Méso-NH turbulent scheme for AROME

CNRM/GMME/Méso-NH

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Parametrised subgrid turbulent processes in Arome

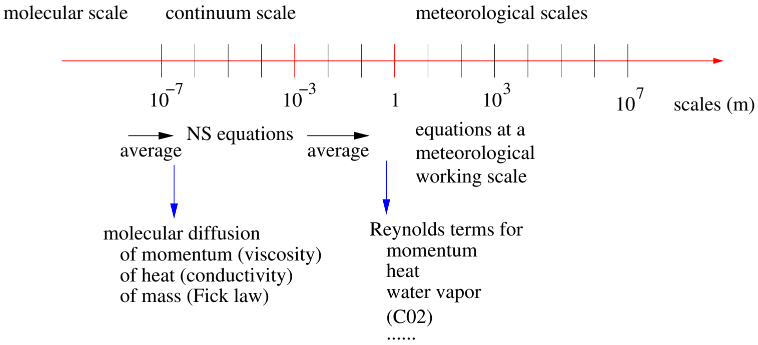
- isotropic turbulence parametrisation : mean isotropic turbulent mixing of cloudy conservative variables (θ_l and r_{np}) + diagnostic « projection » on pronostic variables (θ, r_v and r_c/r_i).
- shallow (non precipitating) cloud convection parametrisation : mean shallow convective updraft mass flux mixing of cloudy conservative variables (θ_l and r_{np}) + diagnostic « projection » on pronostic variables (θ, r_v and r_c/r_i).
- subgrid cloud parametrisation : computation of cloud content and cloud cover for radiation scheme in particular.

Parametrised subgrid turbulent processes in Arome

- CBR turbulence scheme : 45 min
- KFB shallow convection scheme : 45 min
- Subgrid cloud calculations : 45 min
- 1D shallow convective case results : 30 min
- Summary and conclusion : 10 min

Contents

Scales of averaging



Reynolds terms

Average of the NS equations $\longrightarrow$ non-linear terms $\longrightarrow$ turbulent fluxes

For α defined at the scale of the continuum

$$\alpha = \overline{\alpha} + \alpha'$$

Equation for α

$$\frac{\partial \alpha}{\partial t} + u\frac{\partial \alpha}{\partial x} + v\frac{\partial \alpha}{\partial y} + w\frac{\partial \alpha}{\partial z} = S \qquad \text{or} \qquad \frac{D\alpha}{Dt} = S$$

Applying the average operator to the α equation

$$\overline{\frac{\partial \alpha}{\partial t}} + \overline{u\frac{\partial \alpha}{\partial x}} + \overline{v\frac{\partial \alpha}{\partial y}} + \overline{w\frac{\partial \alpha}{\partial z}} = \overline{S}$$

Simplification of $\overline{\alpha}$ equation

The average operator is defined such as $\overline{\overline{\alpha}} = \overline{\alpha}$ and $\overline{\alpha'} = 0$

$$\frac{\partial\overline{\alpha}}{\partial t} + \overline{u}\frac{\partial\overline{\alpha}}{\partial x} + \overline{v}\frac{\partial\overline{\alpha}}{\partial y} + \overline{w}\frac{\partial\overline{\alpha}}{\partial z} + \overline{u'\frac{\partial\alpha'}{\partial x}} + \overline{v'\frac{\partial\alpha'}{\partial y}} + \overline{w'\frac{\partial\alpha'}{\partial z}} = \overline{S}$$

The mean Lagrangian evolution of $\overline{\alpha}$

$$\frac{\overline{D}\overline{\alpha}}{Dt} = -\left[\overline{u'\frac{\partial\alpha'}{\partial x}} + \overline{v'\frac{\partial\alpha'}{\partial y}} + \overline{w'\frac{\partial\alpha'}{\partial z}}\right] + \overline{S}$$

The turbulent or eddy transports

The terms in brackets $\Longrightarrow$ the mean transport of α by motions wich are not described at the scale of the mean meteorological variables

