AN ALTERNATE WAY TO TREAT HELMHOLTZ EQUATION IN THE NHEE MODEL.

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INTRODUCTION.

- We present an alternate way to eliminate equations in the NHEE linear system.
- Elimination will be done in order to have horizontal divergence $D'_{t+\Delta t}$ as unknown in the Helmholtz equation, instead of $d_{t+\Delta t}$.

INTRODUCTION.

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SOME DENOTATIONS.

- M is the mapping factor.
- $\overline{M}$ is a reference mapping factor for semi-implicit computations.
- a is the Earth mean radius.
- $\mathbf{V}$ is the horizontal geographical wind. Components are U and V.
- $D^{'}$ is the reduced divergence of horizontal wind.
- T is the temperature.
- T^* is a vertically-constant reference temperature for the semi-implicit scheme.
- $T_{\rm a}^*$ is a cold vertically-variable reference temperature for the semi-implicit scheme (hidden in linear operators $\mathbf{L}^{**}$, $\mathbf{T}^{**}$).
- Π is the hydrostatic pressure, $\Pi_{\rm s}$ is the hydrostatic surface pressure.
- Π^* is a reference pressure and $\Pi_{\rm s}{}^*$ is a reference surface pressure for the semi-implicit scheme.

SOME DENOTATIONS (CONT'D).

- g is the gravity acceleration constant, assumed to be vertically constant in the current documentation.
- R_{d} the gas constant for dry air.
- $c_{\mathrm{p}_{\mathrm{d}}}$ is the specific heat at constant pressure for dry air.
- c_{vd} is the specific heat at constant volume for dry air.
- $\kappa_{\mathrm{d}} = R_{\mathrm{d}}/c_{\mathrm{p}_{\mathrm{d}}}$.
- ∇' is the reduced first order horizontal gradient.
- p is the total pressure, p_{s} is the surface total pressure.
- $\hat{Q}$ is the pressure departure variable. Expression of $\hat{Q}$ is $\hat{Q} = \log \frac{p}{\Pi}$.
- d is the vertical divergence.
- β : tunable coefficient for the semi-implicit scheme (between 0 and 1).
- γ, τ, ν, μ, $\mathbf{G}^*$, $\mathbf{S}^*$, $\mathbf{N}^*$, $\mathbf{L}^{**}$ (modified Laplacian operator), $\mathbf{T}^{**}$ are generic denotations for linear operators H, C, N are intermediate constants used in the semi-implicit scheme of the NHEE and NHQE models.

DEFINITION OF *COR*.

- When eliminating equations, quantity *COR* appears ($COR = 0$ in the continuous equations).
- Expression of *COR* is :

$$COR = \frac{c_{\mathrm{vd}}}{R_{\mathrm{d}}^2 T^*}\gamma\tau - \frac{c_{\mathrm{vd}}}{R_{\mathrm{d}} c_{\mathrm{pd}}}\gamma - \frac{c_{\mathrm{vd}}}{R_{\mathrm{d}} T^*}\tau + \frac{c_{\mathrm{vd}}}{c_{\mathrm{pd}}}\nu = \frac{c_{\mathrm{vd}}}{c_{\mathrm{pd}}}[\mathbf{G}^*\mathbf{S}^* - \mathbf{S}^* - \mathbf{G}^* + \mathbf{N}^*] \quad (1)$$

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NHEE LINEAR SYSTEM TO BE SOLVED.

$$\log(\Pi_{\mathrm{s}})_{t+\Delta t} + \beta \Delta t \nu (\overline{M}^2 D'_{t+\Delta t}) = \mathcal{P}^* \quad (2)$$

$$D'_{t+\Delta t} + \beta \Delta t \nabla'^2 [\gamma T_{t+\Delta t} - T^*(\gamma \hat{Q}_{t+\Delta t}) + \mu \log(\Pi_{\mathrm{s}})_{t+\Delta t} + R_{\mathrm{d}} T^* \hat{Q}_{t+\Delta t}] = \mathcal{D}'^* \quad (3)$$

$$\hat{Q}_{t+\Delta t} + \beta \Delta t \left[\frac{c_{\mathrm{pd}}}{c_{\mathrm{vd}}} (\overline{M}^2 D'_{t+\Delta t} + d_{t+\Delta t}) - \frac{c_{\mathrm{pd}}}{R_{\mathrm{d}} T^*} \tau (\overline{M}^2 D'_{t+\Delta t}) \right] = \hat{\mathcal{Q}}^* \quad (4)$$

$$d_{t+\Delta t} + \beta \Delta t \frac{g^2}{R_{\mathrm{d}} T^*} (\mathbf{L}^{**} \hat{Q}_{t+\Delta t}) = \hat{\mathcal{D}}^* \quad (5)$$

$$T_{t+\Delta t} + \beta \Delta t \frac{R_{\mathrm{d}} T^*}{c_{\mathrm{vd}}} [\overline{M}^2 D'_{t+\Delta t} + d_{t+\Delta t}] = \mathcal{T}^* \quad (6)$$

