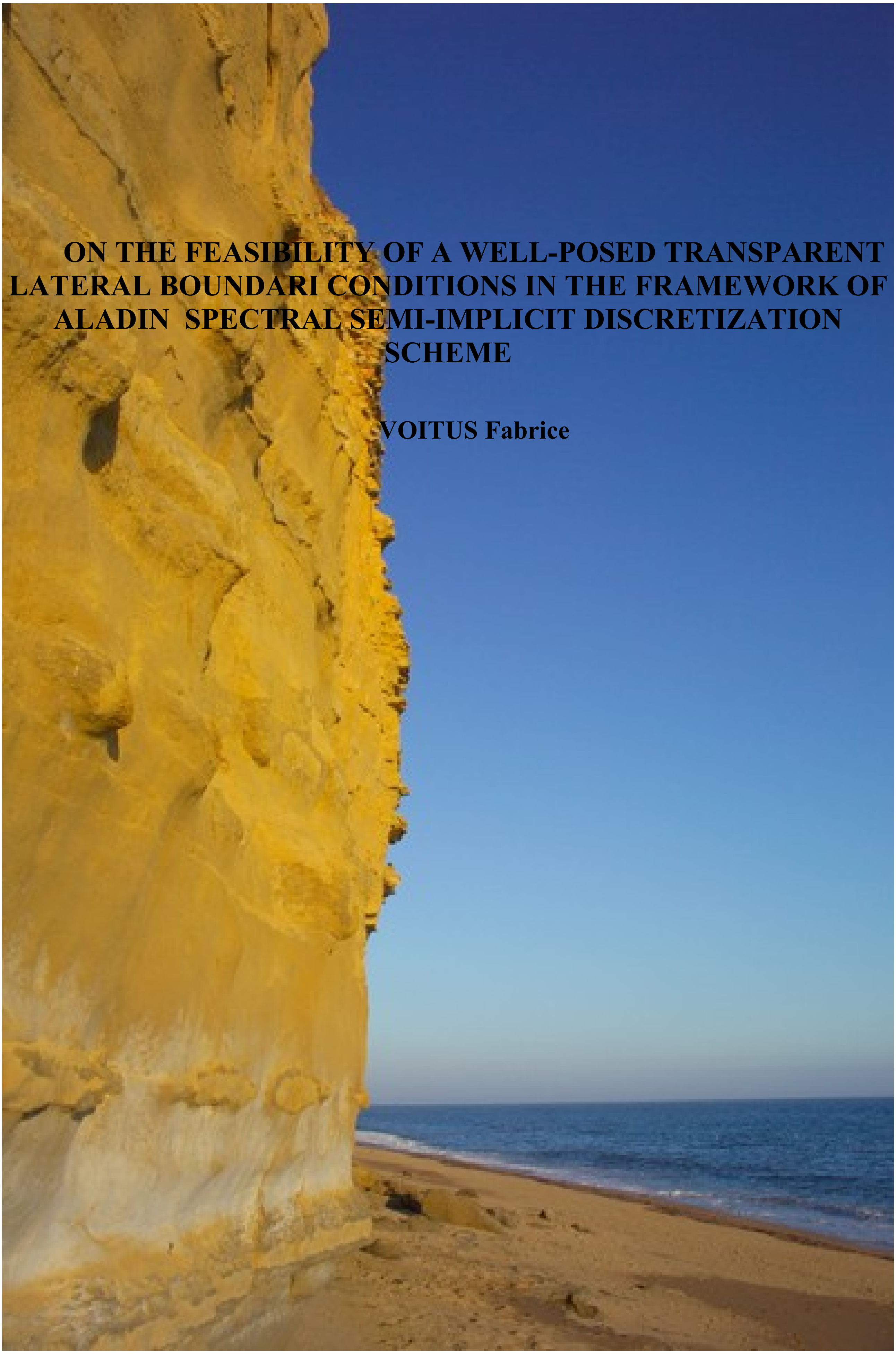

ON THE FEASIBILITY OF A WELL-POSED TRANSPARENT LATERAL BOUNDARI CONDITIONS IN THE FRAMEWORK OF ALADIN SPECTRAL SEMI-IMPLICIT DISCRETIZATION SCHEME

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1. Introduction

A great effort is made in order to construct a well-posed transparent LBC scheme in the view of regional NWP systems. For the HIRLAM semi-implicit grid-point model, the study of a new coupling approach based upon the "characteristics" approach, has been recently performed by McDonald (2000) for finite-difference (FD) discretization and by Hostald and Lie (2001) for finite-element (FE) method, with promising results.