Flux form

Using a Boussinesq or anelastic version of the continuity equation

$$\frac{\partial(\rho_{ref}\, u')}{\partial x} + \frac{\partial(\rho_{ref}\, v')}{\partial y} + \frac{\partial(\rho_{ref}\, w')}{\partial z} = 0$$

the equation for $\overline{\alpha}$ becomes

$$\rho_{ref}\frac{\overline{D}\overline{\alpha}}{Dt} = -\left[\frac{\partial(\rho_{ref}\,\overline{u'\alpha'})}{\partial x} + \frac{\partial(\rho_{ref}\,\overline{v'\alpha'})}{\partial y} + \frac{\partial(\rho_{ref}\,\overline{w'\alpha'})}{\partial z}\right] + \overline{S}$$

Note that the mean variable are often written without the $^{-}$

For mean scales larger than about 1 km, we can usually neglect the gradient of horizontal turbulent fluxes, but the vertical gradient of vertical turbulent fluxes is determinant especially near the surface, .

For AROME, we use a simplified version of the 3D turbulent scheme of Méso-NH with 1D (vertical) turbulent terms only.

Computing the gradients of mean turbulent fluxes

The mean perturbation correlations such as $\overline{w'\alpha'}$ are new unknowns in the equations for the mean state of the atmosphere.
We would need new equations to solve this system.
But equations for the double correlations involve triple correlations such as $\overline{w'w'\alpha'}$ which are again new unknowns

Closure problem

The system may be solve only with a closure hypothesis.

Example : first order closure

Exchange coefficients

Simple expressions of «Fourier law» or «viscosity law» form (molecular diffusion of heat or momentum) may be used to express the turbulent fluxes (second order moment) as a function of the mean variables.

$$\rho_{ref}\,\overline{w'\alpha'} = -\rho_{ref}\,K_\alpha \frac{\partial \overline{\alpha}}{\partial z}$$

K_α is the eddy exchange coefficient for the variable α.

In the simplest first order closure, K_α is taken constant and positive, but more sophisticated first order closure are possible, usually through the definition of a mixing length.

Mixing length

Mixing lengths are defined by analogy with the mean free path in gaz kinetic theory (for example, Prandtl (1925) or Taylor (1915)). When the eddy coefficient is a function of a mixing length L, the closure of the system needs a parametrisation for the computation of this mixing length.
L is supposed to represent the size of the more «efficient» mixing vortices (the more «energetic» ones).

Higher order closures

In a complete second order system, pronostic equations are written for all the second order moments with closure hypothesis to express the third order moments and other unknown terms.
In Méso-NH, the scheme is not a complete second order scheme, but it is intermediate between a first and a second order scheme (1.5 order) :

- pronostic equation only for the isotropic second order moments through a TKE equation ;
- stationarisation of the other second order moments leading to diagnostic expressions.

1.5 order closure

In gaz kinematic theory, the molecular exchange coefficients may be written as

$$\nu = c \ < l > \ v_T$$

where c is a constant, $< l >$ is the mean free path and v_T is the mean speed of thermal agitation.
By analogy, the eddy coefficient for any variable α is defined as :

$$K_\alpha = c_\alpha L \sqrt{e}$$

where

- L is the mixing length
- $e = \overline{u'^2 + v'^2 + w'^2}$ is the mean TKE ($\sqrt{e}$ gives the mean module of the speed fluctuation)

The BL89 mixing length

BL89 postulate that the mixing length can be related to the distance a parcel of air having the initial kinetic energy of its level, can travel upwards (l_{up}) and downwards (l_{down}) before being stopped by buoyancy effects :

$$\begin{aligned} \int_z^{z+l_{up}} \frac{g}{\theta_{v\ ref}}(\theta(z)-\theta(z'))dz' &= -e(z) \\ \int_{z-l_{down}}^{z} \frac{g}{\theta_{v\ ref}}(\theta(z')-\theta(z))dz' &= -e(z) \ and\ l_{down} \leq z \end{aligned}$$

and

$$\frac{1}{L^{2/3}} = \frac{1}{2}\left[\frac{1}{l_{up}^{2/3}} + \frac{1}{l_{down}^{2/3}}\right]$$

The BL89 mixing length

The BL mixing length allows the length scale at any level to be influenced by the stability at this level, but also by the effect of remote stable zones, or the proximity of the ground.

