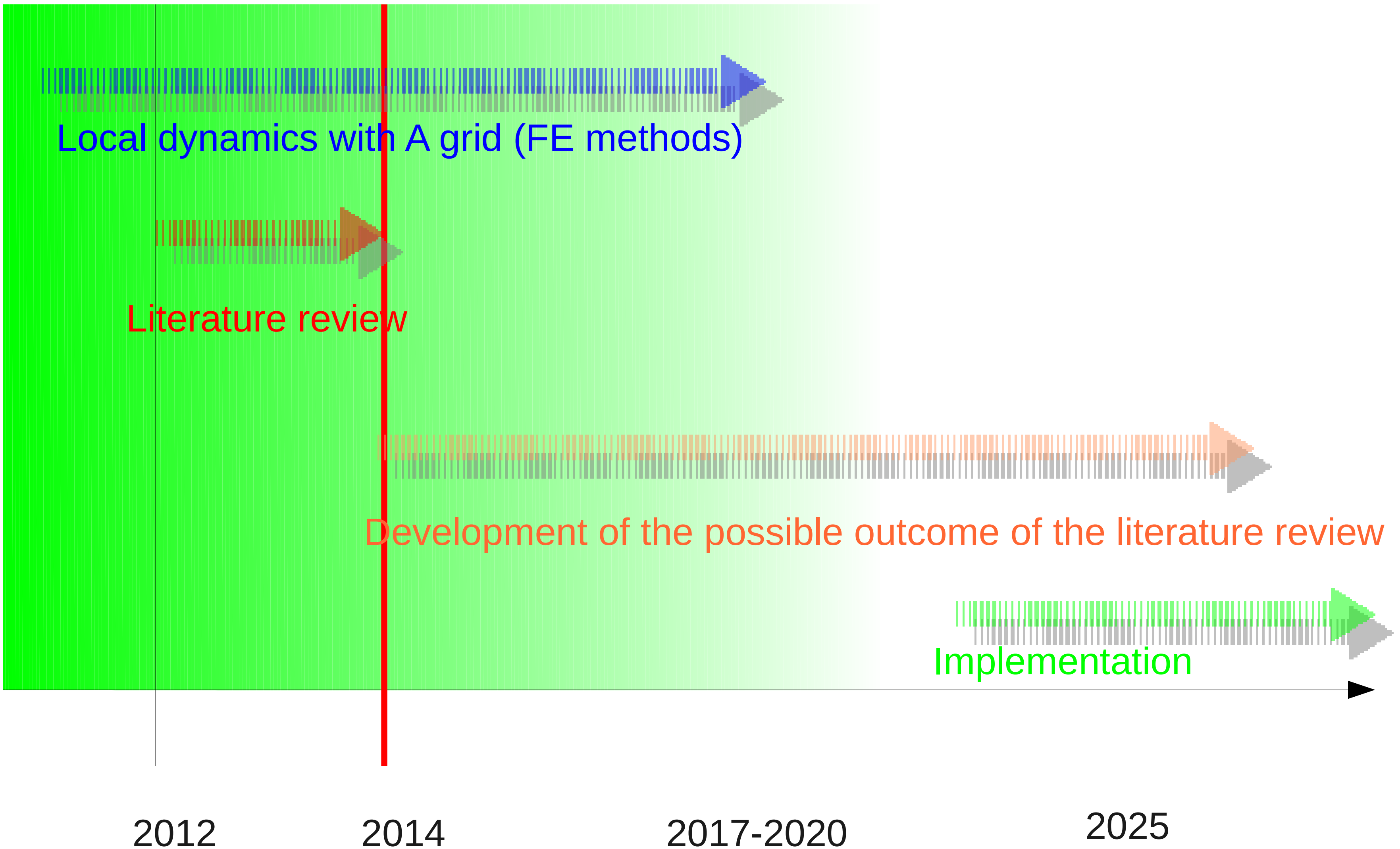
Presentation Workshop 2012: What about local dynamics?