The challenge question is : Can such a strategy be applied in a full Fourier spectral discretization ? Indeed, how can a well-posed transparent boundary treatment be incorporated into the semi-implicit spectral discretization of ALADIN ?

2. Model equation and time discretization

For well-posedness only a certain subset of variables has to be imposed at boundaries. The number of allowable conditions to be applied at a point on the boundary depends on whether there is inflow or outflow at this point. Oliger and Sundstrom (1978) have shown that because of their hyperbolic character, it is theoretically possible to obtain a well-posed continuous problem for both Euler equations and shallow-water equations. But knowing that a continuous problem is well-posed does not necessarily mean that it is obvious how to implement boundary conditions for the associated discrete problem in a satisfactory manner.

To test some alternatives of boundary treatment in Fourier spectral discretization, a one-dimensional linearized shallow-water model is used and numerically solved by a two-time-level semi-Lagrangian semi-implicit time-marching which writes symbolically :

$$u^+ - \frac{\Delta t}{2} f\ v^+ + \frac{\Delta t}{2} C^2 \left(\frac{\partial \Phi}{\partial x}\right)^+ = R^0_{u,d} \tag{1}$$

$$v^+ + \frac{\Delta t}{2} f\ u^+ = R^0_{v,d} \tag{2}$$

$$\Phi^+ + \frac{\Delta t}{2}\left(\frac{\partial u}{\partial x}\right)^+ = R^0_{\Phi,d} \tag{3}$$

R^0_u , R^0_v and R^0_Φ are the explicit RHS (right hand side) terms, the subscript *d* means that quantities are interpolated to the departure point. These fields are first computed at each grid point by :

$$R^0_u = u^0 + \frac{\Delta t}{2} f\ v^0 - \frac{\Delta t}{2} C^2 \left(\frac{\partial \Phi}{\partial x}\right)^0 \tag{4}$$

$$R^0_v = v^0 - \frac{\Delta t}{2} f\ u^0 \tag{5}$$

$$R^0_\Phi = \Phi^0 - \frac{\Delta t}{2}\left(\frac{\partial u}{\partial x}\right)^0 \tag{6}$$

Exponents + and 0 denote respectively the variable state at $t+\Delta t$ and t. The resolution of this implicit problem implies to invert the so-called Helmholtz equation. This inversion is trivially performed in spectral space provided that RHS fields fulfil periodicity condition. Therefore, E-zone extrapolation seems to be unavoidable.

The main difficulty comes from the global character of spectral computations, which introduces a loss of flexibility compared to finite-difference discretization. As a consequence, well-posed boundary treatment in spectral space at $t+\Delta t$ is not obvious at all, the more natural way is to perform the coupling explicitly at t, i.e. at the beginning of the time step. The basic idea consists in constructing the well-posed boundary strategy in the RHS fields in the integration area and using the E-zone extrapolation as an alternative to direct Fourier transform.

3. Explicit boundary treatment

At each grid-point on the lateral boundaries, for each inward pointing characteristic velocity, a field must be externally supplied. Let us assume a subsonic flow pattern, i.e. such that the wind speed is less than the typical gravity waves phase speed ($C > U > 0$), which is usual in atmospheric context. In that case, well-posedness is achieved by imposing only two fields at the western boundary (x=0) boundary and only one at the eastern one (x=L), see Oliger and Sundstrom (1978). The imposed fields are incorporated in the boundary RHS, using a classical one-sided finite-difference scheme to evaluate the derivative at the edges. Afterwards, semi-Lagrangian cubic interpolation is performed, then the E-zone extrapolation is applied for periodicity. Three options have tested, following the various boundary strategies proposed by McDonald :

Option *(i) : Imposing u-field*
One imposes u at both boundaries and v at the western boundary.