Limitations of BL89

- infinite value in strict neutral conditions
- bad formulation in the CLS (amelioration with RMC01)
- sometimes underestimation of the fluxes near strong inversions at the top of the boundary layer (especially in moist or cloudy condition)

The 1D Méso-NH turbulent scheme

- Cuxart (1995)
- Redelsberger and Sommeria (1981)
- Bougeault and Lacarrère (1989)

The (1D) turbulent equations tendencies for the mean variables

$$
\begin{aligned}
\frac{\partial(\rho_{ref}\overline{u})}{\partial t})_{turb} &= -\frac{\partial(\rho_{ref}\overline{w'u'})}{\partial z} \\
\frac{\partial(\rho_{ref}\overline{v})}{\partial t})_{turb} &= -\frac{\partial(\rho_{ref}\overline{v'w'})}{\partial z} \\
\frac{\partial(\rho_{ref}\overline{w})}{\partial t})_{turb} &= 0 \\
\frac{\partial(\rho_{ref}\overline{\theta})}{\partial t})_{turb} &= -\frac{\partial(\rho_{ref}\overline{w'\theta'})}{\partial z} \\
\frac{\partial(\rho_{ref}\overline{r_v})}{\partial t})_{turb} &= -\frac{\partial(\rho_{ref}\overline{w'r_v'})}{\partial z}
\end{aligned}
$$

The (3D) turbulent equations for the first order moments before hypotheses

$\frac{\partial}{\partial t}(b_{ij})$	$+\bar{u}_k \frac{\partial}{\partial x_k}(b_{ij})$	$=$	$-\frac{\partial}{\partial x_k}\left(\overline{b'_{ij} u'_k}\right)$	$-\frac{4}{3} e S_{ij} - \Sigma_{ij} - Z_{ij}$	$+B_{ij}$	$-\Pi_{ij}$	
$\frac{\partial}{\partial t}(e)$	$+\bar{u}_k \frac{\partial}{\partial x_k}(e)$	$=$	$-\frac{\partial}{\partial x_k}\left(\overline{e' u'_k} + \overline{p' u'_k}\right)$	$-\overline{u'_k u'_l}\frac{\partial \bar{u}_k}{\partial x_l}$	$+\beta_k \overline{u'_k \theta'_v}$		$-\epsilon$
$\frac{\partial}{\partial t}(\overline{u'_i \theta'})$	$+\bar{u}_k \frac{\partial}{\partial x_k}(\overline{u'_i \theta'})$	$=$	$-\frac{\partial}{\partial x_k}\left(\overline{u'_k u'_i \theta'}\right)$	$-\overline{u'_k \theta'}\frac{\partial \bar{u}_i}{\partial x_k} - \overline{u'_i u'_k}\frac{\partial \bar{\theta}}{\partial x_k}$	$+\beta_i \overline{\theta' \theta'_v}$	$-\Pi_{i\theta}$	
$\frac{\partial}{\partial t}(\overline{u'_i r'_v})$	$+\bar{u}_k \frac{\partial}{\partial x_k}(\overline{u'_i r'_v})$	$=$	$-\frac{\partial}{\partial x_k}\left(\overline{u'_k u'_i r'_v}\right)$	$-\overline{u'_k r'_v}\frac{\partial \bar{u}_i}{\partial x_k} - \overline{u'_i u'_k}\frac{\partial \bar{r}_v}{\partial x_k}$	$+\beta_i \overline{r'_v \theta'_v}$	$-\Pi_{ir}$	
$\frac{\partial}{\partial t}(\overline{\theta'^2})$	$+\bar{u}_k \frac{\partial}{\partial x_k}(\overline{\theta'^2})$	$=$	$-\frac{\partial}{\partial x_k}\left(\overline{u'_k \theta'^2}\right)$	$-2\overline{u'_k \theta'}\frac{\partial \bar{\theta}}{\partial x_k}$			$-\epsilon_\theta$
$\frac{\partial}{\partial t}(\overline{r'^2_v})$	$+\bar{u}_k \frac{\partial}{\partial x_k}(\overline{r'^2_v})$	$=$	$-\frac{\partial}{\partial x_k}\left(\overline{u'_k r'^2_v}\right)$	$-2\overline{u'_k r'_v}\frac{\partial \bar{r}_v}{\partial x_k}$			$-\epsilon_r$
$\frac{\partial}{\partial t}(\overline{\theta' r'_v})$	$+\bar{u}_k \frac{\partial}{\partial x_k}(\overline{\theta' r'_v})$	$=$	$-\frac{\partial}{\partial x_k}\left(\overline{u'_k \theta' r'_v}\right)$	$-\overline{u'_k \theta'}\frac{\partial \bar{r}_v}{\partial x_k} - \overline{u'_k r'_v}\frac{\partial \bar{\theta}}{\partial x_k}$			$-\epsilon_{\theta r}$
tendency	advection		3^{rd} order	dyn. prod.	therm. prod.	pres.-cor.	dissip.

