Feasability of well-posed transparent LBC with spectral semi-implicit semi-lagrangian discretisation

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Why ?

- Remove main weaknesses of Davies relaxation scheme.

- Adverse effects related to Overspecification
- Not fully transparent

- It Can negatively affect the forecast skills

Recent works

- **Well-posed transparent boundary conditions in the view of NWP model.**

<u>In HIRLAM grid point model:</u>

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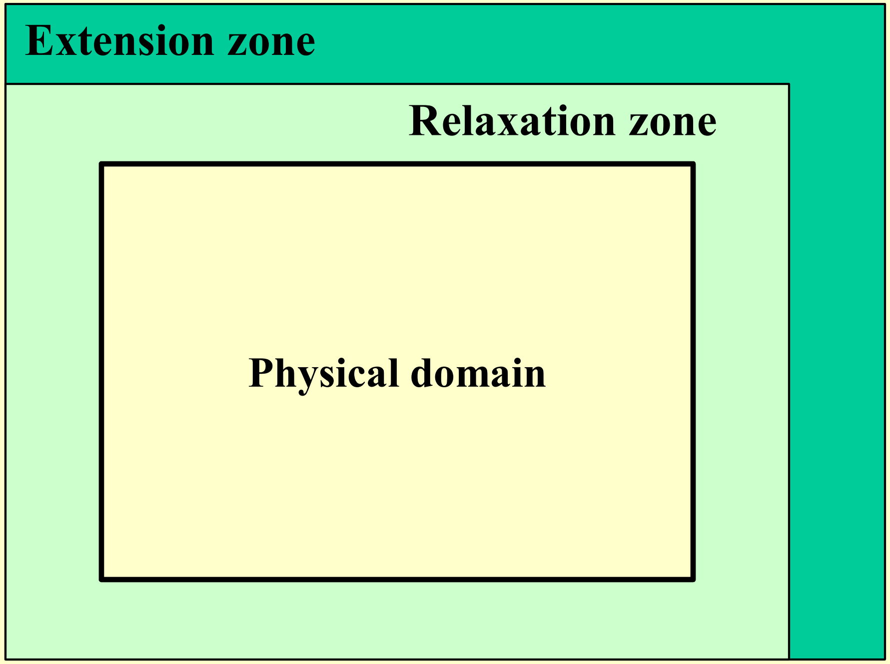
- **McDonald with classical finite difference discretisation**

- **Hostald and Lie with finite element discretisation**

Periodicity and coupling

- Extension zone : for periodicity condition,
- Relaxation zone : for coupling issue



Coupling is used for periodicity at each time step

$$(I-\delta tL)\Psi^{t+\Delta t} = (1-\alpha)RHS(\Psi^{t},\Psi^{t-\Delta t}) + \alpha(I-\delta tL)\Psi_{LS}^{t+\Delta t}$$

Coupling on a line only

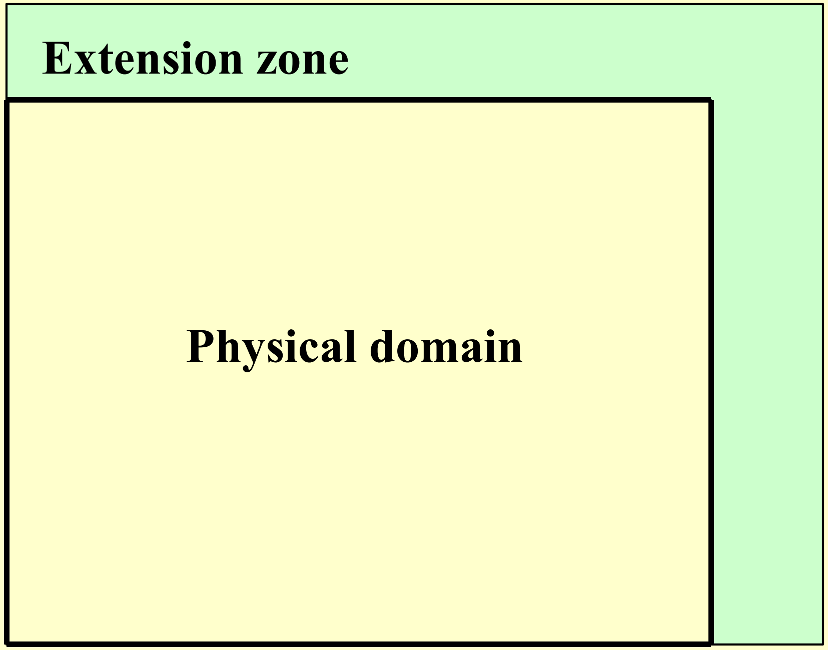
Constraints and Implications :