Steven Caluwaerts

UNIVERSITEIT GENT

KMI

Dynamics: road map presented to our GA

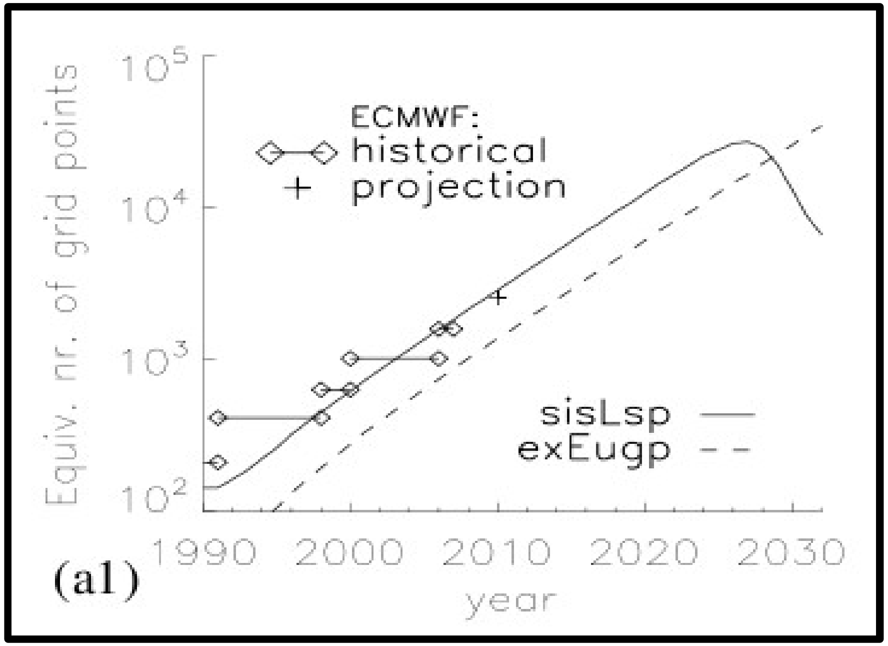


Eliminitating the A grid means we have to overhaul the whole system.
We stay with the current system at least for the term of the current strategy plan (green area).

Study of Cats: comparison between 2 extremes.

Explicit, Eulerian Gridpoint Model ⟷ **Semi-implicit Semi-Lagrangian Spectral Model**

And the winner is...

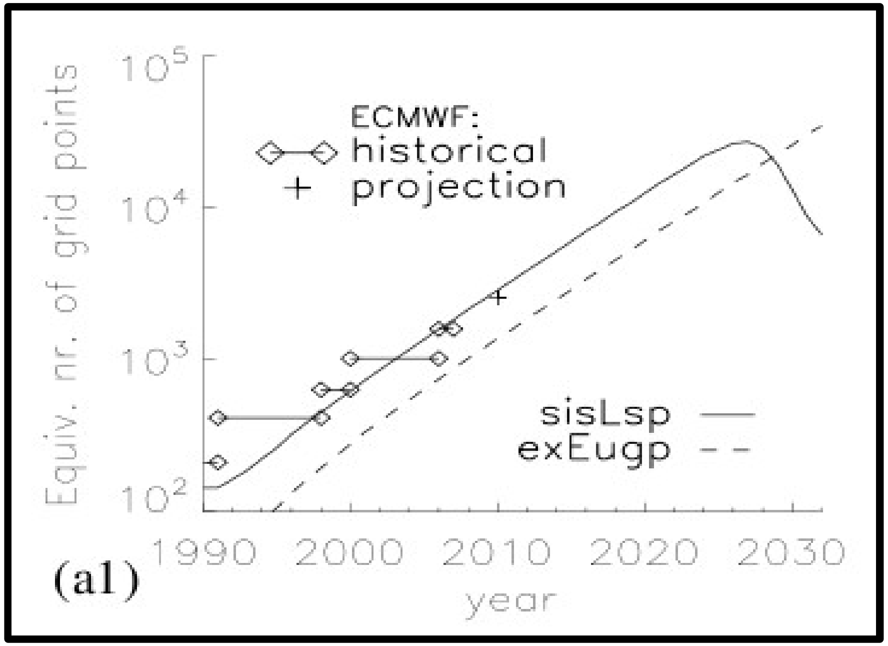


Cats G. 24 More Years of Numerical Weather Prediction: A Model Performance Model (wetensch.rapport KNMI; 2008)

Study of Cats: comparison between 2 extremes.

Explicit, Eulerian Gridpoint Model ↔ **Semi-implicit Semi-Lagrangian Spectral Model**

And the winner is... changing in the future.



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Outline talk

Finite elements

Current timestep-organisation

The need for a reformulation to vorticity-divergence

Finite element timestep-organisation

Conclusions

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Horizontal spatial discretisation: finite differences or spectral or ...

Finite differences

Local, only nearest neighbour interaction

Easy to parallelize

Simple to implement

Spectral discretisation

Global

Lot of communication

Very simple Helmholtz solver

Exact derivatives

Fast Fourier Transforms

Periodic fields needed

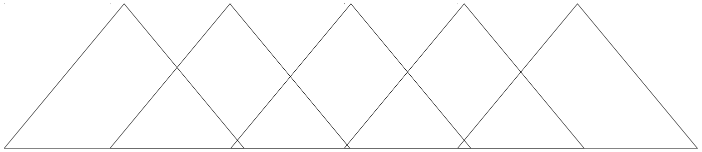
… finite elements. How does it work?

$$\frac{\partial u}{\partial t} = -g\frac{\partial h}{\partial x} + fv$$

1) Write every field as a weighted sum of basis functions.

$$\begin{aligned} u(x,y,t) &= \sum_i u_i(t)\phi_i(x,y) \\ v(x,y,t) &= \sum_i v_i(t)\phi_i(x,y) \\ h(x,y,t) &= \sum_i h_i(t)\phi_i(x,y) \end{aligned}$$

1D-chapeau basis functions



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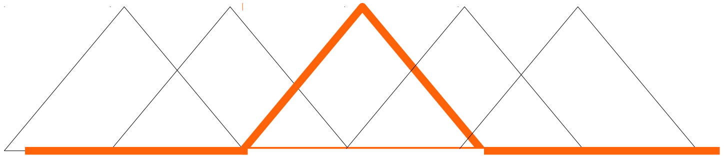

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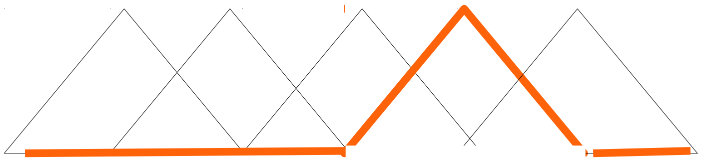

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$$\frac{\partial u}{\partial t} = -g\frac{\partial h}{\partial x} + fv$$

1) Write every field as a weighted sum of basis functions.

