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Localized horizontal discretizations with appropriate adjustment properties for the ALADIN dynamics

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Motivation

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Conclusions

- Aladin/Arome/Harmonie is a semi-implicit, semi-Lagrangian *spectral* model
- When going to higher resolutions and larger domain sizes, we will face some *scientific* and *technical* challenges:
  - Representation of non-smooth fields (e.g. high-resolution orography) is problematic
  - The atmospheric reference state for the SI must be spatially homogeneous
  - Spectral transforms require domain-wide (MPI) communications

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  - The atmospheric reference state for the SI must be spatially homogeneous
  - Spectral transforms require domain-wide (MPI) communications
- Study the replacement of the spectral basis functions with local basis functions (finite elements)
- First focus on scientific impact
- This idea is not original (Staniforth, 1977)

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- **Constraint 2: keep time step organization**

  1. inverse FFT
  2. physics
  3. semi-Lagrangian interpolations
  4. explicit dynamics
  5. LBC treatment
  6. forward FFT
  7. solve Helmholz equation in spectral space

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  keep these! (items 2–5)

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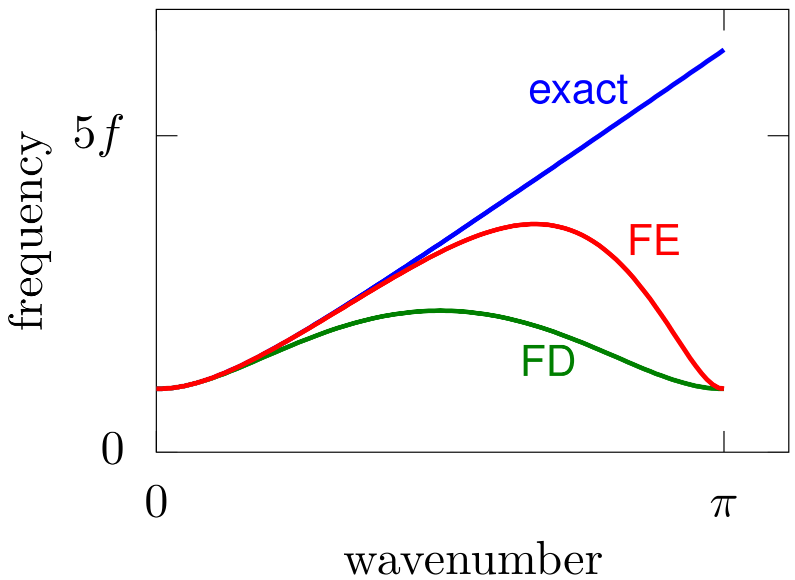
- Considering the linearized SWE equations in (u, v, h)

$$\frac{du}{dt} + g\frac{\partial h}{\partial x} + fv = 0$$
$$\frac{dv}{dt} + g\frac{\partial h}{\partial y} - fu = 0$$
$$\frac{dh}{dt} + H\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0,$$

it is well known that localized schemes (finite differences/finite elements) on A-grid give *bad dispersion relations* for gravity wave propagation



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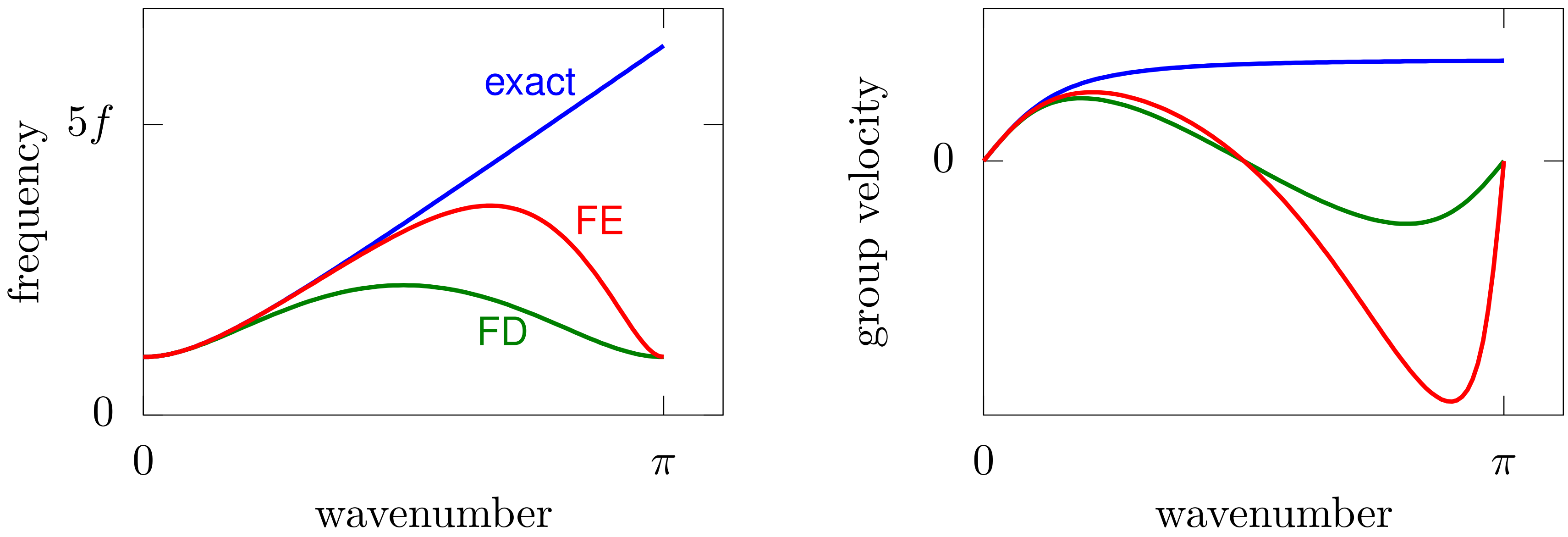