- E-zone extrapolation must be applied at each time step

➡ expensive computational cost

- Keep Helmholtz operator's form.

➡ Prevail explicit boundary treatment



$$(I - \delta t L)\Psi^{t+\Delta t} = RHS(\Psi^{t}, \Psi^{t-\Delta t}, \Psi^{t}_{LS})$$

Test in a 1D linearized shallow water

- Model equations

$$\frac{\partial u}{\partial t} = -U\frac{\partial u}{\partial x} - C^2\frac{\partial \Phi}{\partial x} + fv$$

$$\frac{\partial v}{\partial t} = -U\frac{\partial v}{\partial x} - fu$$

$$\frac{\partial \Phi}{\partial t} = -U\frac{\partial \Phi}{\partial x} - \frac{\partial u}{\partial x}$$

• Time discretisation : Two time level semi-lagrangian semi-implicit scheme.

•.C > U> 0, for well-posedness : only two fields at eastern boundary and one at western .

Explicit boundary treatment : some obvious choices

Modification of RHS at boundaries :

- ➡ **Impose explicitly the right number of fields**
- ➡ **Using finite difference scheme at boundaries**
- ➡ **Apply E-zone extrapolation to the resulting Rhs**

Numerical test

Initial conditions : a geostrophically balanced bell-curve for Φ

At any time :

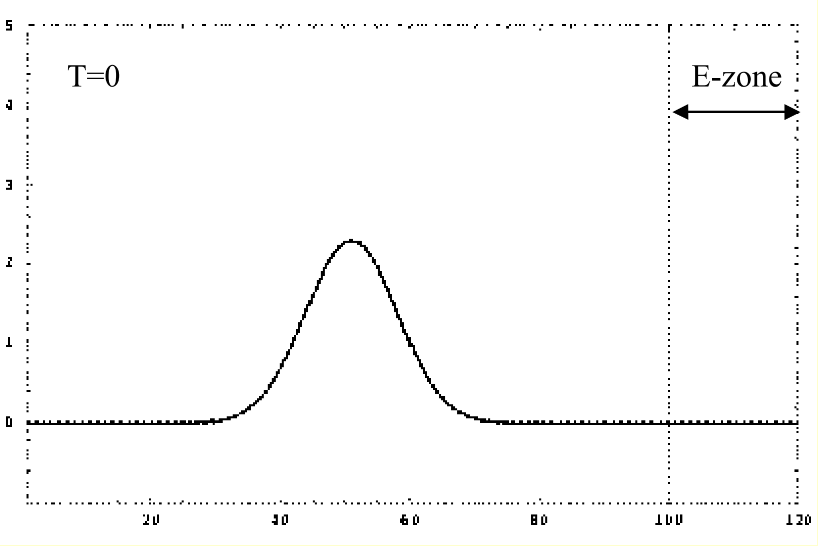
$$fv = C\frac{\partial \Phi}{\partial x}$$

$$u = 0$$

We impose :

