Why high-order A-grid finite differences are the way to go

About testing finite differences schemes within the current ALADIN model.

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Conclusion of my PhD earlier this year...

PhD research was built around analysis (SWE) and real model tests.
The conclusion:

Let us come back to the question that formed the basis of this thesis: *Can we use within the current spectral SISL ALADIN model a local horizontal spatial discretization scheme and how to do this?*.

This thesis provides arguments that a local solver can be added to the ALADIN framework while retaining most of the current code organization. FD spatial discretization methods based on the Z-grid approach suffer from an eigenmode decomposition problem, which mainly manifests itself during the first timesteps. Similar FD tests were undertaken within an A-grid approach and no fingerprint of the spurious waves that are diagnosed in analytical A-grid tests was found. The A-grid approach combined with fourth- or higher-order FD spatial discretization yields results close to the spectral experiments for ALARO tests. Therefore, higher-order A-grid methods are a promising candidate for a modular implementation of local schemes within ALADIN.

In this presentation I will give some background on this conclusion and focus on how the spectral ALADIN framework can be used to model the impact of using finite-differences...

Outline presentation

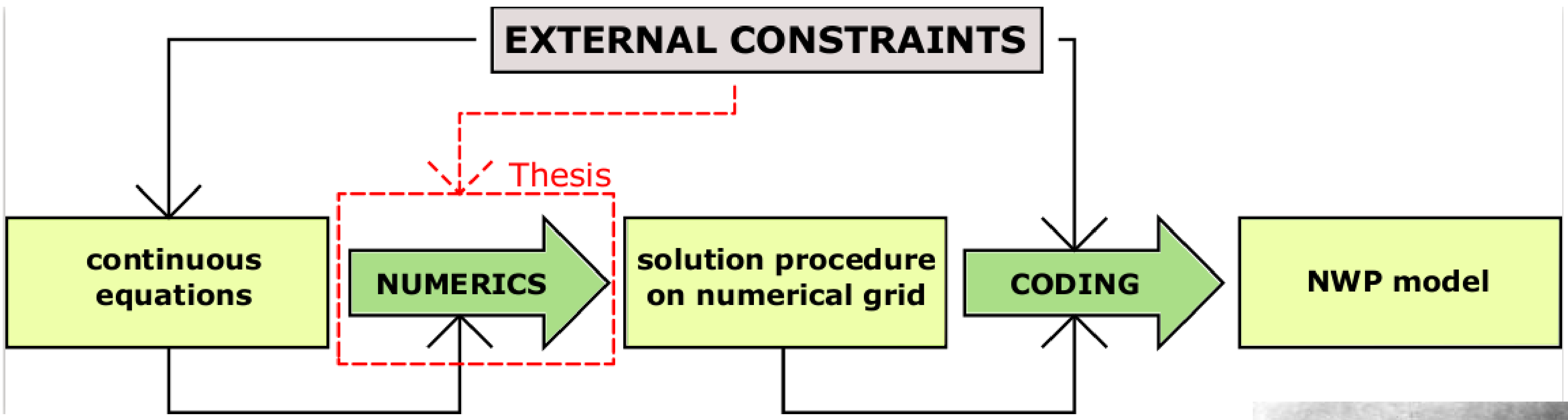
Context of this research and summary of the analysis

The real model FD tests

What’s next?

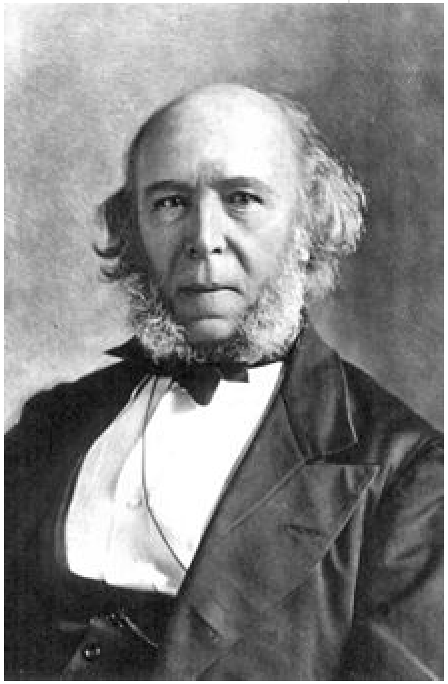
Context of this research

Development of NPW model consists of different steps. The choices made depend on **external constraints**.



One example of a constraint is the available HPC infrastructure.