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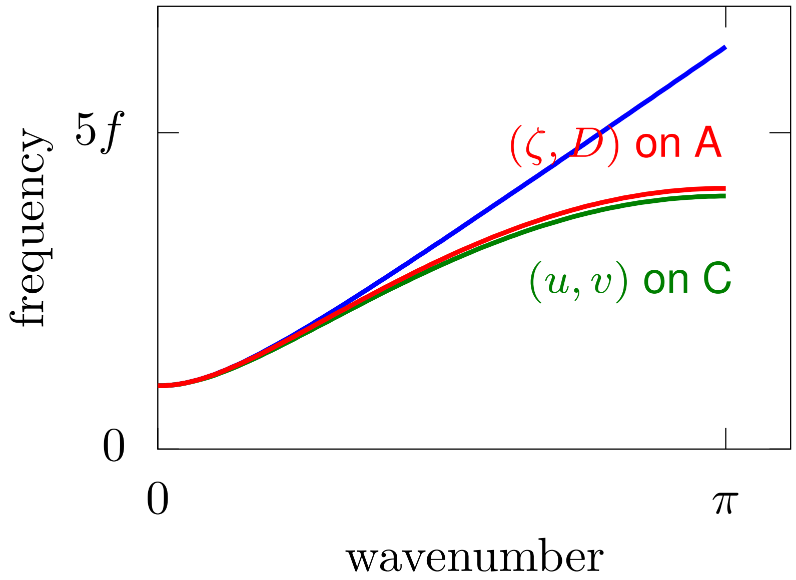
- Two possible solutions:



- Go to a staggered (C-) grid ⇒ not within constraints!
- Reformulate in terms of vorticity/divergence (ζ, D) instead of (u, v)

  ⇒ not entirely within constraints; we want to keep the RHS in (u, v)

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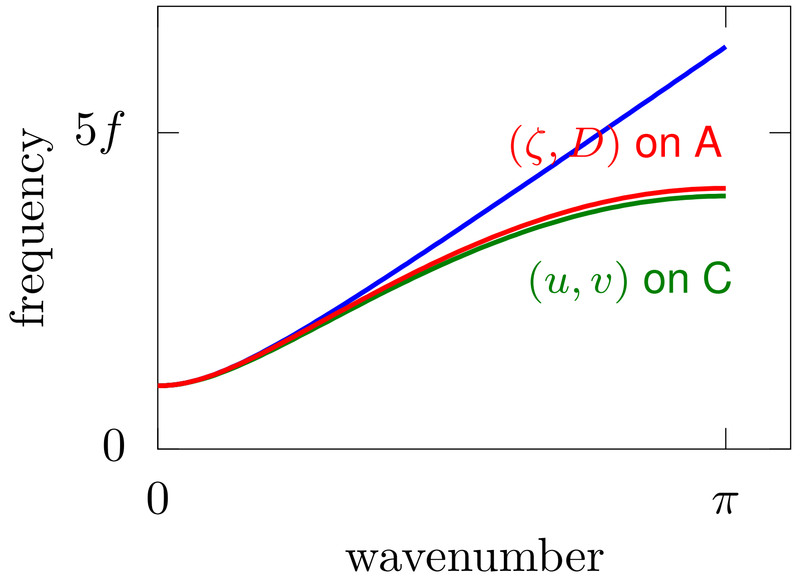

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So we will try a *hybrid* $(u, v)/(\zeta, D)$ approach.

