Reference Multiphase Equations

implementation in AROME and Aladin

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Introduction

multiphase fluid

- aim : consistent representation of interphase interaction (drag,...)
- but : we don't want a full dynamics for each phase
- starting from Sylvie's results (general framework, z coordinate)

Adaptation of AROME equations to multiphase fluid

- Define a consistent set of thermodynamic constants for the parcel
- Define a relevant vertical coordinate
- Derive the set of equations

Be warned :

- You will not see here any description of AROME multi-phase code (this topic has not yet been pushed up to code applications)

Outline

Define consistent thermodynamics constants

Parcel definition

- we consider a parcel containing dry air, water vapour and condensates (liquid)
- corresponding densities ρ_a, ρ_v and ρ_l
- all gases are perfect and liquid has vanishing and constant volume
- for the dry air we have c_{pa}, c_{va} and R_a
- for the water vapour we have c_{pv}, c_{vv} and R_v
- for the liquid water we have c_l

Definition

- c_p : heat needed to increase the temperature of 1 g of the parcel mixture by 1 K under a constant pressure
- c_v : heat needed to increase the temperature of 1 g of the parcel mixture by 1 K at constant volume

definition of c_p

To heat under constant pressure we must :

- heat the gas under constant pressure
- heat the liquid

heat needed to increase T of 1K under constant pressure

$$Q = c_p = \frac{\rho_a c_{pa} + \rho_v c_{pv} + \rho_l c_l}{\rho_a + \rho_v + \rho_l}$$

definition of c_v

At constant volume we must :

- heat the gas at constant volume
- heat the liquid

heat needed to increase T of 1K at $p = cst$:

$$Q = c_v = \frac{\rho_a c_{va} + \rho_v c_{vv} + \rho_l c_l}{\rho_a + \rho_v + \rho_l}$$

state equation

the state equation writes :

$$p = \rho_{av} R_{av} T \quad \text{where} \quad \rho_{av} = \rho_a + \rho_v$$

with :

$$R_{av} = \frac{\rho_a R_a + \rho_v R_v}{\rho_a + \rho_v}$$

we define a "'multiphase"' modified gas constant :

$$R = \frac{\rho_a + \rho_v}{\rho_a + \rho_v + \rho_l} R_{av} = \frac{\rho_{av}}{\rho} R_{av}$$

multiphase Mayer's relationship

with the above definitions we have :

$$\begin{aligned} c_p &= \frac{\rho_a(c_{va}+R_a)+\rho_v(c_{vv}+R_v)+\rho_l c_l}{\rho_a+\rho_v+\rho_l} \\ &= \frac{\rho_a c_{va}+\rho_v c_{vv}+\rho_l c_l}{\rho_a+\rho_v+\rho_l}+\frac{\rho_a R_a+\rho_v R_v}{\rho_a+\rho_v+\rho_l} \\ &= c_v + R \end{aligned}$$

Hence we have the modified multiphase Mayer relationship :

$$c_p - c_v = R$$

And the state equation writes :

$$p = \rho R T$$

Define relevant thermodynamic constants

Summary for this section :

Even in the multiphase case, a physically meaningful set of C_p, C_v and R "constants" can be defined

$\rightarrow$ with this definition we still have $C_p - Cv = R$
$\rightarrow$ C_p, Cv and R can be used as before if needed.

Outline

Define a relevant vertical coordinate

Described in much details in :
http ://www.cnrm.meteo.fr/gmapdoc/modeles/Dynamique/massc.ps

set of hypothesis :

Here we directly assume that :
- the wind is the total barycentric wind of all species
- same wind in SL transport and in momentum variable
- the density is the total density of all species

Consequence :

As outlined by Sylvie, the continuity equation then writes

$$\frac{d\rho}{dt} + \rho D_3 = 0$$

there is no source term in the RHS.

Define a relevant vertical coordinate

When continuity equation has no source :

- classical derivation (Laprise, 1992) of the mass-coordinate remains formally valid
- the mass-coordinate π is defined by :

$$\frac{\partial \pi}{\partial z} = -\rho g$$

mass-based vertical velocity

ω keeps its classical form :

$$\omega = -\int_0^{\pi} \nabla_{\pi} \mathbf{V} d\pi$$

Define a relevant vertical coordinate

definition of hybrid coordinate

- classical derivation (Laprise, 1992) of the hybrid coordinate remains formally valid
- the hybrid mass-coordinate η is defined by :

$$\pi(\eta) = A(\eta) + B(\eta)\pi_s$$

- we still define $m = (\partial\pi/\partial\eta)$

continuity equation

$$\frac{\partial m}{\partial t} + \boldsymbol{\nabla} m\mathbf{V} + \frac{\partial}{\partial\eta}(m\dot{\eta}) = 0$$

Define a relevant vertical coordinate

The surface is no longer a material surface

- because the precipitating part of the parcel goes across the surface
- the gaseous and airborne part does not crosses the surface
- as a consequence for multiphase flows, $\dot{\eta}$ is no longer zero at the surface
- $\dot{\eta}$ was zero at the surface for monophase flows

Define a relevant vertical coordinate

π_s-tendency equation :

$$\frac{\partial \pi_s}{\partial t} = -\int_0^1 \nabla m\mathbf{V} d\eta - [m\dot{\eta}]_s$$

where $[m\dot{\eta}]_s = -g \sum_k F_k$, and F_k are precipitation fluxes ($F_k = 0$ for gases and airborne components).