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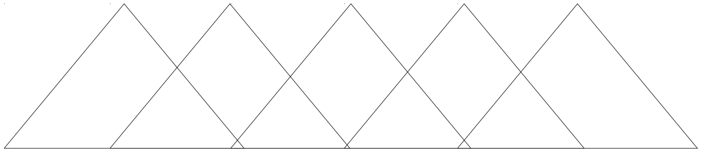


2) Solve the *weak formulation* of the equation.

$$\int \frac{\partial u}{\partial t}\phi_k dxdy = -g\int \frac{\partial h}{\partial x}\phi_k dxdy + f\int v\phi_k dxdy$$

→ results in N expressions with N=number of basis functions

This results in a matrix problem with off-diagonal elements.

3) Work out the equation...

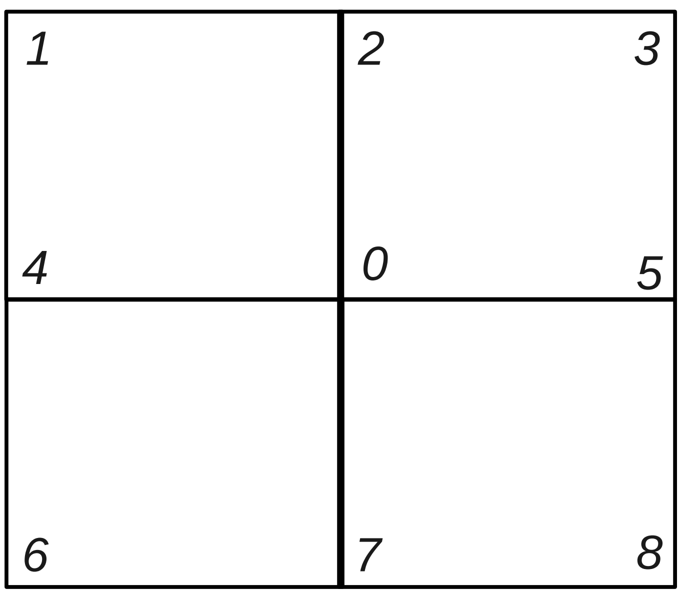
$$\sum_i \frac{du_i}{dt} \int \phi_i \phi_k dx dy = -g \sum_i h_i \int \frac{d\phi_i}{dx} \phi_k dx dy + f \sum_i v_i \int \phi_i \phi_k dx dy$$

This results in a matrix problem with off-diagonal elements.

3) Work out the equation...

$$\sum_i \frac{du_i}{dt}\int \phi_i\phi_k dxdy = -g\sum_i h_i \int \frac{d\phi_i}{dx}\phi_k dxdy + f\sum_i v_i \int \phi_i\phi_k dxdy$$

4) Calculate the different integrals. For example if you use 2D-chapeau functions, you have:



$$\sum_i v_i \int \phi_i\phi_0 dxdy = d^2\left(\frac{4}{9}V_0 + \frac{V_2+V_5+V_7+V_4}{9} + \frac{V_1+V_3+V_8+V_6}{36}\right)$$

off-diagonal elements

Pro's and cons for finite element discretisation.

local method

more accurate derivatives than finite difference, but less accurate than spectral method

solving sparse matrix-problem for Helmholtz equation, more difficult than spectral method

domain with variable resolution possible

→ We will use a finite element discretisation to test local dynamics.

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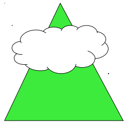

Current timestep-organisation

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Current SISL-spectral timestep organisation.

Spectral fields (u,v,...) at timestep t^n — **Spectral**

↓ Inverse FFT

Calculating SL-trajectories

Calculating physics

Calculating explicit part SI-method

Boundary coupling and periodisation

GP

↓ FFT

Solving in spectral space simple Helmholtzproblem

to find new values (u,v,...) at timestep t^{n+1} — **Spectral**

Current SISL-spectral timestep organisation.

Spectral fields (u,v,...) at timestep t^n — **Spectral**

↓ Inverse FFT

GP

- Calculating SL-trajectories
- Calculating physics
- Calculating explicit part SI-method
- Boundary coupling and periodisation

We absolutely want to keep this block!!!

↓ FFT

Spectral

Solving in spectral space simple Helmholtzproblem

to find new values (u,v,...) at timestep t^{n+1}

Implement in some way finite elements!!!

