SPHERE TO SPHERE TRANSFORMS IN SPECTRAL SPACE
IN THE CYCLE 43 OF ARPEGE/IFS: CONFIGURATION 911.

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Abstract:
This documentation describes configuration formerly numbered 911 which performs dilatation/contraction matrices: these matrices allow to do spectral transformation between a stretched spectral space and between its unstretched counterpart. Some algorithmic aspects and technical aspects are described. Some namelists are provided to use the configuration 911, which is now externalised in the project “utilities”. Such transformations can be used in spherical geometry only, and for METEO-FRANCE purpose only (not at ECMWF).

Résumé:
Cette documentation décrit la configuration anciennement numérotée 911 qui fabrique des matrices de dilatation/rotation. Ces matrices permettent de faire des transformations spectrales entre un espace spectral étiré et sa contrepartie non étirée. On décrit quelques aspects algorithmiques et techniques. On fournit un exemple de namelist pour utiliser cette configuration, qui est maintenant externalisée dans la librairie “utilities”. Cette transformation n’est utilisée qu’à METEO-FRANCE.

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1 Introduction.

This documentation describes the configuration formerly numbered 911.

In the stretched configuration of ARPEGE/IFS the most part of calculations is done on the stretched (computational) geometry. But there are some spectral calculations which have to be done on an unstretched sphere (for example filtering of some derivatives in FULL-POS). So one needs an efficient routine which does direct and inverse transformations between the spectral space of the computational sphere and its unstretched counterpart. Transformation includes a set of dilatation/contraction matrices which take a lot of place of memory, so these matrices are computed and stored separately. These matrices are written on a LFI file.

Dilatation/contraction matrices can also be computed in the set-up part of FULL-POS.

* Distributed memory:
  - 911 works for one level of MPI distribution.
  - Some parts of the code use OpenMP distribution.

* Modifications since cycle 42: none.
2 Dilatation and contraction matrices.

2.1 General considerations and stability.

One uses the algorithm described by Rochas (Rochas, 1990). $c$ is the stretching coefficient. For each zonal wavenumber $m$, the algorithm computes the dilatation matrix $B_m$ (containing the coefficients $\beta_{m,r}$) and the contraction matrix $A_m$ (containing the coefficients $\alpha_{m,r}$). Two truncations are defined: a small truncation $N_s$ corresponding to the spectral representation of the variable resolution computational sphere, and a large truncation $N_c$ corresponding to the spectral representation of the constant resolution geographical sphere. The theoretical value of $N_c$ would be infinite. In practical $N_c$ is slightly greater than $cN_s$. In order to test the stability of the algorithm of matricial computation, one computes for each zonal wavenumber $m$ (varying from 0 to $N_s$) the "L$^\infty$ norm" $|B_mA_m - I|_\infty$ which is equal to the maximum of the absolute values of the coefficients of matrix $B_mA_m - I$, where $I$ is the identity matrix.

* Computation of the norm $|B_mA_m - I|_\infty$.

  - The most voluminous matrices are obtained for $m=0$.
  - For small truncations, the most unstable algorithm (the highest value of $|B_0A_0 - I|_\infty$) are obtained for $m=0$. This property is no longer valid for higher truncations ($N_s > 1000$).
  - Norm is lower when calculations are done with 128 bits. All the following results are given for
    - 128 bits calculations when computing the Legendre polynomials used in the matrices.
    - 64 bits calculations when computing the matrices.
  - For $N_c = cN_s$, $|B_0A_0 - I|_\infty$ is $O(0.01)$ (so not negligible relative to 1) for the cases for which computation of such a norm has been done (for $c=2, 3$ or $4$). The consequence is that a higher value of $N_c$ is necessary to obtain more negligible values of $|B_0A_0 - I|_\infty$, some calculations show that a satisfactory value is $N_c \approx 1.7cN_s$ for $N_s \approx 20$ and $N_c \approx 1.15cN_s$ for $N_s \approx 200$ to obtain a norm lower than $10^{-13}$, if the stretching coefficient is between 1.5 and 4. Table (2.1) gives some examples of values of $N_c$ which have to be used for different stretching coefficients between 1.5 and 4.0 and a list of admissible truncations. Values of $N_c$ are taken preferably among admissible values (i.e. which allow use of fast Fourier transforms). See documentation (IDTS) about spectral transforms for more details about admissible truncations.
  - $\log(|B_0A_0 - I|_\infty)$ is nearly proportional to the quantity $N_c/(c * N_s)$ when over $10^{-13}$.
  - For a given truncation $N_s$, the ratio $N_c/(c * N_s)$ which gives a norm of $10^{-13}$ (or any other threshold) converges towards 1 when $c$ converges towards 1.
  - For a given stretching coefficient $c$, the ratio $N_c/(c*N_s)$ which gives a norm of $10^{-13}$ (or any other threshold) converges towards 1 when the lower truncation $N_s$ becomes infinite.
  - Algorithm can be used even for high stretching coefficients ($c \approx 10$) and for high truncations ($N_c \approx 1000$).
Table 2.1: Optimal value of $N_c$ to be taken for $N_c$ between 30 and 599.
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Table 2.1b: Optimal value of $N_c$ to be taken for $N_a$ between 624 and 2999.
2.2 General algorithm to compute the upper truncation $N_c$: subroutine SUNCMAX.