Formulation for SWE

1. 3TL time discretization is done in (u, v)

$$
\begin{aligned}
u^{+} + g\Delta t \left(\frac{\partial h}{\partial x}\right)^{+} &= R_u \\
v^{+} + g\Delta t \left(\frac{\partial h}{\partial y}\right)^{+} &= R_v \\
h^{+} + H\Delta t \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)^{+} &= R_h
\end{aligned}
$$

2. We switch to (ζ, D) without touching R_u, R_v and R_h

$$
\begin{aligned}
D^{+} + g\Delta t \nabla^2 h^{+} &= \frac{\partial R_u}{\partial x} + \frac{\partial R_v}{\partial y} \\
\zeta^{+} &= \frac{\partial R_v}{\partial x} - \frac{\partial R_u}{\partial y} \\
h^{+} + H\Delta t D^{+} &= R_h
\end{aligned}
$$

3. We solve this system with FE to $(\zeta^{+}, D^{+}, h^{+})$
4. We transform (ζ, D) back to (u, v), using FE

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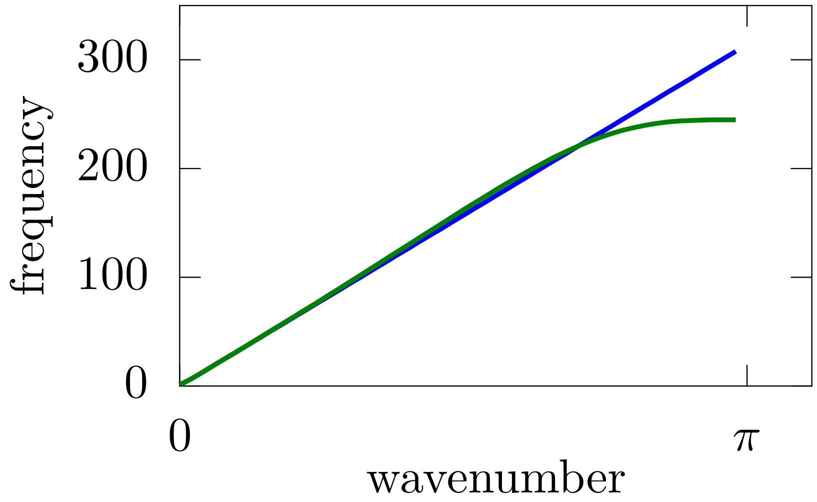

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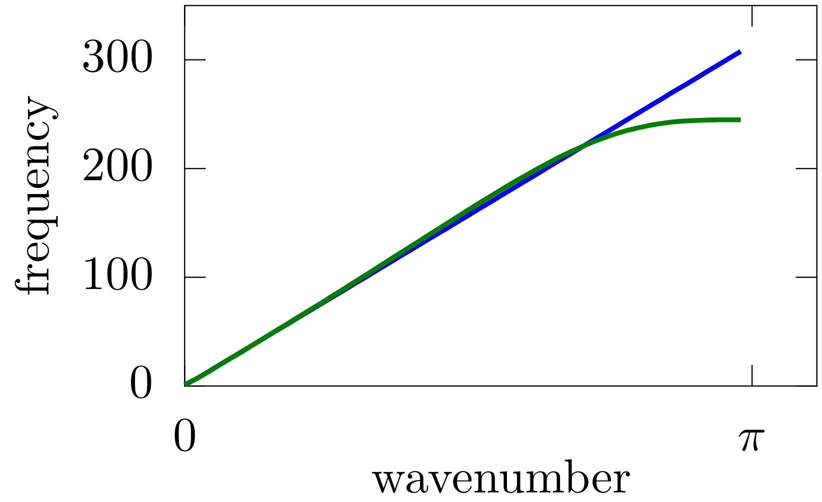
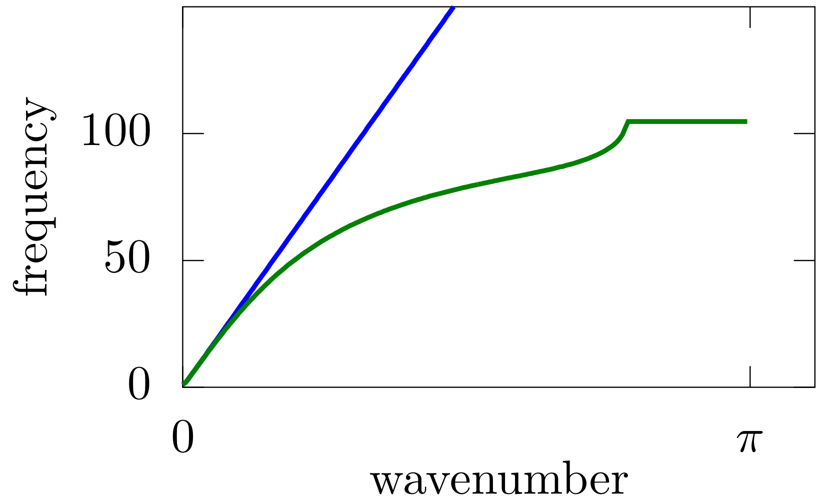