Timestep organisation current SISL-spectral code

Calculating SL-trajectories

$$\frac{u^{n+1} - u_*^n}{\Delta t} = -g\frac{\frac{\partial h}{\partial x}^{n+1} + \frac{\partial h}{\partial x}_*^n}{2} + f\frac{v^{n+1} + v_*^n}{2}$$

$$\frac{v^{n+1} - v_*^n}{\Delta t} = -g\frac{\frac{\partial h}{\partial y}^{n+1} + \frac{\partial h}{\partial y}_*^n}{2} - f\frac{u^{n+1} + u_*^n}{2}$$

$$\frac{h^{n+1} - h_*^n}{\Delta t} = \frac{-H}{2}\left(\frac{\partial u}{\partial x}^{n+1} + \frac{\partial u}{\partial x}_*^n + \frac{\partial v}{\partial y}^{n+1} + \frac{\partial v}{\partial y}_*^n\right)$$

$$u^{n+1} + \frac{g\Delta t}{2}\frac{\partial h}{\partial x}^{n+1} - \frac{f\Delta t}{2}v^{n+1} = u_*^n - \frac{g\Delta t}{2}\frac{\partial h}{\partial x}_*^n + \frac{f\Delta t}{2}v_*^n$$

$$v^{n+1} + \frac{g\Delta t}{2}\frac{\partial h}{\partial y}^{n+1} + \frac{f\Delta t}{2}u^{n+1} = v_*^n - \frac{g\Delta t}{2}\frac{\partial h}{\partial y}_*^n - \frac{f\Delta t}{2}u_*^n$$

$$h^{n+1} + \frac{H\Delta t}{2}\left(\frac{\partial u}{\partial x}^{n+1} + \frac{\partial v}{\partial y}^{n+1}\right) = h_*^n - \frac{H\Delta t}{2}\left(\frac{\partial u}{\partial x}_*^n + \frac{\partial v}{\partial y}_*^n\right)$$

Solve Helmholtzproblem in h^{n+1} : $\left(\nabla^2 + k\right) h^{n+1} = F$

and calculate u^{n+1} and v^{n+1}

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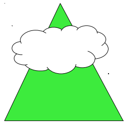


The need for a reformulation to vorticity-divergence

Finite element timestep-organisation

Conclusions

Mesinger and Arakawa found bad dispersion relations for finite differences...

Let us assume a wavelike solution: $u(x,t) = \mathbf{U}e^{i\omega t + ikx}$

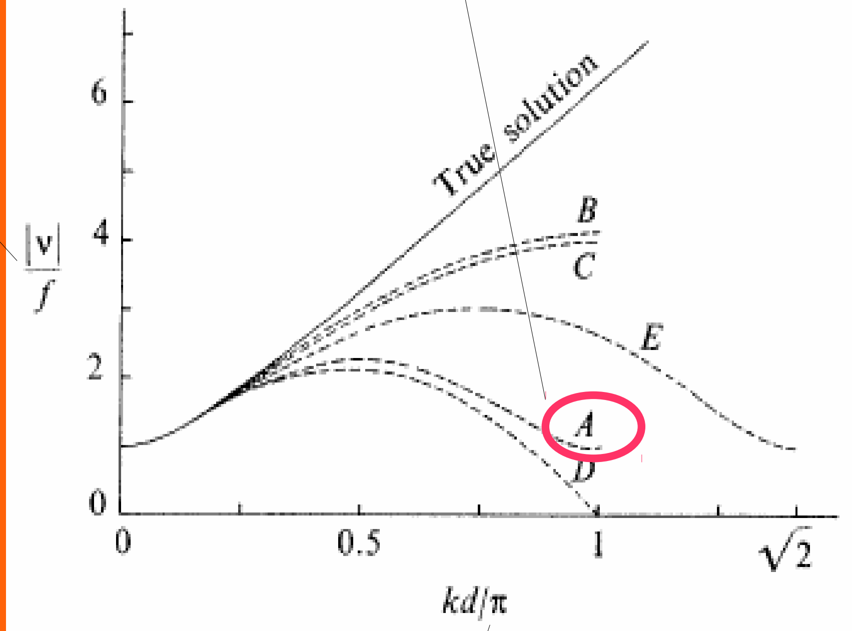
SWE

$$\frac{\partial u}{\partial t} = -g\frac{\partial h}{\partial x} + fv$$
$$\frac{\partial v}{\partial t} = -g\frac{\partial h}{\partial y} - fu$$
$$\frac{\partial h}{\partial t} = -H\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$$

Finite difference discretisation

frequency

Wrong group velocity

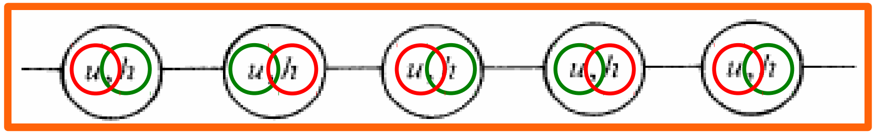


wavenumber

As a first solution one can go to a staggered grid.

