Operational version of turbulence scheme
TOUCANS

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TOUCANS

T - Third
O - Order moments (TOMs)
U - Unified
C - Condensation
A - Accounting and
N - N-dependent
S - Solver (for turbulence and diffusion)
Turbulent diffusion

Turbulent flux

\[ w'\bar{\psi}' = -K_\psi \frac{\partial \bar{\psi}}{\partial z} \] - upper air

\[ w'\bar{\psi}' = C_\psi \sqrt{u^2 + v^2 (\psi - \psi_s)} \] - surface layer

- \( \bar{\cdot} \) - average, \( \cdot' \) - fluctuation, \( u, v, w \) - wind components, \( z \) - height, \( \psi \) - diffused variable
Diffused variables

- \( u, v \) - horizontal wind components
- \( q_t \) - total specific moisture
- \( s_{sL} = c_{pd} \left( 1 + \left[ \frac{c_{pv}}{c_{pd}} - 1 \right] q_t \right) T + g z - (L_v q_l + L_s q_i) \) - moist static energy
- \( K_M, K_H \) - coefficients of turbulent diffusion for momentum and heat
- \( C_D, C_H \) - drag coefficients for momentum and heat

- \( q_{l/i} \) - specific humidity of air for liquid or ice water, \( L_{s/v} \) latent heat latent heat of sublimation/vaporization, \( g \) - acceleration of gravity, \( T \) - air temperature, \( c_{pd} \) and \( c_{pv} \) specific heat values for dry air and water vapour
Turbulent diffusion

\[
\frac{\partial \psi}{\partial t} = -g \frac{\partial \rho \psi' w'}{\partial p}
\]

Upper air:
\[
\frac{\partial \psi}{\partial t} = -g \frac{\partial \rho K_{\psi} \frac{\partial \psi}{\partial z}}{\partial p},
\]

Surface:
\[
\frac{\partial \psi}{\partial t} = -g \frac{\partial \rho C_{\psi} (\psi_n - \psi_s)}{\partial p}
\]

\[
\tilde{\psi}_i^+ - \tilde{\psi}_i^- = -g \frac{\Delta t}{\delta p_i} [ K'_{\psi,i} (\tilde{\psi}_i^+ - \tilde{\psi}_{i+1}^+) + K'_{\psi,i-1} (\tilde{\psi}_{i-1}^+ - \tilde{\psi}_i^+) ]
\]

leads to inversion of 3-diagonal matrix

\[
K'_{\psi} = -\frac{\rho K_{\psi}}{\Delta z}
\]

\( \Delta t \) - time step, \( \rho \) - density, \( p \) - pressure

vertical levels

\[ ... \]

\[ \sim 0 \]

\[ \sim 1 \]

\[ 1 \]

\[ ... \]

\[ \sim i \]

\[ \sim i+1 \]

\[ i \]

\[ ... \]

\[ \sim n-1 \]

\[ \sim n \]

variables:

\[ u, v, S_{\text{SL}}, q_t \]

\[ K_M, K_H \]

\[ C_D, C_H \]

\[ T_s, q_s \]
Turbulent diffusion

Exchange coefficients

\[ K_M = \frac{\nu^4}{C_\epsilon} L \chi_3 \sqrt{e_k}, \quad K_H = C_3 \frac{\nu^4}{C_\epsilon} L \phi_3 \sqrt{e_k} \]

- \( \chi_3(\Pi, \nu, C_\epsilon, C_3, O_\lambda), \phi_3(\Pi, \nu, C_\epsilon, C_3, O_\lambda) \) - stability dependency functions
  (Bašták, Š. I., J.-F. Geleyn, and F. Váňa, 2014)
- \( e_k = \frac{1}{2} (u'u' + v'v' + w'w') \) - Turbulence Kinetic Energy
- \( \Pi = \frac{e_t - e_k}{e_k} \), \( e_t \) - Turbulence Total Energy
  - based on Zilitinkevich et al. (2013)
- \( \nu, C_\epsilon, C_3, O_\lambda \) - free parameters
- \( L \) - length scale
Drag coefficients

\[ C_D = \left( \frac{\kappa}{\ln \left( 1 + \frac{z}{z_0} \right)} \right)^2 F_M(Ri_B), \]
\[ C_H = \left( \frac{\kappa^2}{\ln \left( 1 + \frac{z}{z_0} \right) \ln \left( 1 + \frac{z}{z_{0h}} \right)} \right)^2 F_H(Ri_B) \]
\[ F_M(Ri_B) = \chi_3(Ri_B) \cdot f(Ri_B) \]
\[ F_H(Ri_B) = C_3 \frac{\phi_3(Ri_B)}{\chi_3(Ri_B)} \cdot F_M(Ri_B) \]
\[ Ri_B > 0 : f(Ri_B) = \chi_3(Ri_B) - C_3 \phi_3(Ri_B) Ri_B \]
\[ Ri_B \leq 0 : f(Ri_B) = 1 \]