Constraints can evolve in time and a NWP model should be ready to adapt…



Why should we care about local horizontal spatial discretization methods?

<u>Strength spectral method:</u>

Combining a **spectral spatial** approach with a **SISL time** discretization permits stable, long timestep integrations while solving efficiently the implicit Helmholtz problem.

Spectral spatial discretization is the core of IFS, ARPEGE, ALADIN,… and it helped to make this models highly succesful.

<u>But:</u>

→ not very flexible (e.g. impossible to get horizontally inhomogeneous terms in SI solver)
→ needs global communication but what on massively parallel computer architectures?

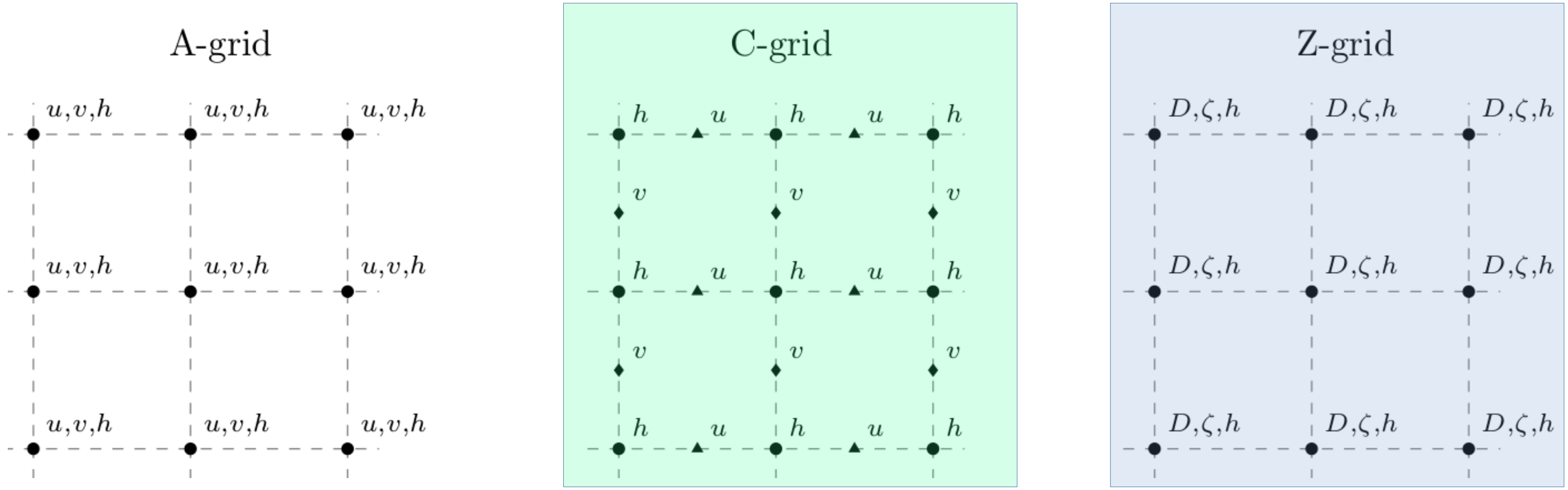
We should investigate local spatial discretization alternatives (e.g. finite differences) but **modularity** is crucial. We need to keep as many building blocks as possible!
Not only for practical reasons but also to permit ‘scientifically clean’ tests.

Retain maximally the timestep organization

	ALADIN time step organization	
1	transform fields: spectral $\rightarrow$ grid point	
2	calculate physics in arrival points	$\mathcal{P}(\mathbf{U}_A^0)$
3	update tendencies	
4	compute SL departure points D and do interpolations	
5	compute explicit part dynamics	$(\mathcal{I} + \frac{\Delta t}{2}\mathcal{L}^*)\mathbf{U}_D^0 + \Delta t(\mathcal{M} - \mathcal{L}^*)\left(\tilde{\mathbf{U}}\right)$
6	add all tendencies	$\mathbf{R}_{lam}$
7	lateral boundary coupling	$\mathbf{R}_{tot} = \alpha\mathbf{R}_{host} + (1-\alpha)\mathbf{R}_{lam}$
8	transform fields: grid point $\rightarrow$ spectral	
9	solve for updated fields	$\mathbf{U}_A^+ = (\mathcal{I} - \frac{\Delta t}{2}\mathcal{L}^*)^{-1}\mathbf{R}_{tot}$