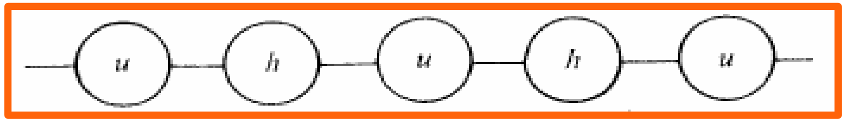
1D

A-grid

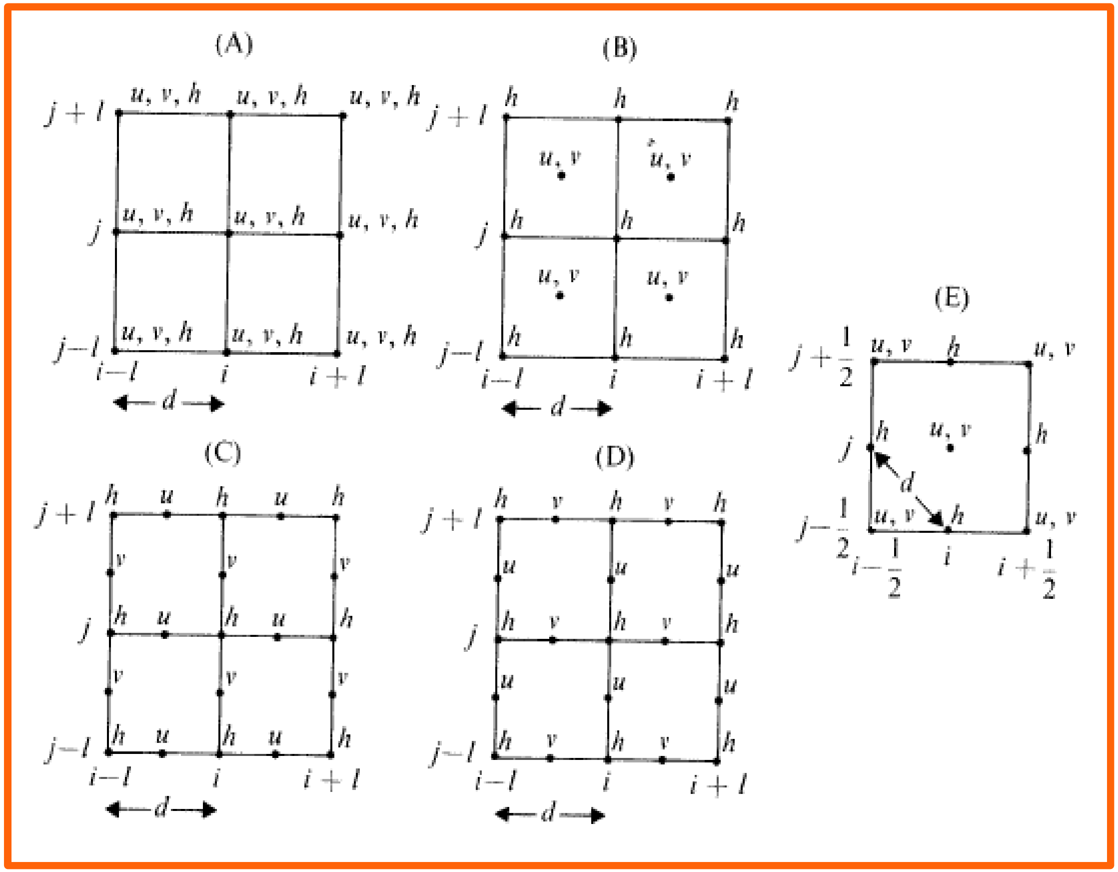


→ 2 decoupled solutions

C-grid



2D



We have to go a divergence=vorticity formulation of the equations.