u **at both boundaries**

v **at eastern boundary**

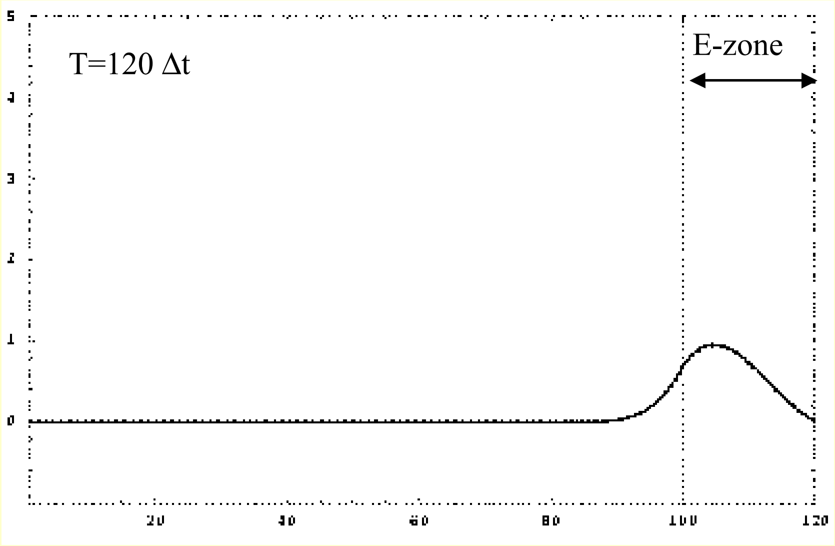


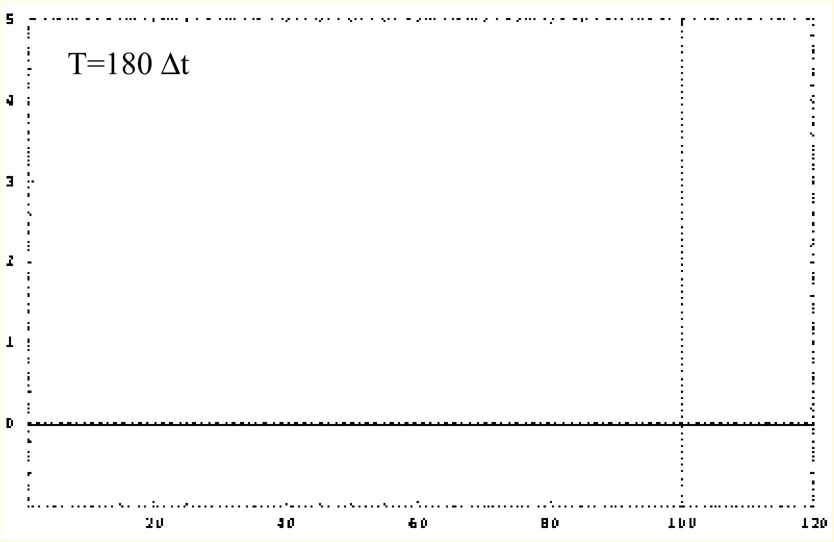
Numerical test

Settings :