η-based vertical velocity :

$$m\dot{\eta} = \left[B \int_0^1 \nabla m\mathbf{V} d\eta' - \int_0^\eta \nabla m\mathbf{V} d\eta' \right] + B[m\dot{\eta}]_s$$

$$\omega = \mathbf{V}\nabla\pi - \int_0^\eta \nabla m\mathbf{V} d\eta'$$

Define a relevant vertical coordinate

Transformation rules :

they remain formally unchanged :

$$\frac{\partial}{\partial \pi} = \frac{1}{m}\frac{\partial}{\partial \eta}$$

$$\nabla_\pi = \nabla_\eta - (\nabla_\eta \pi)\frac{1}{m}\frac{\partial}{\partial \pi}$$

and :

$$\frac{\partial}{\partial z} = -\frac{gp}{RT}\frac{\partial}{\partial \pi}$$

$$\nabla_z = \nabla_\pi + \frac{gp}{RT}(\nabla_\pi z)\frac{\partial}{\partial \pi}$$

Define a relevant vertical coordinate

Summary for this section :

- If the wind is the total barycentric wind,
- If the density is the total density

$\longrightarrow$ then we can define a vertical coordinate which has formally the same properties as in the monophase case.

Comment :

If other choices are made, this leads to source terms $\dot{R}$ in Cont. Eq.

$\longrightarrow$ then we can still define a vertical coordinate but it has modified properties
$\longrightarrow$ extra terms linked to $\dot{R}$ appear in most prognostic equations (not shown here - see paper)
$\longrightarrow$ deeper modification of the code

Outline

Horizontal momentum equation

Starting point – Sylvie found :

$$\rho \frac{d\mathbf{V}}{dt} = -\rho f \mathbf{k} \times \mathbf{V} - \boldsymbol{\nabla}_z p + [\text{viscous tensor term}]$$

transformation to mass coordinate

Applying transformation rules we simply find :

$$\frac{d\mathbf{V}}{dt} = -f \mathbf{k} \times \mathbf{V} - \frac{1}{\rho} \boldsymbol{\nabla}_\pi p - \frac{\partial p}{\partial \pi} \boldsymbol{\nabla}_\pi \phi + [\text{viscous tensor term}]$$

where still :

$$\phi = \int_{\eta}^{\eta_0} \frac{mRT}{p} d\eta$$

Vertical momentum equation

Starting point – Sylvie found :

$$\rho \frac{dw}{dt} = -g - \frac{1}{\rho}\frac{\partial p}{\partial z} - \frac{1}{\rho}\frac{\partial \left(\sum_k \rho_k \widetilde{w}_k^2\right)}{\partial z} + \frac{1}{\rho}\mathrm{div}(\sigma_w)$$

transformation to mass coordinate

Applying transformation rules we simply find :

$$\frac{dw}{dt} = g\left(\frac{\partial p}{\partial \pi} - 1\right) + g\frac{\partial \left(\sum_k \rho_k \widetilde{w}_k^2\right)}{\partial \pi} + \frac{1}{\rho}\mathrm{div}(\sigma_w)$$

Thermodynamics equation

Starting point – Sylvie found :

$$\frac{dT}{dt} = \frac{1}{\rho c_p}\left[\frac{dp}{dt} - \sum_k \rho_k \widetilde{w}_k \frac{\partial c_{pk} T}{\partial z} + \epsilon + L_v(T)\dot{\rho}_l + L_i(T)\dot{\rho}_i\right]$$

which, using the state equation, rewrites :

$$\frac{dT}{dt} = \frac{RT}{c_p}\frac{1}{p}\frac{dp}{dt} + \frac{Q_1}{c_p}$$

Besides, from the state equation $p = \rho RT$,

$$\frac{1}{p}\frac{dp}{dt} = -D_3 + \frac{1}{R}\frac{dR}{dt} + \frac{1}{T}\frac{dT}{dt}$$

Thermodynamics equation

Finally :

$$\frac{dT}{dt} = -\frac{RT}{c_v} D_3 + \frac{Q_1}{c_v} + \frac{T}{c_v}\frac{dR}{dt}$$

Pressure equation

Combining the state equation and the thermodynamic equation

$$\frac{dp}{dt} = -\frac{c_p}{c_v} p\, D_3 + \frac{p\, c_p}{R c_v} \frac{dR}{dt} + \frac{p\, Q_1}{c_v\, T}$$

Deriving equations

Summary for this section :

The modification of the system for the planned multiphase representation implies the addition of extra diagnostic terms from place to place. Some of them could be simply neglected, bases on analysis or practice.

Outline

Conclusion

Aim of the exercise

- Allow a representation of main multiphase effects (drag, interphase thermal equilibrium, etc.)
- But keep a single-variable representation of the dynamical field (wind)

Working hypothesis

- Use the total density and the total barycentric wind
- (other hypotheses are possible, examined but not retained)

Transformation to mass-coordinate context

- the working hypothesis implies a unique relevant definition of the coordinate
- the transformation seems to rise no particular problem
- as in z coordinate extra diagnostic terms to be added (or ignored)