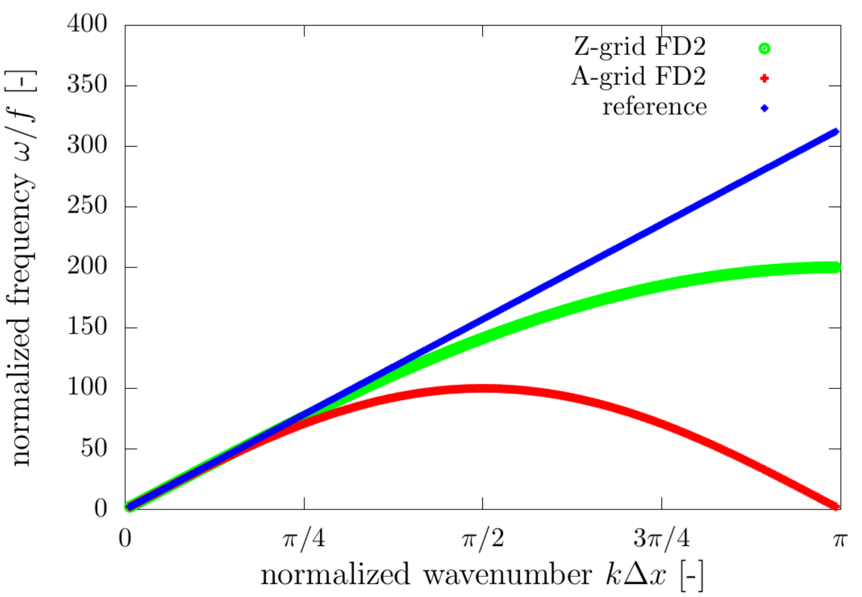
This is only one illustration of the benefits of modularity.

Stay on a collocation grid



No option, due to modularity

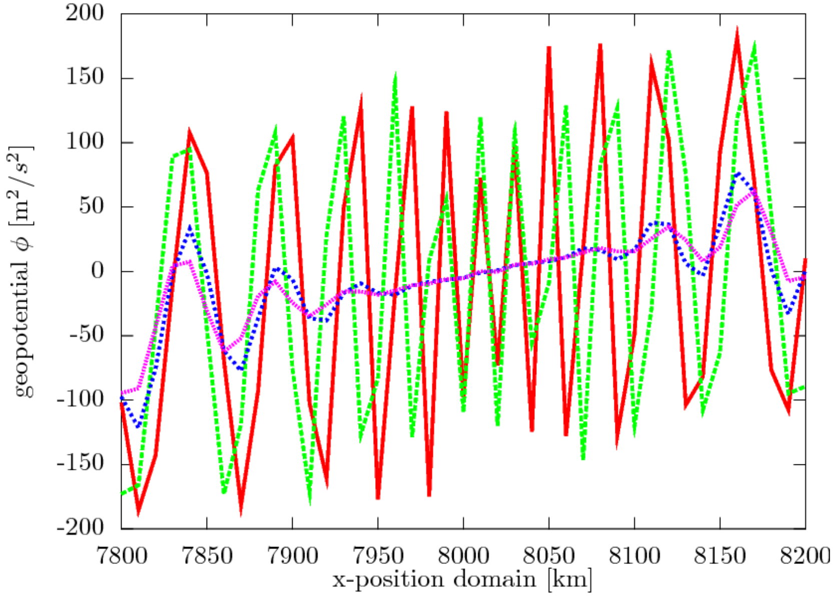
In fact this is what is used currently with the spectral spatial discretization



Dispersion analysis on the SWE shows that the FD A-grid approach results in negative group velocity for the shortest waves.