- But for larger timestep:



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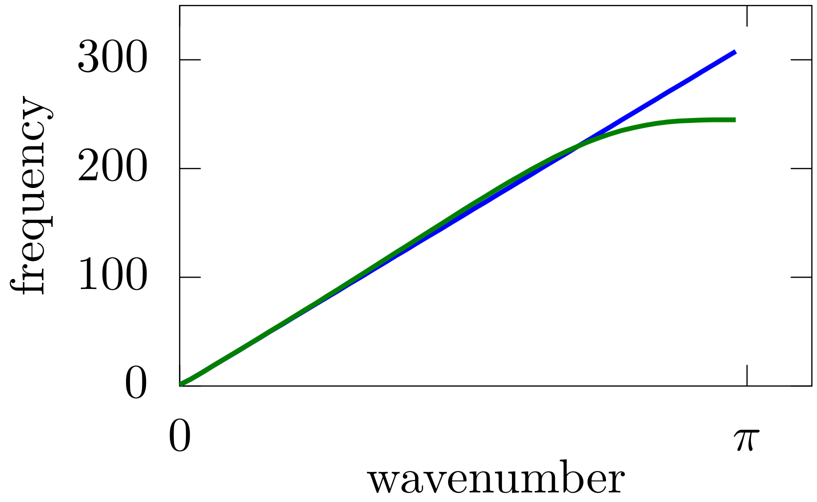

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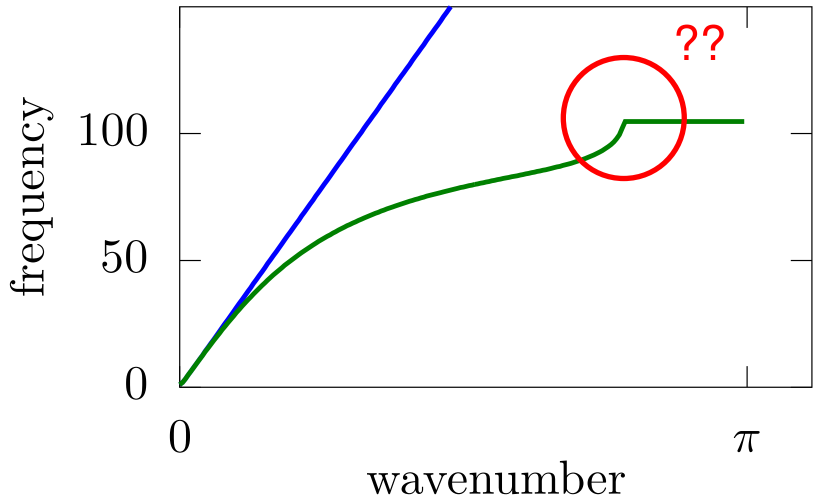

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- This behavior turns out to be the consequence of

1. with finite elements,

$$\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial x}\right) \neq \frac{\partial^2 \psi}{\partial x^2}$$

2. there exists a time asymmetry in our scheme:

LHS contains	$\nabla^2 h^+$
RHS contains	$\frac{\partial R_u}{\partial x} + \frac{\partial R_v}{\partial y}$

with R_u containing $\frac{\partial h^-}{\partial x}$ and R_v containing $\frac{\partial h^-}{\partial y}$.

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- **Solution**: modify the pressure gradient terms in R_u and R_v to make the scheme symmetric again.

- **Remark**: such an asymmetry is also present in the Staniforth (1986) scheme, which performs poorly for *small* Δt.