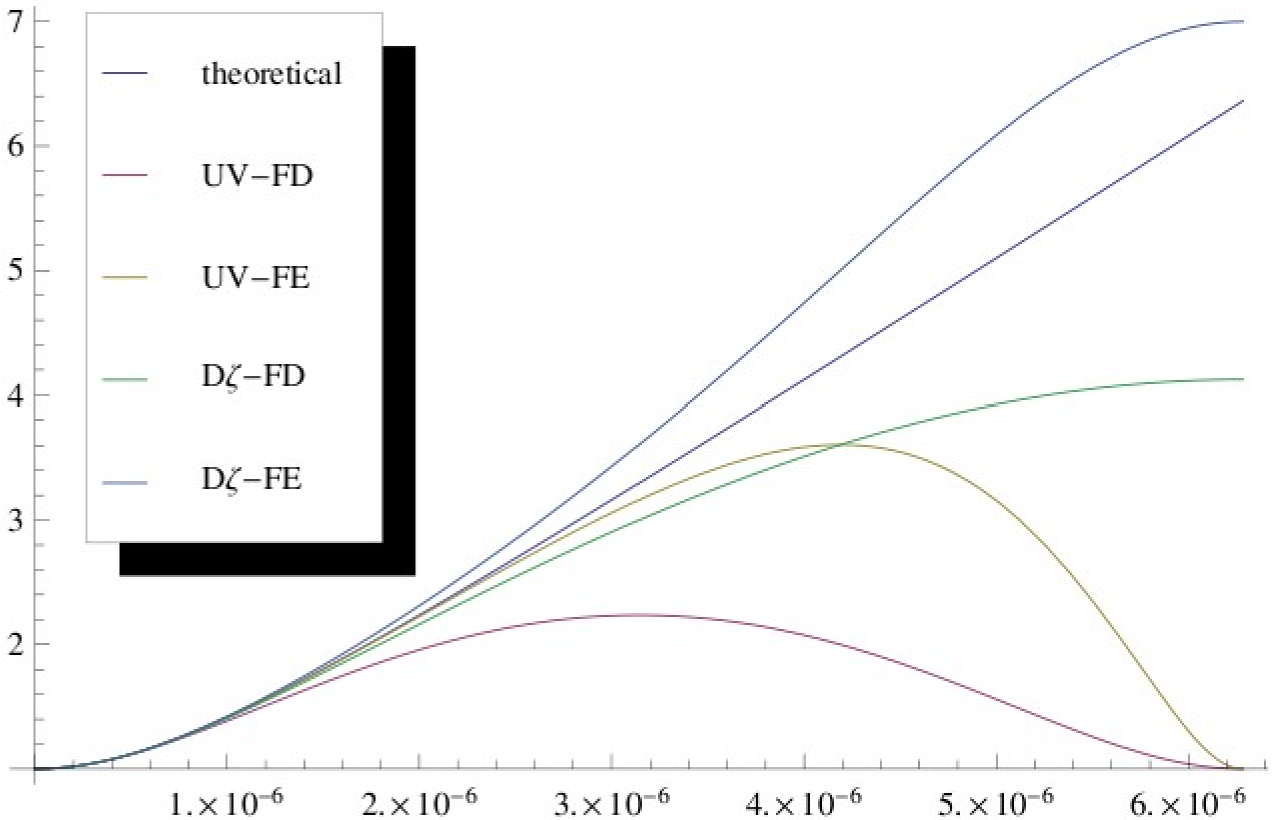
$$\frac{\partial u}{\partial t} = -g\frac{\partial h}{\partial x} + fv$$
$$\frac{\partial v}{\partial t} = -g\frac{\partial h}{\partial y} - fu$$
$$\frac{\partial h}{\partial t} = -H\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$$

$$\frac{\partial \zeta(x,y,t)}{\partial t} + fD(x,y,t) = 0$$
$$\frac{\partial D(x,y,t)}{\partial t} - f\zeta(x,y,t) = -g\nabla^2 h(x,y,t)$$
$$\frac{\partial h(x,y,t)}{\partial t} = -HD(x,y,t)$$

possible on classical A-grid !!

$$D = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$
$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

Dispersion relations for finite differences/elements for both formulations.



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As discussed earlier, we start from exactly the same point, but...

$$\frac{u^{n+1} - u_*^n}{\Delta t} = -g\frac{\frac{\partial h}{\partial x}^{n+1} + \frac{\partial h}{\partial x}^n_*}{2} + f\frac{v^{n+1} + v_*^n}{2}$$

$$\frac{v^{n+1} - v_*^n}{\Delta t} = -g\frac{\frac{\partial h}{\partial y}^{n+1} + \frac{\partial h}{\partial y}^n_*}{2} - f\frac{u^{n+1} + u_*^n}{2}$$

$$\frac{h^{n+1} - h_*^n}{\Delta t} = \frac{-H}{2}\left(\frac{\partial u}{\partial x}^{n+1} + \frac{\partial u}{\partial x}^n_* + \frac{\partial v}{\partial y}^{n+1} + \frac{\partial v}{\partial y}^n_*\right)$$

Calculating SL-trajectories

$$u^{n+1} + \frac{g\Delta t}{2}\frac{\partial h}{\partial x}^{n+1} - \frac{f\Delta t}{2}v^{n+1} = u_*^n - \frac{g\Delta t}{2}\frac{\partial h}{\partial x}^n_* + \frac{f\Delta t}{2}v_*^n = K$$

$$v^{n+1} + \frac{g\Delta t}{2}\frac{\partial h}{\partial y}^{n+1} + \frac{f\Delta t}{2}u^{n+1} = v_*^n - \frac{g\Delta t}{2}\frac{\partial h}{\partial y}^n_* - \frac{f\Delta t}{2}u_*^n = L$$

$$h^{n+1} + \frac{H\Delta t}{2}\left(\frac{\partial u}{\partial x}^{n+1} + \frac{\partial v}{\partial y}^{n+1}\right) = h_*^n - \frac{H\Delta t}{2}\left(\frac{\partial u}{\partial x}^n_* + \frac{\partial v}{\partial y}^n_*\right) = M$$

… we change the solution of the implicit part!

Rewriting into vorticity-divergence

$$
\begin{aligned}
D^{n+1} + \frac{g\Delta t}{2}\left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2}\right)^{n+1} - \frac{f\Delta t}{2}\zeta^{n+1} &= \frac{\partial K}{\partial x} + \frac{\partial L}{\partial y} \\
\zeta^{n+1} + \frac{f\Delta t}{2} D^{n+1} &= \frac{\partial L}{\partial x} - \frac{\partial K}{\partial y} \\
h^{n+1} + \frac{H\Delta t}{2} D^{n+1} &= M
\end{aligned}
$$

Solve Helmholtzproblem in h^{n+1} : $\left(\nabla^2 + k\right) h^{n+1} = F$

and calculate D^{n+1} and ζ^{n+1}

Calculate the wind fields with the Poisson equations:

$$
\begin{aligned}
\frac{\partial^2 u(x,y)}{\partial x^2} + \frac{\partial^2 u(x,y)}{\partial y^2} &= \frac{\partial D(x,y)}{\partial x} - \frac{\partial \zeta(x,y)}{\partial y} \\
\frac{\partial^2 v(x,y)}{\partial x^2} + \frac{\partial^2 v(x,y)}{\partial y^2} &= \frac{\partial D(x,y)}{\partial y} + \frac{\partial \zeta(x,y)}{\partial x}
\end{aligned}
$$

Our proposal starts from the current organisation.