* Calculation of $N_c$: The default value of $N_c$ is computed as follows:

- A threshold $S_0$ is given for $| B_0 A_0 - I |_\infty$ (this threshold is set to $10^{-13}$ in the code).
- One uses the property: $\log(| B_0 A_0 - I |_\infty)$ is nearly proportional to the quantity $N_c/(c N_s) - 1$ for a given threshold $S_0$ when $c$ and $N_s$ are given:
  
  The ratio:
  
  $$ a_2 = \frac{\log(| B_0 A_0 - I |_\infty)}{N_c/(c N_s) - 1} $$
  
  can be fitted by a curve:
  
  $$ a_2 = \alpha(c) N_s^r + \beta(c) $$
  
  - Exponent $r$ is quasi-independent from $c$ and a value of 2/3 is satisfactory; this value has been taken in the code.
  - $\alpha$ only depends on the stretching coefficient $c$ and has been fitted by the following function:
    
    $$ \alpha = \frac{-1.08}{(c - 1)^{0.86}} - 6.54 $$
  
  - $\beta$ only depends on the stretching coefficient $c$ and has been fitted by the following function:
    
    $$ \beta = \frac{0.65}{(c - 1)^{0.86}} + 5.91 $$
  
  - One computes $\alpha$, then $\beta$, then $a_2$, then the integer immediately above:
    
    $$ (c N_s) \left( 1 + \frac{a_2}{S_0} \right) $$
  
  This integer is not necessary an admissible truncation; so one takes for $N_c$ the admissible truncation immediately above this integer.
  
- For $S_0 = 10^{-13}$ the previous algorithm generally gives the same values as in table 2.1, with some isolated exceptions. So one does a correction to obtain in all cases the values of table 2.1 and a monotonic solution: $N_c$ has to be a monotonic function of $S_0$, $c$ and $N_s$.

* Subroutine SUNCMAX: SUNCMAX is called by MASTER911 to compute an acceptable default value for the upper truncation $N_c$. SUNCMAX calls SUADMI and SUNCET13.

- SUADMI says if a truncation is admissible or not, for stretched geometry and returns the adjacent admissible truncations.
- SUNCET13 returns the values of $N_c$ given by table 2.1 for a subset of admissible truncations and stretching coefficients.

2.3 Matrices size.

Two files of identical size are made, the first one containing dilatation matrices, the second one containing contraction matrices. Each file contains a number of values equal to:

$$ \frac{(N_s + 1)(N_s + 2)}{2} \cdot \frac{(N_c + 1)(N_c + 2)}{6} $$

This number has to be multiplied by 8 to obtain the number of octets. Local name of file is MATDILA for dilatation, MATCONT for contraction.

2.4 Description of subroutine SUDIL.

Most of the code is now in the library “utilities” (directory “rdc”). Some “ifsaux”, “algor” and “trans” routines may be required.
MASTER911 ->
  * SUNCMAX -> SUADMI and SUNCET13
  * SUDIL ->
    - SETUP_TRANS0 and SETUP_TRANS
    - TRANS_INQ
    - DILAT_MAPPING -> MXMAOP
    - DILAT_CALC
    - DILAT_DEVIATION
    - DILATB -> MXMAOP and LFIECR
    - DILAT_CONTROL
    - the processor communication routines MPL_RECV, MPL_SEND, MPL_BARRIER.
    - Some LFI.. routines.