$$U = 100\, ms^{-1}, C = 300\, ms^{-1}, f = 10^{-4} s^{-1}.$$

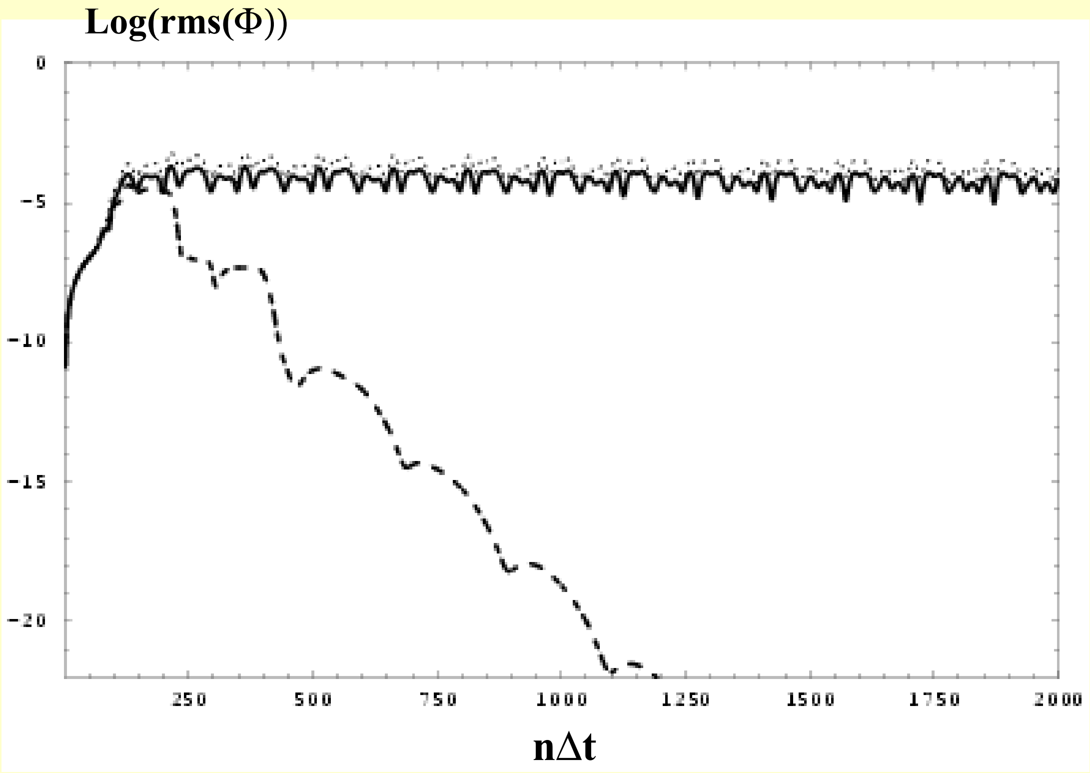
$$\Delta x = 10km, \Delta t = 50s, L = 1200km,$$