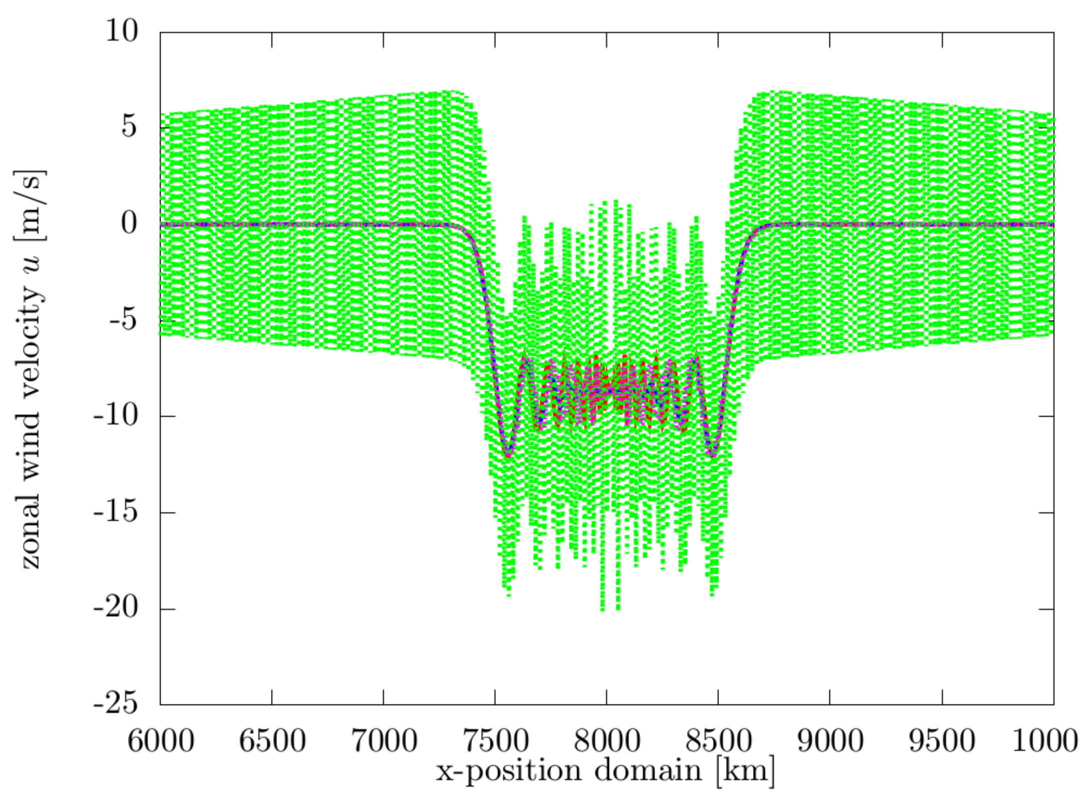
Conclusion: go for FD Z-grid

But analysis reveals two drawbacks of FD Z-grid



Z-grid **eigenvectors** are different from the analytical eigenvectors at the short scale end of the spectrum. This is a fundamental property of Z-grid schemes and spoils even symmetric SI Z-grid schemes.

Introduction of **asymmetries** distorts the appropriate Z-grid geostrophic adjustment behaviour. A solution consists of constructing symmetric Z-grid schemes but they come at an extra cost...



Conclusion after analysis

Both FD A-grid and Z-grid schemes suffer from problems. No local method can beat the spectral approach in terms of dispersion analysis.

Only real model ALADIN tests with the different local alternatives can determine wich of the local approaches (A-grid or Z-grid) is most suitable.

But how to do such real model tests without having to implement new solvers?

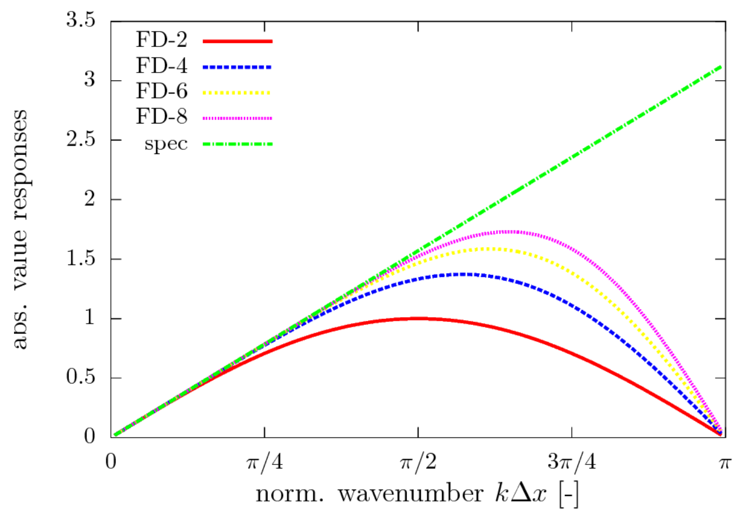
We can mimic a FD spatial discretization in the spectral ALADIN model by changing the responses.