ELIMINATION OF VARIABLES.

- Elimination of T, $\hat{Q}$ and $\log(\Pi_{\rm s})$ between equations (2), (3), (4) and (6) leads to equation (7) :

$$D'_{t+\Delta t} - (\beta\Delta t)^2 \nabla'^2 [\{R_{\rm d} T^* (\frac{\gamma}{R_{\rm d}} - 1)(\frac{c_{\rm p\,d}}{R_{\rm d} T^*}\tau - \frac{c_{\rm p\,d}}{c_{\rm v\,d}}) + \frac{R_{\rm d} T^*}{c_{\rm v\,d}}\gamma + R_{\rm d} T^* \nu\} \overline{M}^2 D'_{t+\Delta t}$$
$$+ \{-R_{\rm d} T^* \frac{c_{\rm p\,d}}{c_{\rm v\,d}} (\frac{\gamma}{R_{\rm d}} - 1) + \frac{R_{\rm d} T^*}{c_{\rm v\,d}}\gamma\} d_{t+\Delta t}]$$
$$= \mathcal{D}'^* + \beta\Delta t \nabla'^2 [R_{\rm d} T^* (\frac{\gamma}{R_{\rm d}} - 1)\hat{\mathcal{Q}}^* - \gamma \mathcal{T}^* - R_{\rm d} T^* \mathcal{P}^*] \quad (7)$$

$\mathcal{D}'^{**}$ is defined by equation (8) :

$$\mathcal{D}'^{**} = \mathcal{D}'^* + \beta\Delta t \nabla'^2 \left[R_{\rm d} T^* (\frac{\gamma}{R_{\rm d}} - 1)\hat{\mathcal{Q}}^* - \gamma \mathcal{T}^* - R_{\rm d} T^* \mathcal{P}^* \right] \quad (8)$$

We use the relationship $c_{\rm p\,d} - c_{\rm v\,d} = R_{\rm d}$, the definition of C and we isolate the term COR (see equation (1)) : this equation can be rewritten :

$$[-(\beta\Delta t)^2 \nabla'^2 (C^2 - T^*\gamma)] d_{t+\Delta t} + [\mathrm{I} - (\beta\Delta t)^2 \nabla'^2 (C^2(1 + COR)) \overline{M}^2] D'_{t+\Delta t} = \mathcal{D}'^{**} \quad (9)$$

ELIMINATION OF VARIABLES (CONT'D).

- Elimination of $\hat{Q}$ between equations (4) and (5) leads to equation (10) :

$$d_{t+\Delta t} - (\beta \Delta t)^2 \left[\frac{\mathbf{L}^{**}}{H^2}(-c_{\mathrm{pd}}\tau + C^2)\overline{M}^2 D'_{t+\Delta t} + \frac{C^2}{H^2}\mathbf{L}^{**} d_{t+\Delta t} \right] = \hat{\mathcal{D}}^{**} \quad (10)$$

where $\hat{\mathcal{D}}^{**}$ is defined by :

$$\hat{\mathcal{D}}^{**} = \hat{\mathcal{D}}^{*} + \beta \Delta t \left[-\frac{g}{H} \mathbf{L}^{**} \hat{\mathcal{Q}}^{*} \right] \quad (11)$$

OLD WAY : HELMHOLTZ EQUATION WITH d AS UNKNOWN.

