot uniquely resolve $k^i_r$, $K^i_x$, $S^i$ without further constraint.
- Inversion yields $k^i_r S^i$ and $K^i_x / S^i$ or just stencil $a$ and $b$, predicting flows $(Q_R)$:

<table>
<thead>
<tr>
<th>Data</th>
<th>Boundary conditions</th>
<th>Inversion yields:</th>
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</thead>
<tbody>
<tr>
<td>$\bar{\psi}_x$, $Q_R$</td>
<td>$\psi_1^1, G_0^m = 0$</td>
<td>$k^i_r S$, $K^i_x / S$</td>
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<td>$\psi_x$, $Q_R$</td>
<td>$G_1^1, G_0^m = 0$</td>
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<tr>
<td>$Q_R$</td>
<td>$\psi_1^1, G_0^m = 0$</td>
<td>$k^i_r S$, $K^i_x / S$ *</td>
</tr>
<tr>
<td>$Q_R$</td>
<td>$G_1^1, G_0^m = 0$</td>
<td>stencil $a$, $b$</td>
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* except layer $n$