The scientific impact of local schemes can be tested by replacing the spectral responses by finite differences responses.

	detail of ALADIN timestep organization	
9	solve for updated fields	$\mathbf{U}_A^+ = (\mathcal{I} - \frac{\Delta t}{2}\mathcal{L}^*)^{-1}\mathbf{R}_{tot}$

operator	second-order FD	spectral	linear FE
$\mathcal{P}f$	f_x	f_x	$\frac{1}{6}\left[f_{x+\Delta x} + 4f_x + f_{x-\Delta x}\right]$
$\mathcal{P}_x f$	$\frac{1}{2\Delta x}\left[f_{x+\Delta x} - f_{x-\Delta x}\right]$	$\left(\frac{df}{dx}\right)_x$	$\frac{1}{2\Delta x}\left[f_{x+\Delta x} - f_{x-\Delta x}\right]$
$\mathcal{P}_{xx} f$	$\frac{1}{\Delta x^2}\left[f_{x+\Delta x} - 2f_x + f_{x-\Delta x}\right]$	$\left(\frac{d^2f}{dx^2}\right)_x$	$\frac{1}{\Delta x^2}\left[f_{x+\Delta x} - 2f_x + f_{x-\Delta x}\right]$

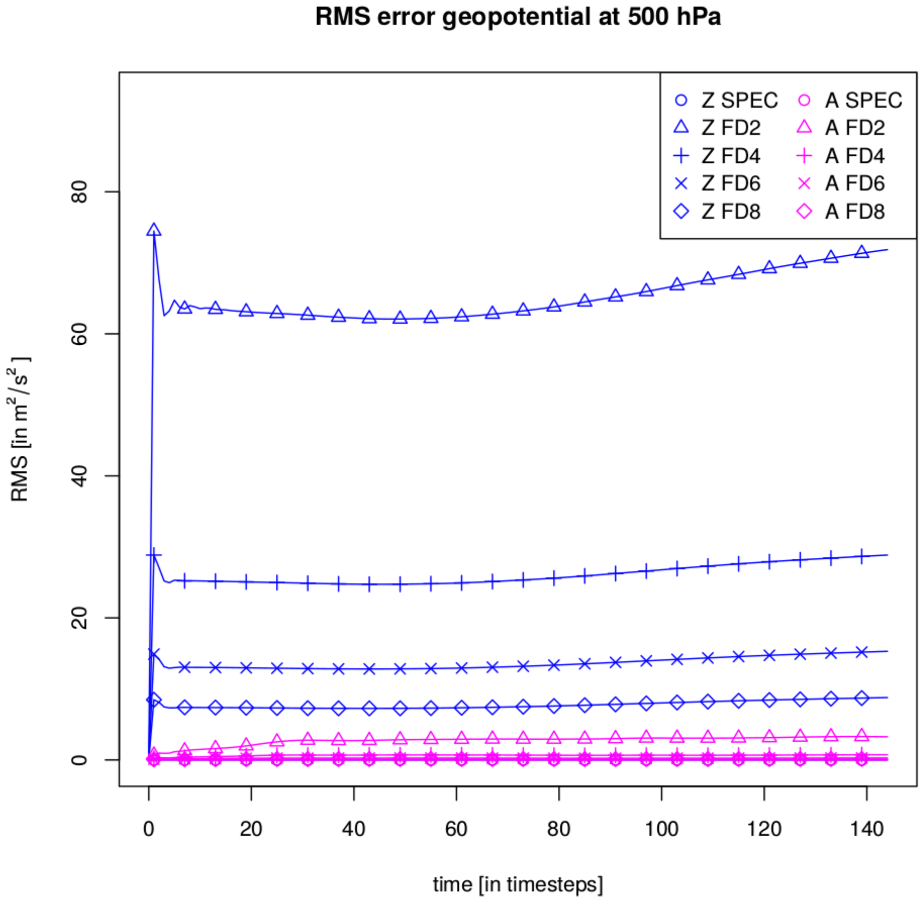
response	second-order FD	spectral	linear FE
p	1	1	$\frac{1}{3}\left[2 + \cos(k\Delta x)\right]$
p_x	$\frac{1}{\Delta x} ik \sin(k\Delta x)$	ik	$\frac{1}{\Delta x} ik \sin(k\Delta x)$
p_{xx}	$\frac{2}{\Delta x^2}\left[\cos(k\Delta x) - 1\right]$	$-k^2$	$\frac{2}{\Delta x^2}\left[\cos(k\Delta x) - 1\right]$



Different response functions for 1st order derivative

→ Implementation is trivial but the approach is very powerful and 'scientifically clean'. ALARO provides a unique testbed!

FD A-grid gives better results than the FD Z-grid.



ALADIN run in hydrostatic mode at 7km horizontal resolution.