- When $COR = 0$ (constraint C1), elimination of $D^{'}$ between equations (9) and (10) leads to Helmholtz equation (12) :

$$(\mathtt{I} - \beta^2 \Delta t^2 \; \mathtt{B} \; \overline{M}^2 \nabla^{'2}) d_{t+\Delta t} = \mathcal{R} \tag{12}$$

where :

$$\mathtt{B} \; = C^2 \left(\mathtt{I} - \beta^2 \Delta t^2 C^2 \frac{\mathbf{L}^{**}}{H^2} \right)^{-1} \left(\mathtt{I} + \beta^2 \Delta t^2 N^2 \mathbf{T}^{**} \right) \tag{13}$$

and :

$$\mathcal{R} = \left(\mathtt{I} - \beta^2 \Delta t^2 C^2 \frac{\mathbf{L}^{**}}{H^2} \right)^{-1} \left[(\mathtt{I} - \beta^2 \Delta t^2 C^2 \overline{M}^2 \nabla^{'2}) \hat{\mathcal{D}}^{**} + \beta^2 \Delta t^2 \frac{\mathbf{L}^{**}}{H^2} (-c_{\mathrm{p_d}} \tau + C^2) \overline{M}^2 \mathcal{D}^{'**} \right. \tag{14}$$

OLD WAY : HELMHOLTZ EQUATION WITH d AS UNKNOWN.

If we compare with the LHS of the Helmholtz equation in the hydrostatic case :

- The unknown is $d_{t+\Delta t}$.
- The order of $\overline{M}^2$ and ∇'^2 is inverted.
- B now depends on Δt, it must be recomputed each time the timestep is changed.

Additional remarks :

- Operator $\mathbf{T}^{**}$ appears, which is simple when constraint C2 is matched (ex : VFD with NDLNPR=1) and more tricky otherwise (ex : VFE-NH).

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OLD WAY (CONT'D).

If constraint C1 is not matched (non zero COR).

- This is the case for example for VFE-NH.
- An iterative algorithm has been implemented, with **NITERHELM** iterations.
- Predictor step : replace COR by 0 and proceed as above.
- Corrector steps : the term containing COR is put in the RHS, it is multiplied by $D'_{t+\Delta t}$ at the previous iteration. The LHS is unchanged, so the elimination and the Helmholtz solving can be done like in the predictor step.
- We rather take as unknowns the increments between the current iteration and the predictor step, that allows to simplify the calculation of the RHS for the corrector step.

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NEW WAY : HELMHOLTZ EQUATION WITH D' AS UNKNOWN.

General formalism for D' and d equations :

- Equation (9) has the following shape :

$$[\mathrm{I} + (\beta\Delta t)^2 \nabla'^2 \mathrm{B}_1 \overline{M}^2] D'_{t+\Delta t} + [(\beta\Delta t)^2 \nabla'^2 \mathrm{B}_2] d_{t+\Delta t} = \mathcal{D}'^{**} \quad (15)$$

- Equation (10) has the following shape :

$$[(\beta\Delta t)^2 \mathrm{B}_3 \overline{M}^2] D'_{t+\Delta t} + [\mathrm{I} + (\beta\Delta t)^2 \mathrm{B}_4] d_{t+\Delta t} = \hat{\mathcal{D}}^{**} \quad (16)$$

- where :

$$\mathrm{B}_1 = -C^2(1 + COR)$$
$$\mathrm{B}_2 = -(C^2 - T^*\gamma)$$
$$\mathrm{B}_3 = -(1/H^2)\mathbf{L}^{**}(C^2 - c_{\mathrm{p_d}}\tau)$$
$$\mathrm{B}_4 = -(C^2/H^2)\mathbf{L}^{**}$$

NEW WAY (CONT'D).

Elimination :

- The following combination is used :

$$(\text{EQ } 15) - [(\beta\Delta t)^2 \nabla'^2 \mathrm{B}_2][\mathrm{I} + (\beta\Delta t)^2 \mathrm{B}_4]^{-1} (\text{EQ } 16)$$

- And additionally we use the fact that ∇'^2 commute with B_1, B_2, B_3 and B_4.
- One obtains the following Helmholtz equation :

$$(\mathrm{I} - \beta^2 \Delta t^2 \; \mathrm{B} \; \nabla'^2 \overline{M}^2) D'_{t+\Delta t} = \mathcal{R} \quad (17)$$

where :

$$\mathrm{B} \; = -\mathrm{B}_1 + (\beta\Delta t)^2 \mathrm{B}_2 [\mathrm{I} + (\beta\Delta t)^2 \mathrm{B}_4]^{-1} \mathrm{B}_3 \quad (18)$$

and :

$$\mathcal{R} = \mathcal{D}'^{**} - (\beta\Delta t)^2 \nabla'^2 \mathrm{B}_2 [\mathrm{I} + (\beta\Delta t)^2 \mathrm{B}_4]^{-1} \hat{\mathcal{D}}^{**} \quad (19)$$