Spectral fields (u,v,...) at timestep t^n

Spectral

Inverse FFT

Calculating SL-trajectories

Calculating physics

Calculating explicit part SI-method

Boundary coupling and periodisation

GP

FFT

Solving in spectral space simple Helmholtzproblem

to find new values (u,v,...) at timestep t^{n+1}

Spectral

Our proposal starts from the current organisation.

Spectral fields (u,v,...) at timestep t^n

~~Spectral~~ **coefficient space**

↓ ~~Inverse FFT~~ transform to gridpoint space

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↓ ~~FFT~~ transform to coeff space

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to find new values ~~(u,v,...)~~ (D,Zeta) at timestep t^{n+1}

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~~Spectral~~ **coefficient space**

↓ ~~Inverse FFT~~ transform to gridpoint space

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GP

Solve Poisson equation to find (u,v) at t^{n+1} ↑

↓ ~~FFT~~ transform to coeff space

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to find new values ~~(u,v,...)~~ (D,Zeta) at timestep t^{n+1}

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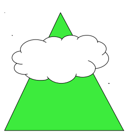


Conclusions

Stable method found, now testing...

We constructed a **numerically stable method** that integrates localized dynamics (= finite elements) into our **current timestep organisation**.

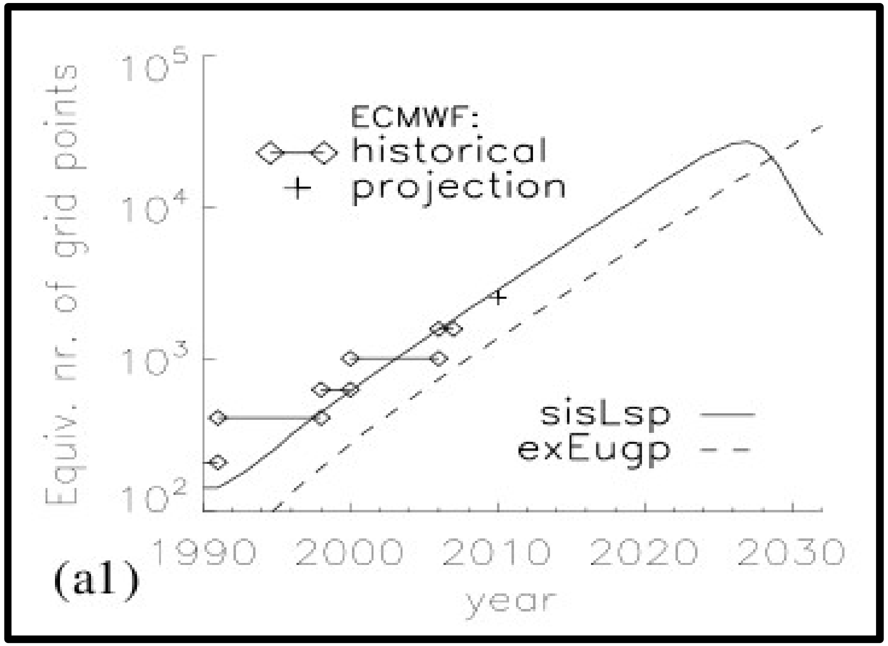
→ And now?

- test orography-behaviour of new method on 2D SWE-model (Alembix)
- do the complete analysis for the 3D non-hydrostatic model
- think about different kind of finite elements
-....

Remarks, ideas, questions???

Resolution increases year after year

→ **What kind of dynamical core will perform best in the future?**



Opmerken dat het eigenlijk vooral om orografie gaat...

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Shallow water equations (SWE)

