Regional Cooperation for Limited Area Modeling in Central Europe

LACE – DEVELOPMENT IN DYNAMICS

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Hirlam coop.: Juan Simarro, Alvaro Subias (AEMET)
1. Finite element method in vertical discretization of NH model (J.Vivoda, P.Smolíková)

- based on hydrostatic version of VFE (being developed by A.Untch, M.Hortal)
- cooperation with HIRLAM colleagues (J.Simarro, A.Subias)
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2. Physical tendency of vertical velocity in NH (D.Lancz, I.Bašták-Durán, P.Smolíková)

3. ENO technique for SL interpolations (J.Mašek, A.Craciun)
VFE in NH

**Current status:** there is a working implementation of the VFE method in the NH model in the cycle CY40T1

**How well does it work?**

1. **Idealized tests** with known analytical solution in 2D vertical plane

2. **Real case study in high resolution** to show that the proposed method is stable and robust

3. **Accuracy study** of the vertical FE operators to show the advantage of FE discretization over FD one

4. **Problems**
Idealized tests:

1. **Baldauf-Brdar**: the linear expansion of sound and gravity waves in a channel induced by a weak warm bubble

   - originally in height based coordinate
   - with the boundary conditions: \( w=0 \) at top and bottom boundary
   - the solution: a set of waves that propagate horizontally
   - **in mass based vertical coordinate**: no fixed top boundary \( \Rightarrow \) atmosphere can evolve freely and move up and down at the top, we may impose strict sponge conditions, but not to avoid the vertical propagation of waves
   - vertical resolution 125m, regular levels in height up to 10km
Idealized tests:

1. **Baldauf-Brdar**: the linear expansion of sound and gravity waves in a channel induced by a weak warm bubble.
VFE in NH

Idealized tests:

1. **Baldauf-Brdar**: the linear expansion of sound and gravity waves in a channel induced by a weak warm bubble

Potential temperature:

4\textsuperscript{th} level in 0.5km, 40\textsuperscript{th} level in 5km, 76\textsuperscript{th} level in 9.5km


downarrow

7 April 2015, Helsingør, Denmark
2. **Simarro’s test**: the linear 3-dim expansion of sound and gravity waves induced by a weak warm bubble

- initial perturbation localized in the lower atmosphere
- propagation of a set of waves horizontally and vertically
- it will take some time to reach the upper atmosphere
- the vertical velocity at the top is zero in the analytical solution and there is no need to impose artificially the condition of the vertical velocity being zero at the top

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VFE in NH

8 April 2015, Helsingør, Denmark
2. Simarro’s test: the linear 3-dim expansion of sound and gravity waves induced by a weak warm bubble

Initial field of $\Theta$ vs. $w$ after 600s ~ 1200 time steps

- **VFE in NH**

9 April 2015, Helsingør, Denmark
2. Simarro’s test: the linear 3-dim expansion of sound and gravity waves induced by a weak warm bubble

Relative $\ell_2$-error for $w$.

- black: FD
- red: FE
- blue: FE approx
Real simulation: domain over Alpina region, 1.25km resolution, 87 vertical levels, 2tl PC + LGWADV, 2 series – January 2014 and July 2014, once per day from 00UTC + 24hours, timestep = 50s
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Results:
- VFE scheme used in NH with proper setting of FE parameters and “well defined” vertical levels placement may be as stable as FD scheme
- no clear impact of vertical discretisation method on objective scores
- there is an impact of the vertical discretisation method on the precipitation field
  - confirms results obtained for coarser horizontal resolution
Real simulation: stability measured by the average spectral norm of pressure departure variable.
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VFE in NH
Real simulation: stability measured by the average spectral norm of pressure departure variable
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**CZ 87 LEVELS**
- **VFD**
- **VFE**

**MIX 94 LEVELS**
- **VFD**
- **VFE**
**VFE in NH**

**Real simulation**: statistical analysis of 1 hour cumulated precipitation – **winter period**

Rescaled results - only 10% of common events are taken with the remaining number of events in which two series differ.