\( Ri_B \) - bulk Richardson number, \( \kappa \) - Von Karman constant, \( z_0/z_{0h} \) - roughness lengths
Framework of stability dependency functions

Stability functions $\chi_3, \phi_3$:

- derived from TKE and TTE equations at equilibrium
- no existence of critical $Ri - Ri_{cr}$
- anisotropy of turbulence:
  - $\frac{\partial \chi_3}{\partial Ri} \neq 0$
  - $\phi_3 = \phi_Q(Ri) \left( 1 - \frac{2 O_\lambda e_k}{C_4 w'^2 \Pi} \right)$, $\frac{\partial \phi_Q}{\partial Ri} \neq 0$

  anisotropy energy conversion

- valid for whole range of $Ri$

($Ri$ - gradient Richardson number, $C_4$ - coefficient)
Framework of stability functions:

- based on Cheng et al. (2002) scheme:
  - anisotropy included
  - with $R_i_{cr}$

- modification that avoid existence of $R_i_{cr}$ result in general shape:

$$
\chi_3(R_i) = \frac{1 - \frac{R_i_f}{R}}{1 - R_i_f}, \quad \phi_3(R_i) = \frac{1 - \frac{R_i_f}{P}}{1 - R_i_f},
$$

$$
\phi_Q(R_i) = \frac{1 - \frac{R_i_f}{Q}}{1 - R_i_f}, \quad \frac{R_i}{R_i_f} = \frac{P(R - R_i_f)}{C_3 R (P - R_i_f)}
$$

$$
0 < \lim_{R_i \to \infty} P = R_i_{fc} < 1, \quad R_i_{fc} < \lim_{R_i \to \infty} R \equiv R_\infty \leq 1, \quad R_i_{fc} \leq \lim_{R_i \to \infty} Q \equiv Q_\infty \leq 1.
$$

($R_i_f = R_i K_H/K_M$ - flux Richardson number, $R_i_{fc} = \lim_{R_i \to \infty} R_i_f$ - critical $R_i_f$)
Framework of stability functions:

- the turbulent scheme then depends on:
  - 4 free parameters - \((C_\epsilon, C_3, \nu, O_\lambda)\),
    - following Schmidt and Schumann (1989)
    we assume: \(C_\epsilon = \pi \nu^2\)
  - 3 “functional dependencies” \((P, R, Q)\)

- 4 possible realisations:

<table>
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<th>eeEFB</th>
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<td>(P)</td>
<td>Const.</td>
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<td>Ri fun.</td>
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<td>(R)</td>
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<td>(Q)</td>
<td>Const.</td>
<td>Ri fun.</td>
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- emulation and extension
  eeQNSE : of QNSE - Sukoriansky et al. (2005)
  eeEFB : of EFB - Zilitinkevich et al. (2013)
Prognostic TKE - $e_k$ equation:

\[
\frac{de_k}{dt} = -g \frac{\partial}{\partial p} \left( \rho K_{e_k} \frac{\partial e_k}{\partial z} \right) + I + II - \frac{2 e_k}{\tau_k}
\]

\[
I = -u'w' \frac{\partial u}{\partial z} - v'w' \frac{\partial v}{\partial z},
\]

\[
II = E_{s_{sL}} (SCC) \overline{w'}s'_{sL} + E_{q_t,s_{sL}} (SCC) \overline{w'}q'_t
\]

- Influence of moisture via $E_{s_{sL}} (SCC)$ and $E_{q_t,s_{sL}} (SCC)$ - weights according to Marquet and Geleyn (2013)
- **SCC** - Shallow Convection Cloudiness - currently implicitly given by shallow convection parametrisation according to Geleyn (1987)
- **SCC** used also in turbulent diffusion of $q_{l/i}$