Hypotheses for the resolution of the first order moments

- pronostic equation for the TKE
- 1.5 order closure
- Stationarization of the first order moment (except e)
- Isotropy and homogeneity
- No 3^{rd} order moments (except for e)
- Closure hypotheses for the presso-correlation and dissipation

In 1D :

- $\partial \bar{\ } / \partial x = \partial \bar{\ } / \partial y = 0$ (horizontal homogeneity)
- $\overline{w} = 0$ (horizontal homogeneity, except for e advection)

The (1D) system for the first order moments after hypotheses

An equation for e +

$$
\begin{aligned}
0 &= -C_m \frac{\sqrt{e}}{L}\left(\overline{u'^2} - \frac{2}{3}e\right) \\
0 &= -C_m \frac{\sqrt{e}}{L}\left(\overline{v'^2} - \frac{2}{3}e\right) \\
0 &= -C_m \frac{\sqrt{e}}{L}\left(\overline{w'^2} - \frac{2}{3}e\right) \\
0 &= \overline{u'v'} \\
0 &= -\frac{4}{15} e \frac{\partial \overline{u}}{\partial z} - C_m \frac{\sqrt{e}}{L}\overline{u'w'} \\
0 &= -\frac{4}{15} e \frac{\partial \overline{v}}{\partial z} - C_m \frac{\sqrt{e}}{L}\overline{v'w'}
\end{aligned}
$$

The (1D) system for the first order moments after hypotheses

$$
\begin{aligned}
0 &= \overline{u'\theta'} \\
0 &= \overline{v'\theta'} \\
0 &= -\frac{2}{3}e\frac{\partial\overline{\theta}}{\partial z} + \frac{2}{3}\beta E_\theta\overline{\theta'^2} + \frac{2}{3}\beta E_{moist}\overline{\theta' r_v'} - C_h\frac{\sqrt{e}}{L}\overline{w'\theta'} \\
0 &= \overline{u' r_v'} \\
0 &= \overline{v' r_v'} \\
0 &= -\frac{2}{3}e\frac{\partial\overline{r_v}}{\partial z} + \frac{2}{3}\beta E_\theta\overline{\theta' r_v'} + \frac{2}{3}\beta E_{moist}\overline{r_v'^2} - C_h\frac{\sqrt{e}}{L}\overline{w' r_v'} \\
0 &= -2\frac{\partial\overline{\theta}}{\partial z}\overline{w'\theta'} - 2C_\theta\frac{\sqrt{e}}{L_\epsilon}\overline{\theta'^2} \\
0 &= -2\frac{\partial\overline{r_v}}{\partial z}\overline{w' r_v'} - 2C_\theta\frac{\sqrt{e}}{L_\epsilon}\overline{r_v'^2} \\
0 &= -\frac{\partial\overline{r_v}}{\partial z}\overline{w'\theta'} - \frac{\partial\overline{\theta}}{\partial z}\overline{w' r_v'} - 2C_\theta\frac{\sqrt{e}}{L_\epsilon}\overline{\theta' r_v'}
\end{aligned}
$$