18 April 2015, Helsingør, Denmark
**VFE in NH**

**Real simulation:** statistical analysis of 1hour cumulated precipitation – **summer period**

Rescaled results - only 10% of common events are taken with the remaining number of events in which two series differ

19 April 2015, Helsingør, Denmark
Summary for real simulations:

1. VFE scheme with proper setting of parameters and vertical levels is as stable as VFD scheme in 1.25km resolution.

2. It is difficult to find any benefit from FE used in vertical discretization concerning objective scores.

3. The precipitation field is modified by FE in such a way that there is bigger number of grid points without rain (cumulated precipitations for 1 hour < 0.1mm) and bigger number of grid points with highest values of cumulated precipitations (>30mm/hour). Consequently, there is smaller number of grid points with modest rain between 0.1 and 30mm.
VFE in NH

Problems:

1. saturation with higher orders and vertical resolution
2. remaining FD features
3. ...
VFE in NH

- Vertical operators applied on a smooth function $\sin(\pi \eta)^3 \cos(\pi \eta)$
- Regular eta levels
- Saturation for higher orders
- Saturation for higher vertical resolutions

$\leq$ rounding error and high number of operations

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**Order of splines**

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**DER**

**INTEGRAL**

**SEC.DER**

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22 April 2015, Helsingør, Denmark
Work in progress:
- remaining FD features
- transformations $w \leftrightarrow d$ (Alvaro Subias invertible operators)
- BC in vertical Laplacian - study for $\sigma$ coordinate and 5 vertical levels (J.Vivoda):

the choice of BBC and TBC between $f=0$ and $\frac{\partial f}{\partial \sigma}=0$ for 2 vertical Laplacian definitions:

$$L_v^* X = \sigma \frac{\partial}{\partial \sigma} \left( \sigma \frac{\partial}{\partial \sigma} + 1 \right) X$$
$$L_v^* X = \sigma \left( 2 \frac{\partial}{\partial \sigma} + \sigma \frac{\partial^2}{\partial \sigma^2} \right) X$$

=> conclusion: form 2 with BBC imposed gives real and negative eigenvalues, other choices are wrong
ENO technique in SL interp.

ENO (Essentially Non-Oscilatory)/WENO (Weighted ENO) techniques (Alexandra Craciun and Ján Mašek)

Idea of Ján Mašek (inspired by literature):

to explore alternative interpolators which are
- less overshooting than Lagrange polynomials close to discontinuities
- more accurate than their quasi-monotonic versions

⇒ interpolation depending on the smoothness of the interpolated field
ENO technique in SL interp.

Second order interpolation scheme (quadratic) needs 3 points to find, we have 4 points stencil: we may choose the first triplet => interpolated value $p_1$
ENo technique in SL interp.

Second order interpolation scheme (quadratic) needs 3 points to find $p_2$, we have 4 points stencil: or the second triplet $\Rightarrow$ interpolated value $p_2$
ENO technique in SL interp.

ENOS chooses the smoothest solution \((w_1=0\ or\ w_2=0)\)

WENO weighted combination based on smoothness

\[
S_i = |y_{i+1} - 2y_i + y_{i-1}|
\]

Linear/cubic \(p_1, p_2\) interpolated with linear/cubic Lagrange polynomial, weights depend on smoothness

\[
y = p_1 \cdot w_1 + p_2 \cdot w_2, \quad w_1 + w_2 = 1
\]
ENO technique in SL interp.

Third order interpolation scheme (cubic) needs 4 points to find $\bullet$:
ENO technique in SL interp.

Third order interpolation scheme (cubic) needs 4 points to find $p_1$: 

\[ y_0, y_1, y_2, y_3, y_4, y_5 \]
ENO technique in SL interp.

Third order interpolation scheme (cubic) needs 4 points to find $p_2$.
ENO technique in SL interp.

Third order interpolation scheme (cubic) needs 4 points to find \( \bullet \):

\[
y = w_1 p_1 + w_2 p_2 + w_3 p_3
\]
ENO technique in SL interp.

Third order interpolation scheme (cubic) needs 4 points to find $\bullet$:

$\Rightarrow$ 6 points stencil needed for ENO/WENO interpolations !!!
ENO technique in SL interp.

**Toy model** – 1D linear advection of rectangular pulse in a periodic domain (courtesy of Ján Mašek)

- **quadratic Lagrange**
- **quadratic ENO**
- **quadratic WENO**

33 April 2015, Helsingor, Denmark
ENO technique in SL interp.

**Toy model** – 1D linear advection of rectangular pulse in a periodic domain (courtesy of Ján Mašek)
ENO technique in SL interp.

Robert’s test in 2D model:
- warm bubble (+0.5K)
in the field of potential
temperature (300K)
advected with the
wind speed 2m/s
(courtesy of A.Craciu)
ENO technique in SL interp.

Conclusions:

- Quadratic interpolator too smoothing to work well

- Cubic ENO/WENO technique promising, but technically and computationally demanding (number of cubic interpolations increased from 7 to 21 !!!, usage of NSTENCILWIDE=3 ???)

- Combined linear/cubic interpolation may be easily tested and gives promising results – controlled damping depending on the interpolated field

⇒ 2 last points worth to be tested in 3D – planned for future work
Tak for din opmærksomhed !!!
Hvis du har spørgsmål ...