SWE

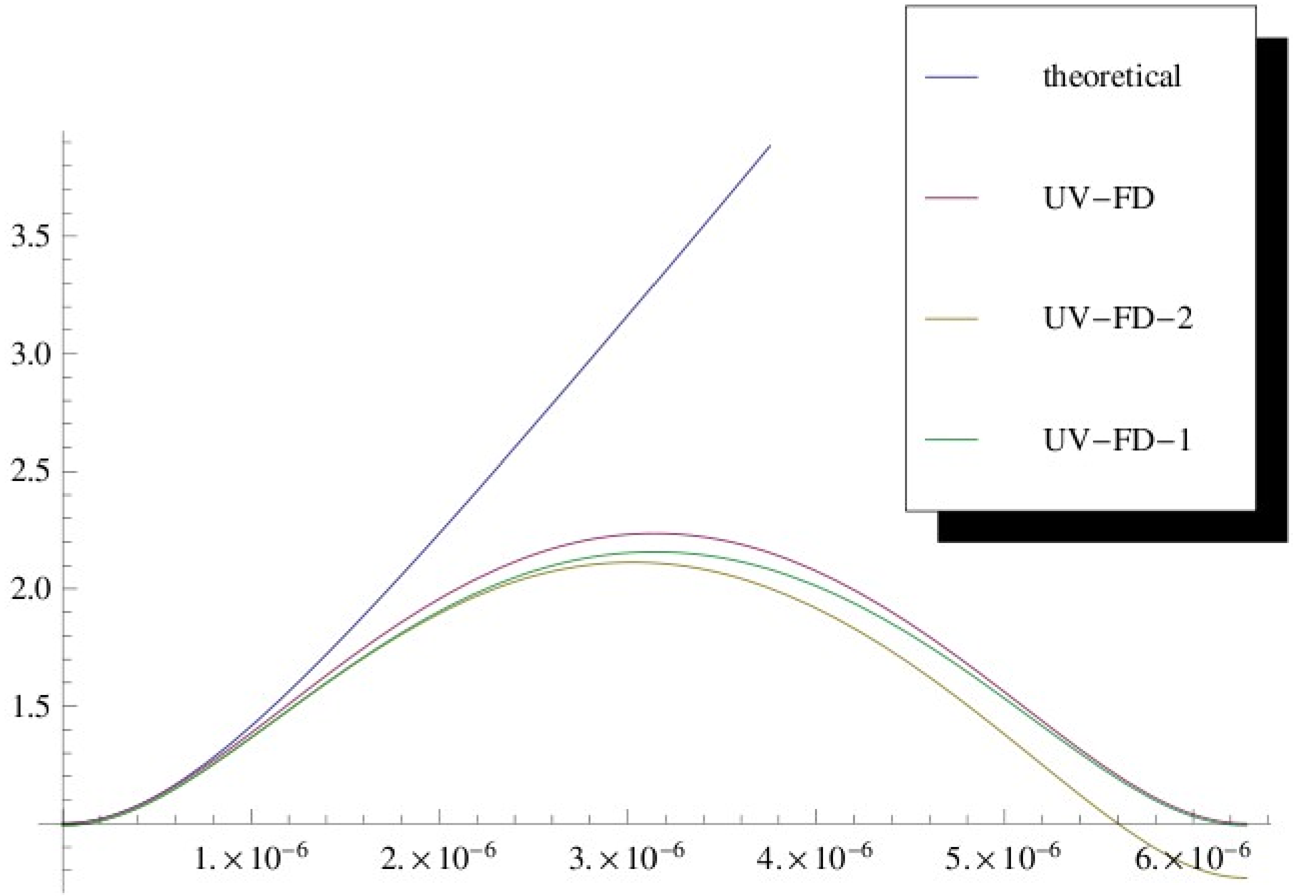
$$\frac{Du}{dt} = -g\frac{\partial h}{\partial x} + fv$$
$$\frac{Dv}{dt} = -g\frac{\partial h}{\partial y} - fu$$
$$\frac{Dh}{dt} = -h\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$$

$$u = u'$$
$$v = v'$$
$$h = H + h'$$

Linearized SWE

$$\frac{\partial u}{\partial t} = -g\frac{\partial h}{\partial x} + fv$$
$$\frac{\partial v}{\partial t} = -g\frac{\partial h}{\partial y} - fu$$
$$\frac{\partial h}{\partial t} = -H\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$$

Dispersion relations depends on the exact way of evaluating the equations.



Gravity waves: Explicit vs Semi-Implicit

$$\frac{\partial u}{\partial t} = -g\frac{\partial h}{\partial x} + fv$$
$$\frac{\partial v}{\partial t} = -g\frac{\partial h}{\partial y} - fu$$
$$\frac{\partial h}{\partial t} = -H(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y})$$

→ **Gravity waves (100m/s)** are possible solution

Explicit

$$\frac{u^{n+1} - u^n}{\Delta t} = -g\frac{\partial h^n}{\partial x} + fv^n$$

CFL- timestep limitation

Semi-Implicit

$$\frac{u^{n+1} - u^n}{\Delta t} = -\frac{g}{2}\left(\frac{\partial h^n}{\partial x} + \frac{\partial h^{n+1}}{\partial x}\right) + \frac{f}{2}\left(v^n + v^{n+1}\right)$$

Unconditionally stable, timestep not limited by gravity waves but Helmholtzproblem needs to be solved:

$$\left(\nabla^2 + k\right) h^{n+1} = F$$

Advection: Eulerian or Semi-Lagrangian

If advection is handled explicitly, the timestep is again limited by the CFL-criterion. (**Eulerian**)

→ If you follow air parcels during their motion (= Lagrangian approach), your method is unconditionally stable

One can use a **semi-lagrangian** method = calculate along trajectories of parcels at gridpoints

$$\frac{u_A^{n+1} - u_*^n}{\Delta t} = -\frac{g}{2}\left(\left(\frac{\partial h}{\partial x}\right)_*^n + \left(\frac{\partial h}{\partial x}\right)_A^{n+1}\right) + \frac{f}{2}\left(v_*^n + v_A^{n+1}\right)$$