($K_{e_k}$ - turbulent exchange coefficients for TKE, $\tau_k = \frac{2L}{C_\epsilon \sqrt{e_k}}$ - dissipation time scale for TKE)
Stability dependent adjustment for turbulent energy modelling - TKE:

\[
\frac{de_k}{dt} = -g \frac{\partial}{\partial p} \left( \rho K_{e_k} \frac{\partial e_k}{\partial z} \right) + \frac{1}{\tau_\epsilon} (\tilde{e}_k - \hat{e}_k),
\]

\[
\tilde{e}_k = \frac{\tau_k}{2} (I + II), \quad K_{e_k} = \frac{\nu^2}{C_\epsilon} L \sqrt{\hat{e}_k},
\]

\[
K_M = \frac{\nu^4}{C_\epsilon} L \chi_3(Ri) \sqrt{e_k^+}, \quad K_H = C_3 \frac{\nu^4}{C_\epsilon} L \phi_3(Ri) \sqrt{e_k^+},
\]

full level variables, half level variables

(\(\tau_\epsilon = 0.5\tau_k\) -relaxation time scale, \(\overbrace{\text{overbrace}}\) operator - interpolation from half levels to full levels, \(\hat{\text{hat}}\) operator - interpolation from full levels to half levels.)
Stability dependent adjustment for turbulent energy modelling - TKE:

- enables a fully consistent treatment of the prognostic TKE
- enables longer times steps
- avoids difficulties with vertical staggering
  - TKE placed on full levels
Prognostic TTE/TPE

- based on Zilitinkevich et al. (2013)
- addition of second prognostic turbulent energy: Turbulent Potential Energy (TPE), or Turbulent Total Energy (TTE) = TKE + TPE
- stability parameter based on energy ratio (linked to fluxes) rather than on local gradients ($Ri$)
- enables modelling of counter-gradient heat transport maintained by velocity shear in very stable stratification
Prognostic TTE - $e_t$ equation:

\[
\frac{de_t}{dt} = -g \frac{\partial}{\partial p} \left( \rho K_{e_t} \frac{\partial e_t}{\partial z} \right) + l - \frac{2 e_t}{\tau_t}
\]

\[
\tau_t \equiv \tau_k \frac{C_4 (1 + \Pi)}{C_4 + 2 C_3 \Pi},
\]

\[
\Pi \equiv \frac{e_p}{e_k} = \frac{e_t}{e_k} - 1,
\]

\[
e_p = \frac{g}{\theta} \frac{\theta'^2}{2 \frac{\partial \theta}{\partial z}}
\]

\[
Ri_f = \frac{\Pi}{\frac{C_4}{2 C_3} + \Pi},
\]

\[
K_M = \frac{\nu^4}{C_\epsilon} L_3(Ri_f) \sqrt{e_k},
\]

\[
K_H = C_3 \frac{\nu^4}{C_\epsilon} L_\phi_3(Ri_f) \sqrt{e_k}
\]

($e_p$ - TPE, $K_{e_t}$ - turbulent exchange coefficient for TTE, $\tau_t$ - dissipation time scale for TTE, $Ri_f = Ri K_H/K_M$ - flux Richardson number, $C_4$ - coefficient)
Stability dependent adjustment for turbulent energy modelling - TTE:

\[ \frac{d \tilde{e}_t}{dt} = -g \frac{\partial}{\partial p} \left( \rho K_{et} \frac{\partial e_t}{\partial z} \right) + \frac{2}{\tau_t} (\tilde{e}_t - \hat{e}_t) \]

\[ \tilde{e}_t = \frac{\tau_t}{2} I, \quad K_{et} = \frac{\nu^2}{C_\epsilon} L \sqrt{\hat{e}_k} \]

\[ \tau_t = \frac{C_4 \left( 1 + \hat{\Pi} \right)}{C_4 + 2 C_3 \hat{\Pi}}, \quad \text{Ri}_f = \frac{\hat{\Pi}}{\frac{C_4}{2 C_3} + \hat{\Pi}} \]

\[ K_M = \frac{\nu^4}{C_\epsilon} L \chi_3(Ri_f) \sqrt{\hat{e}_k}, \quad K_H = C_3 \frac{\nu^4}{C_\epsilon} L \phi_3(Ri_f) \sqrt{e_k} \]