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Properties of B :

- After calculations (not detailed) using expression of *COR* and the relationship $c_{p_d} - c_{v_d} = R_d$, B can be rewritten :

$$\mathrm{B} = \mathrm{B}_{\mathrm{hyd}} + \mathrm{B}_2[(\beta\Delta t)^2[\mathrm{I} + (\beta\Delta t)^2\mathrm{B}_4]^{-1} - \mathrm{B}_4{}^{-1}]\mathrm{B}_3 \quad (20)$$

  where :

$$\mathrm{B}_{\mathrm{hyd}} = \gamma\tau + R_d T^* \nu$$

- An alternate way to write B is :

$$\mathrm{B} = \mathrm{B}_{\mathrm{hyd}} + \frac{1}{C^2}[C^2 - T^*\gamma][(\beta\Delta t)^2 \frac{C^2}{H^2}[\mathrm{I} + (\beta\Delta t)^2\mathrm{B}_4]^{-1}\mathbf{L}^{**} + \mathrm{I}][C^2 - c_{p_d}\tau] \quad (21)$$

  Term *COR* and constraint C1 have disappeared.
- For large timesteps, B converges towards $\mathrm{B}_{\mathrm{hyd}}$.
- For very small timesteps, B converges towards $C^2(1 + COR)$.

NEW WAY (CONT'D).

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Remarks and advantages :

- Easier to code than initially expected.
- The code is significantly simpler than with the old method.
- Closer to hydrostatic model design.
- Options **LSIDG** and **LIMPF** are simpler to implement (easier to re-use hydrostatic model pieces of code).
- No C1 constraint, quantity *COR* does not appear any longer.
- No iterative algorithm for VFE-NH.
- Operator $\mathbf{T}^{**}$ does not appear any longer.
- B easily writes as a sum of hydrostatic and anhydrostatic contributions.
- Calculations use quantity $[\mathrm{I} + (\beta\Delta t)^2 \mathrm{B}_4]^{-1}$ which is already computed with the old method (array **SIFACI**).
- **It is actually surprising that this solution has not been thought and coded before year 2007 ! ! !**

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REPRODUCIBILITY BETWEEN NEW WAY RESULTS AND OLD WAY RESULTS :

Some results :

- Constraint C1 matched (VFD) : small numerical differences.
- Constraint C1 not matched (VFE-NH) : the old algorithm with NITERHELM>10 converges towards the solution given by the new one.

FIGURES :

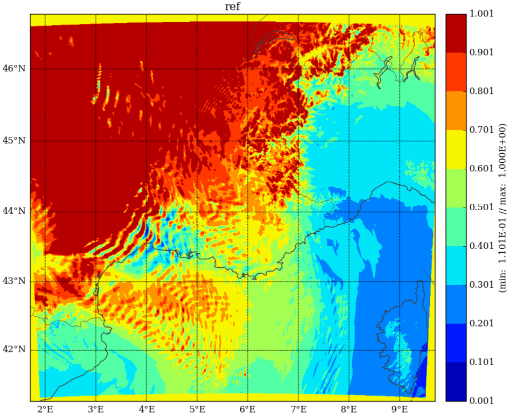

Figure – OC0500 : HU850 range 6h ; REF (init 31/03/2015 00TU).

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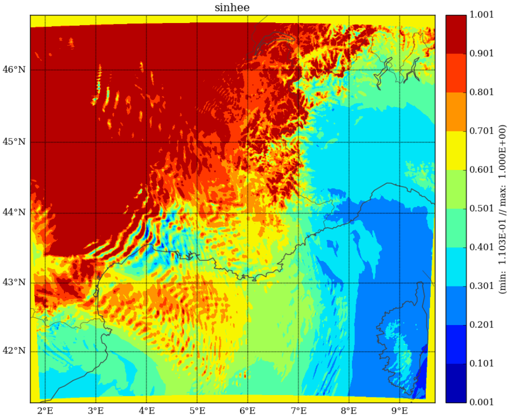

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NEW WAY : CALENDAR OF IMPLEMENTATION.

Calendar :

- It is coded on the top of CY45T1 for both global and LAM models.
- Expected to enter the code in CY46T1 and CY47.

THANK YOU / MERCI.