Action of each routine:
- **SUDIL**: head routine.
- **SETUP_TRANS0** and **SETUP_TRANS**: calculation of the useful Legendre polynomials. One computes the Legendre polynomials corresponding to the truncation \(N_c\) on the variable resolution sphere latitudes which are associated to the Gaussian latitudes of the geographical sphere. One computes the Legendre polynomials corresponding to the truncation \(N_c\) only for the zonal wavenumbers \(m\) below or equal than \(N_s\), on the Gaussian latitudes of the sphere of truncation \(N_c\).
- **TRANS_INQ** gets the Gaussian weights and the useful Legendre polynomials.
- **LFIOUV** opens LFI files which will contain matrices.
- **LFIFER** closes LFI files.
- **LFIECR** writes on LFI files.
- **DILAT_MAPPING**: pre-calculations.
- **DILAT_CALC**: calculation of dilatation and contraction matrices.
- **DILAT_DEVIATION**: calculation of product contraction times dilatation, computes deviation from identity matrix.
- **DILATB**: write matrices on LFI files; in some cases prints matrices coefficients.
- **DILAT_CONTROL**: prints deviation from identity matrix.

2.5 Important namelist parameters.

* Main namelist parameters: Configuration 911 has now its own namelist containing only one element: NAM911.
  - **INSMAX** \((N_c)\), **INSMIN** \((N_s)\) and **INDGLG** (number of latitudes corresponding to a linear grid associated with truncation \(N_c\)).
  - **ZSTRET** (stretching coefficient \(c\)).
  - **LLPROVIDED_NSMAX**: says if **INSMAX** and **INDGLG** must be provided in the namelist (otherwise they are automatically computed).
  - **IPRINTLEV**: level of printings.
  - **LLIO_PNM**: T/F: PNM are stored/not stored in LFI files.

The number of MPI processors (INPROC) is not in NAM911 but taken from the script environment.

* Additional namelist parameters: Other namelist variables are provided, for example to control memory, execution time, level of printings.

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<th>Variable</th>
<th>Description</th>
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<td>IMPL_OUTPUT</td>
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<tr>
<td>IMBX_SIZE</td>
<td>user-provided mailbox size</td>
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<td>IMP_TYPE</td>
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<td>2=buffered (MPI_BSEND/MPI_BRECV)</td>
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<td>3=immediate (MPI_ISEND/MPI_IRECV)</td>
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<td>intermediate truncation for optimisations, assumed to be (&gt;) INSMIN.</td>
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<td>if INSMIN &lt;= INSLIM &lt; INSMAX, DILA/CONT articles are split</td>
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!  into WNUM and WNUC.
!  LLOPT_NSLIM : active only if INSMIN <= INSLIM < INSMAX; in this case
!  articles named WNUC are not written.
!  ITEST : For testing in DILATS (between 0 and 2).
!  ITESTIO : For testing : 0 = No I/Os ; 1 = with I/Os
!  IULOUT : Logical unit.

* Remark about granularity factor: the granularity factor (local variable IFACTD in
MASTER911) is computed inside the code as an internal intermediate variable. Optimal factor is proportional
to the size of matrices; The default value is the integer closest to
\[
\left\lfloor \frac{(N_s + 1)(N_s + 2)}{2} - \frac{(N_s)(N_s + 1)(N_s + 2)}{6} \right\rfloor /6232525
\]
bounded by 1 and 120. For stretching coefficients lower than 4, IFACTD is equal to 1 if \( N_s < 66 \). IFACTD is
equal to 120 if \( N_s >= 485 \).

* Default values: see in main routine MASTER911.

2.6 Example of namelist for configuration 911 of ARPEGE.

The example provided below is the namelist allowing to compute the dilatation/contraction matrices for the
resolution TL1278c2.4 (linear grid).

```
&NAM911
  IPRINTLEV=0,
  IMPL_OUTPUT=0,
  IMBX_SIZE=2048000000,
  INSMIN=1278,
  ZSTRET=2.4,
  INSMAX=3199,
  INDGLG=3200,
  LLPROVIDED_NSMAX=.TRUE.,
/
```

3 References.

* Internal notes:
  • (TDECTEC) 2015: IFS technical documentation (CY41R1). Part VI: technical and computational
    procedures. Available at “https://software.ecmwf.int/wiki/display/IFS/Official+IFS+Documentation”.
  • Rochas, 1990: Dilatation dans l’espace spectral (internal note in French, included in the “note ARPEGE
    numéro 19”).
  • (IDBAS) Yessad, K., 2016: Basics about ARPEGE/IFS, ALADIN and AROME in the cycle 43 of
    ARPEGE/IFS (internal note).
  • (IDDM) Yessad, K., 2016: Distributed memory features in the cycle 43 of ARPEGE/IFS (internal note).
  • (IDTS) Yessad, K., 2016: Spectral transforms in the cycle 43 of ARPEGE/IFS (internal note).