**full level variables**,  **half level variables**
Third Order Moments (TOMs) parametrisation

- based on Canuto et al., (2007)
- enables modelling of distant turbulent transport of heat caused by presence of semi-organised large eddies
- two step approach - local (down-gradient term only) solution is a reference
- stable and accurate algorithm immune against singularities
- requires iterations to improve accuracy
- influence of time-tendency terms parametrized via scaling factor
TOMs parametrisation:

\[
\begin{align*}
\overline{w' s'_{sL}} + A_t \frac{\partial \overline{w' s'_{sL}}}{\partial t} &= -K_H \frac{\partial s_{sL}}{\partial z} + A_{sL}^{sL} \frac{\partial w'^3}{\partial z} + A_{sL}^{sL} \frac{\partial w' s'^2_{sL}}{\partial z} + A_3 \frac{\partial w'^2 s'_{sL}}{\partial z} \\
\overline{w' q'_{t}} + A_t \frac{\partial \overline{w' q'_{t}}}{\partial t} &= -K_H \frac{\partial q_t}{\partial z} + A_{q_t}^{q_t} \frac{\partial w'^3}{\partial z} + A_{q_t}^{q_t} \frac{\partial w' q'^2_{t}}{\partial z} + A_3 \frac{\partial w'^2 q'_{t}}{\partial z} \\
\overline{w'^2 s'^2_{sL}} &= -\tau_k w'^2 \frac{\partial w' s'_{sL}}{\partial z}, \\
\overline{w'^2 q'^2_{t}} &= -0.3 \tau_k w'^2 \frac{\partial w' q'_{t}}{\partial z}, \\
\overline{w'^3} &= -0.06 \tau_k^2 w'^2 \left( E_{sL} \frac{\partial w' s'_{sL}}{\partial z} + E_{q_t,s_{sL}} \frac{\partial w' q'_{t}}{\partial z} \right) \\
\end{align*}
\]

(\(\overline{w'^2}\) - twice the vertical component of TKE, \(A_{sL}^{sL}, A_{q_t}^{q_t}, A_3, A_t\) - weights resulting from equations for \(\overline{w'^2}, \overline{w' \theta'}\) and \(\overline{\theta'^2}\))
TOMS - two step solver

- local diffusion: \( \frac{\partial \theta_{\text{loc}}}{\partial t} = \frac{\partial}{\partial p} \left( -g \rho K H \frac{\partial \theta_{\text{loc}}}{\partial z} \right) \)

- TOM's contribution computed in terms of:

  \( \delta s_{sL}^+ = s_{sL}^+ - s_{sL}^{\text{loc}} \)

  \( \delta q_t^+ = q_t^+ - q_t^{\text{loc}} \)
Code organisation

- Length scale $L$
- $\tilde{e}_k, K_{e_k}, \tau_\epsilon, \tilde{e}_t, K_{e_t}, \tau_t$
- TKE/TTE equation $e_k^+, e_t^+$
- $e_k^+, e_t^+ \rightarrow \Pi \rightarrow \chi_3(\Pi), \phi_3(\Pi)$
- ex. coefficients $K_{M/H}$
- diffusion equation
- TOMs contribution
- $e_k^-, e_t^- \rightarrow \Pi$
- $\chi_3(\Pi), \phi_3(\Pi)$
- drag coefficients $C_{D/H}$
- tendency of prognostic variables
Scores

TOUCANS+ACANE+B2+update of microphysics

- period: 03-23.1.2015, start: 0UTC
- black: ALARO0, red: ALARO1
- score: BIAS

Temperature 2m

Temperature cross section
TOUCANS + ACANE B2 + update of microphysics

- period: 03-23.1.2015, start: 0UTC
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Relative humidity 2m

Relative humidity cross section
TOUCANS+ACANEB2+update of microphysics

- period: 03-23.1.2015, start: 0 UTC
- black: ALARO0, red: ALARO1
- score: RMSE

Relative humidity 2m

Relative humidity cross section
Summary:

- operation setup with TOUCANS scheme
- stability dependency functions $\chi_3$ and $\phi_3$
  - include anisotropy, no $Ri_c r$
- 2 prognostic turbulent energies - TKE and TTE
- moisture influence via definition of moist buoyancy term - SCC influence
- TOMs parametrisation
  - in winter cases increases mixing in PBL - prog. TTE + stab. functions
Thank you for your attention!
Summary