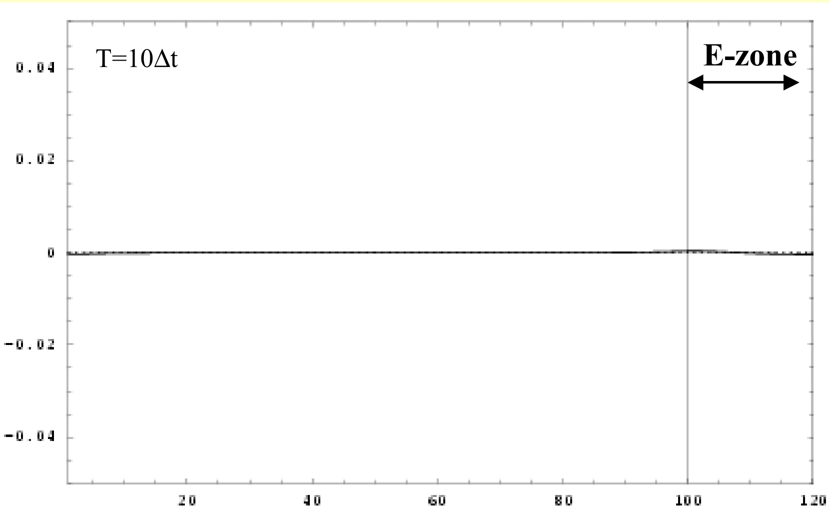


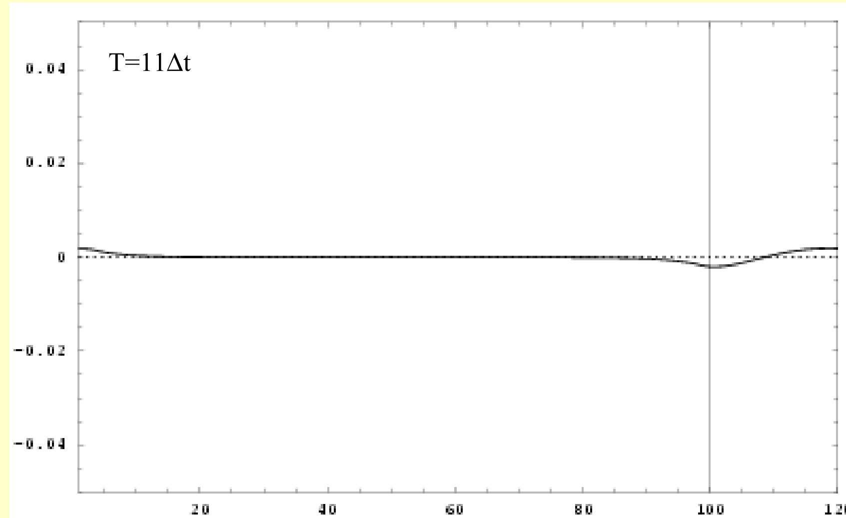
Benefit from characteristics

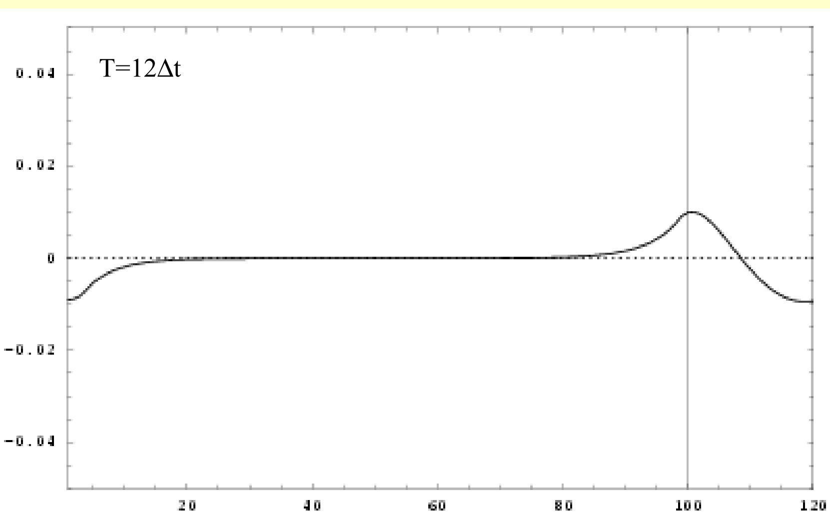
It reduces false reflections magnitudes

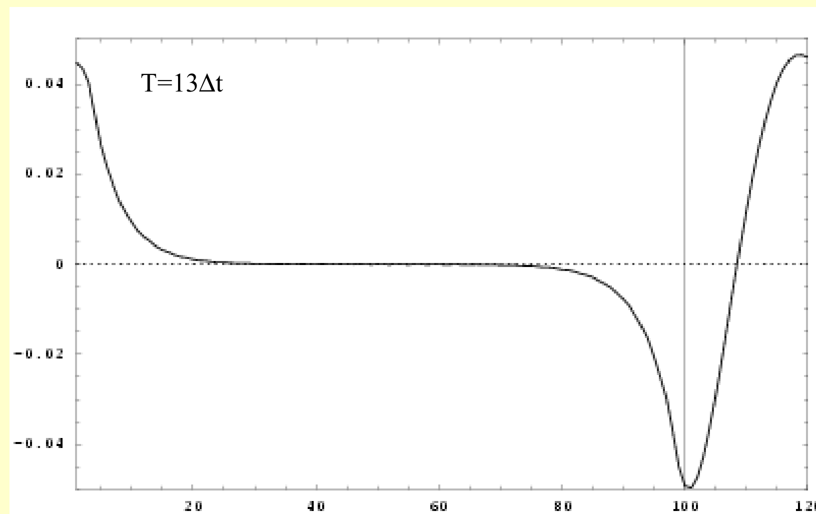


Instabilities induced by explicit treatment and E-zone extrapolation









Experimental analysis

- Explicit boundary treatment + E-zone extrap = strong instabilities

- If we remove one of these two ingredients :

  ⇨ Stable, but unconsistent solution

- To fix it, try some absorbing boundaries approach in the E-zone

Next works

- Explore implicit boundary treatment

- Try some other spectral formulations.