1. Increasing the order of accuracy does decrease the RMS with respect to the spectral reference run

2. The A-grid FD methods do a better job than their Z-grid counterparts

3. The jump in RMS of the Z-grid method is present from the first timestep and should thus be related to the decomposition on the eigenvectors

Conclusion: the A-grid dispersion issue is not found back in real model tests whereas the Z-grid schemes do suffer from problems.
It is better to develop a (higher-order) FD A-grid scheme for ALADIN.

What's next?

1. We are currently doing a more extended impact study of the different FD options in the ALADIN model → paper in 2017

domain
- 2 different grid resolutions; 12km and 4km
- 46 vertical levels
- consider both linear as well as quadratic truncation

Finite difference parameters:

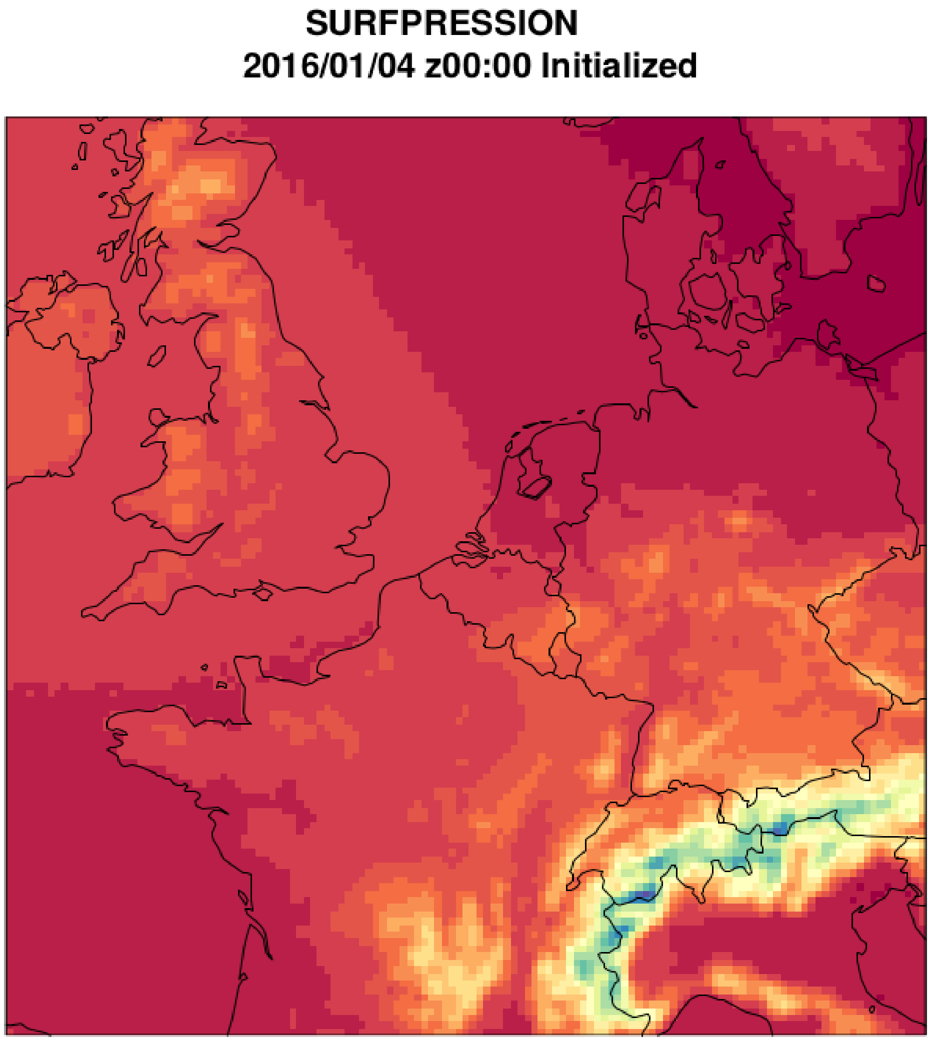
Simulated finite difference methods: A grid and Z grid
Orders of accuracy: 2,4,6 and 8

Other parameters considered:

with DFI/without DFI

Forecast periods:

Investigate 2 periods of 7 consecutive days in different seasons, e.g. 1/1/2016 to 7/1//2016 and 20/6/1016 to 26/6/2016.



domain used for the study

2. Redo the FD tests in ALADIN but with implementation of a local solver instead of the spectral responses trick

What’s next?