The second order moment diagnostic equations

$$
\begin{aligned}
\overline{w'\theta'} &= -\frac{2}{3}\frac{L}{C_h}e^{\frac{1}{2}}\frac{\partial\overline{\theta}}{\partial z}\phi_3 \\
\overline{w'r'_v} &= -\frac{2}{3}\frac{L}{C_h}e^{\frac{1}{2}}\frac{\partial\overline{r_v}}{\partial z}\psi_3 \\
\overline{u'\,v'} &= 0 \\
\overline{u'\,w'} &= -\frac{4}{15}\frac{L}{C_m}e^{\frac{1}{2}}\frac{\partial\overline{u}}{\partial z} \\
\overline{v'\,w'} &= -\frac{4}{15}\frac{L}{C_m}e^{\frac{1}{2}}\frac{\partial\overline{v}}{\partial z}
\end{aligned}
$$

$$
\begin{aligned}
\overline{u'^2} &= \frac{2}{3}e \\
\overline{v'^2} &= \frac{2}{3}e \\
\overline{w'^2} &= \frac{2}{3}e
\end{aligned}
$$

Others moments

$$
\begin{aligned}
\overline{\theta' r_v'} &= \frac{1}{2}\,CL^2\left(\frac{\partial\overline{\theta}}{\partial z}\frac{\partial\overline{r_v}}{\partial z}\right)(\phi_3+\psi_3) \\
\overline{\theta'^2} &= CL^2\left(\frac{\partial\overline{\theta}}{\partial z}\frac{\partial\overline{\theta}}{\partial z}\right)\phi_3 \\
\overline{r_v'^2} &= CL^2\left(\frac{\partial\overline{r_v}}{\partial z}\frac{\partial\overline{r_v}}{\partial z}\right)\psi_3
\end{aligned}
$$

The stability functions ϕ_3 and ψ_3 describe the enhancement or inhibition of turbulent transfers by stability effects.
In the 1D system :

$$\phi_3 = \psi_3 = \frac{1}{C(R_\theta + R_r)}$$

$$\begin{aligned} C &= \frac{2}{3}\frac{1}{C_h\, C_\theta} \\ C_h &= 4 \\ C_m &= 4 \\ C_\theta &= 1.2 \end{aligned}$$

$$\begin{aligned} R_\theta &= \frac{g}{\theta_{v\ ref}}\frac{L^2}{e}E_\theta\frac{\partial\overline{\theta}}{\partial z} \\ R_r &= \frac{g}{\theta_{v\ ref}}\frac{L^2}{e}E_{moist}\frac{\partial\overline{r_v}}{\partial z} \end{aligned}$$

where $E_\theta = \frac{\overline{\theta_v}}{\overline{\theta}}$ and $E_{moist} = 0.61\overline{\theta}$

The TKE equation

$$\frac{\partial e}{\partial t} = -\frac{1}{\rho_{ref}}\frac{\partial}{\partial z}(\rho_{ref}\, e\overline{w}) - \overline{u_i' w'}\frac{\partial \overline{u_i}}{\partial z} + \frac{g}{\theta_{v\,ref}}\overline{w'\theta_v'}$$
$$+\frac{1}{\rho_{ref}}\frac{\partial}{\partial z}\left(C_{2m}\rho_{ref}\, L e^{\frac{1}{2}}\frac{\partial e}{\partial z}\right) - C_\epsilon \frac{e^{\frac{3}{2}}}{L}$$

The source terms appearing in this equation are respectively the advection of TKE, the shear production, the buoyancy production ($\overline{w'\theta_v'} = E_\theta \overline{w'\theta'} + E_{moist}\overline{w' r_v'}$), the diffusion, and the dissipation. The additionnal numerical constants take the values (RS81) $C_{2m} = 0.2$ and $C_\epsilon = 0.7$.

Turbulence of cloud water r_c and ice r_i

Idea : to use variables which are conversative in the condensation process

$$\vec{u} \longrightarrow \vec{u} \qquad (1)$$

$$\theta \longrightarrow \theta_l = \theta - \frac{L_v}{C_{ph}} {\Pi_{ref}}^{-1} r_c - \frac{L_s}{C_{ph}} {\Pi_{ref}}^{-1} r_i \qquad (2)$$

$$r_v \longrightarrow r_{np} = r_v + r_c + r_i \qquad (3)$$

Formally, the scheme is unchanged, except for the computation of E_θ and E_{moist} (weight of condensed water in buoyancy production and ϕ_3 and ψ_3 computation)

The eddy fluxes are computed for conservative variables $\overline{w'r'_{np}}$ et $\overline{w'\theta_l}$

The eddy fluxes for non-conservative variables are then deduced from these conservative variable fluxes using a parametrisation of the cloud water/ice fluxes.