Consequences of asymmetry

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- Resulting symmetrized hybrid $(u, v)/(\zeta, D)$ FE scheme:

$$\begin{aligned}
\mathcal{S}_{xy} D^+ + g\Delta t\left(\mathcal{Q}_x + \mathcal{Q}_y\right) h^+ &= \mathcal{L}_x \tilde{R}_u + \mathcal{L}_y \tilde{R}_v \\
\mathcal{S}_{xy} \zeta^+ &= \mathcal{L}_x \tilde{R}_v - \mathcal{L}_y \tilde{R}_u \\
\mathcal{S}_{xy} h^+ + H\Delta t \mathcal{S}_{xy} D^+ &= \mathcal{S}_{xy} R_h \\
\mathcal{S}_{xy} D^+ &= \mathcal{L}_x u^+ + \mathcal{L}_y v^+ \\
\mathcal{S}_{xy} \zeta^+ &= \mathcal{L}_x v^+ - \mathcal{L}_y u^+
\end{aligned}$$

Consequences of asymmetry

- Resulting symmetrized hybrid $(u, v)/(\zeta, D)$ FE scheme
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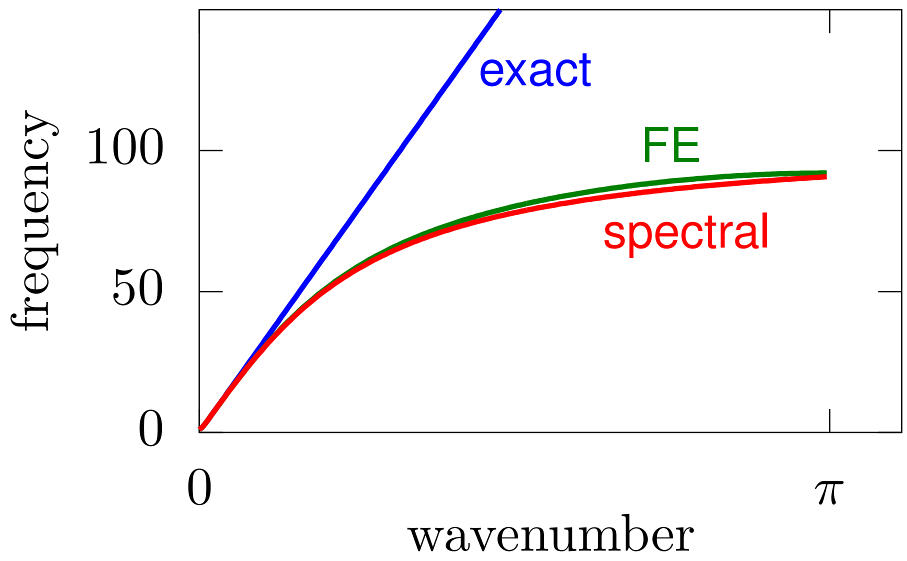

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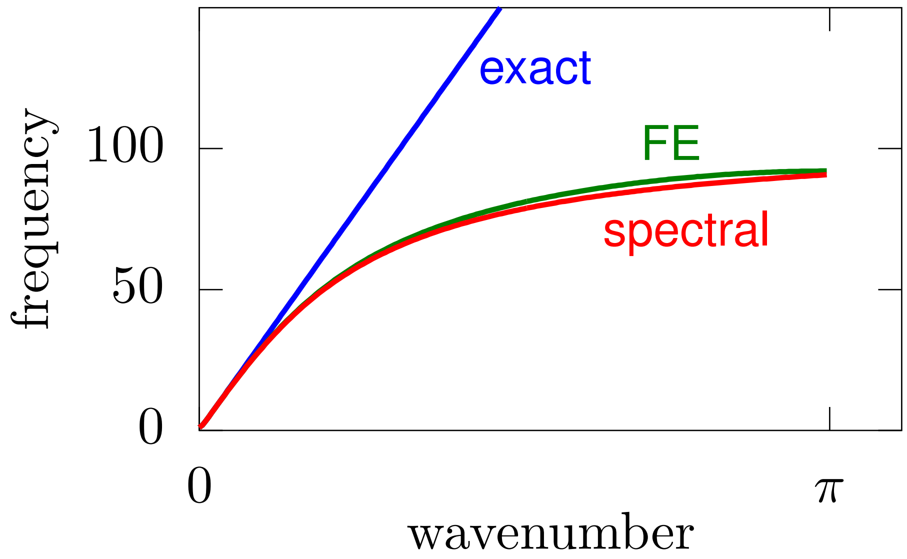

- Note that a 2TL variant of the scheme can also be derived.

Conclusions

We present a localized horizontal discretization scheme:

- finite-element based
- on our non-staggered A-grid
- which fits into the ALADIN semi-implicit, semi-Lagrangian algorithmics
- doesn't require modifications to gridpoint calculations (physics!)
- has an excellent dispersion relation for gravity waves
- opens the way for answering scientific questions like a non-homogeneous reference state and influence of steep orography

Thank you !