- In a «all or nothing» adjustment process, $\overline{w'r'_c}$ and $\overline{w'r'_i}$ are directly given by a first order adjustment process.
- With a sub-grid condensation scheme, the shape of the cloud water fluxes depends on the second-order correlation $\overline{s'r'_c}$.

More details in the sub-grid condensation lecture ...

Steps of computation

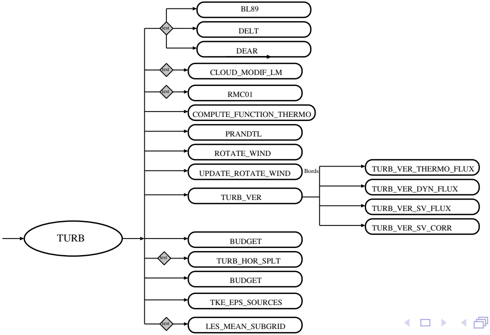
- Compute L from e^-
- Compute ϕ_i and ψ_i from the α_i^-, e^- and L
- Compute the turbulent tendencies (with some possible implicitness) for the α_i (turbulent contribution for the computation of the α_i^+)
- Compute the TKE tendency and obtain e^+.

Algorithm

Input/output

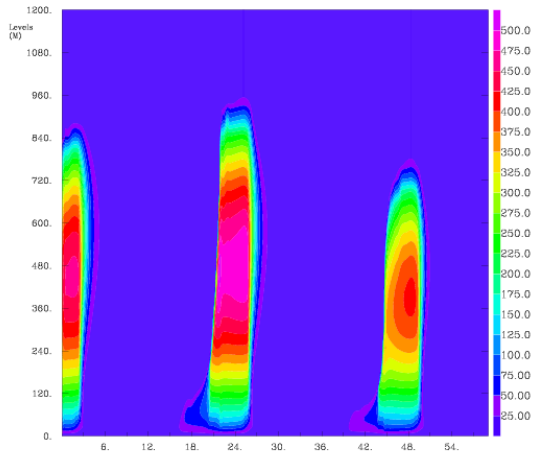
inputs : turbulent surface fluxes (constant fluxes in the CLS)

outputs : turbulent tendencies for non precipitating pronostic variables



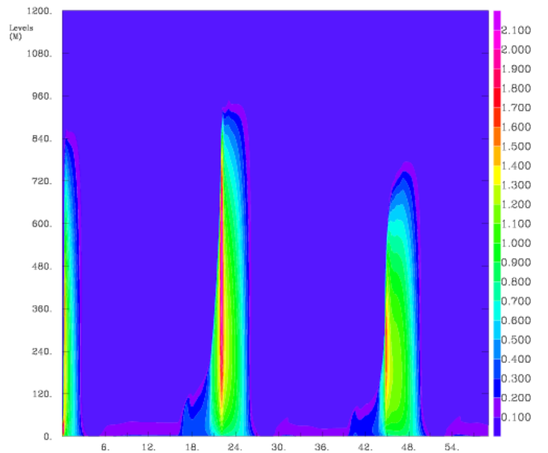
GABLS2 : 1D diurnal evolution

BL mixing length



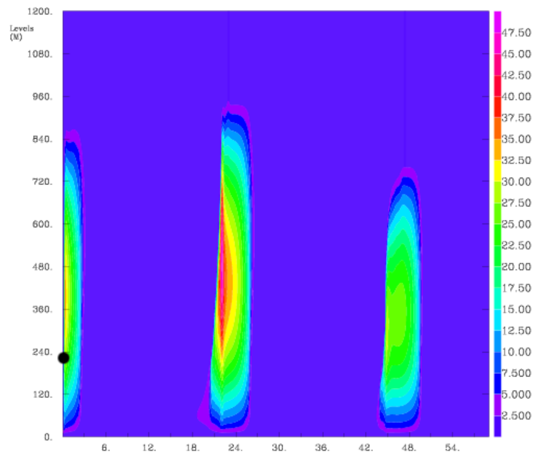
GABLS2 : 1D diurnal evolution

TKE



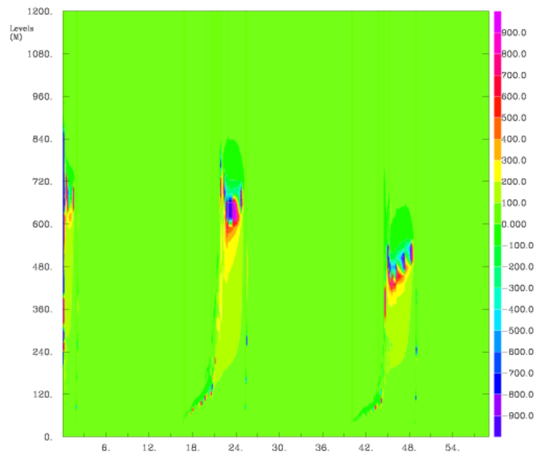
GABLS2 : 1D diurnal evolution

K_m



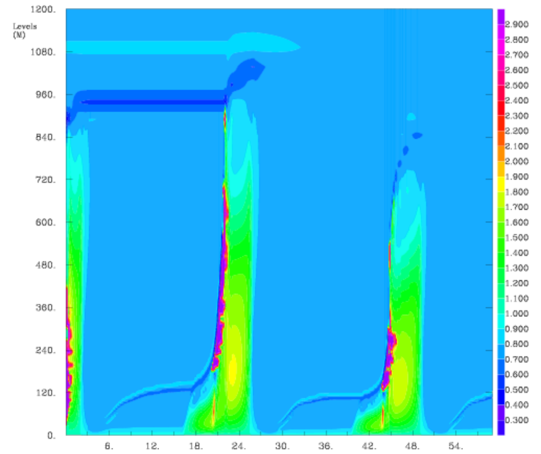
GABLS2 : 1D diurnal evolution

K_h



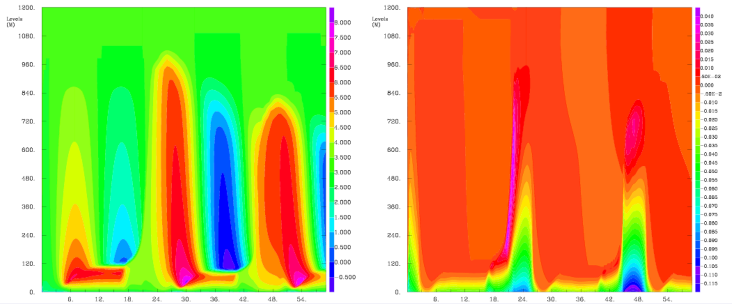
GABLS2 : 1D diurnal evolution

$\phi_3 = \psi_3$



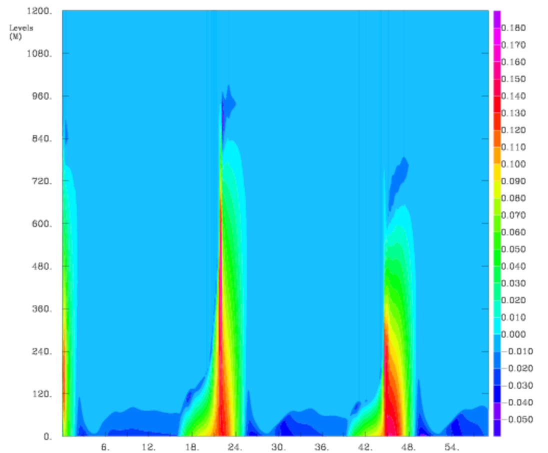
GABLS2 : 1D diurnal evolution

$\overline{u}$ and $\overline{w'u'}$



GABLS2 : 1D diurnal evolution

$\overline{w'\theta'_l}$



GABLS2 : 1D diurnal evolution

$\overline{w' r'_{np}}$