Option *(ii) : Characteristics boundary condition*
The fields corresponding to the ingoing characteristics $u+C\Phi$ and $u-C\Phi$ are imposed and the outgoing characteristics are extrapolated from the interior. In subsonic case, $u+C\Phi$ and v must be externally supplied at western boundary, while $u-C\Phi$ is extrapolated from inside. At the eastern boundary, only $u-C\Phi$ is supplied, one extrapolates the other fields, see Elvius and Sundstrom (1979).

Option *(iii) : Semi-transparent boundary condition*
Semi-transparent boundary conditions are derived from the Engquist and Madja (1977) theory, see also McDonald (2002, 2003) for more details. These non-reflecting boundary conditions are more accurate than the characteristic one. Interestingly, they are non-local in space, and so rather well-matched with spectral computation. It can be expressed as following :

$$B_{\pm}^{1} = C\left(\frac{\partial \Phi}{\partial x}\right)^{0} - \left(\frac{f}{C}\right)v^{0} \pm \left(\frac{\partial \Phi}{\partial x}\right)^{0} \qquad (7)$$

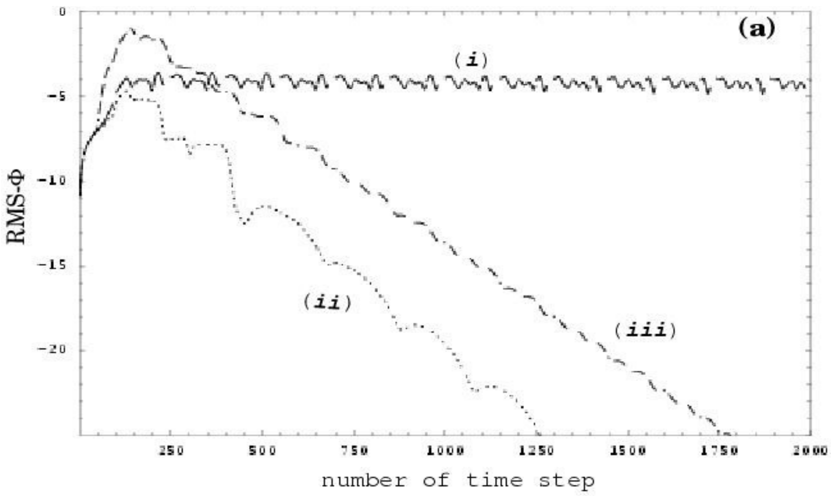
B_{+}^{1} is imposed at western boundary and B_{-}^{1} at eastern boundary. Finite-difference evaluation of the derivative at boundaries is no longer necessary. v is subject to the same treatment as in *(i)* and *(ii)*. It can be noticed that in a geostrophic pattern $B_{\pm}^{1}=0$.

4. Numerical tests

We start with a geostrophically balanced bell-shape placed at the centre of the domain and moving in the positive direction at mean velocity U without changing shape. The settings for this experiment are : $\Delta x=10\ km, U=100\ ms^{-1}, C=300\ ms^{-1}, f=10^{-4}s^{-1}$.

This test has given some encouraging results for small time-steps, Δt=50 s, with option *(i)* : the bell-shape leaves the integration domain with some spurious unbalanced gravity waves, but a priori not large enough to be troublesome. Better results from the standpoint of transparency are obtained when imposing the characteristics fields *(ii)* and semi-transparent boundary *(iii)* conditions, see the figure below. But characteristics boundary conditions remains the best strategy. Unfortunately all these boundary explicit treatments are strongly unstable with larger time-steps, e.g. Δt=400 s.

The associated unstable mode corresponds to a large unbalanced disturbance with a $2\Delta t$ time-frequency. It first appears at the eastern boundary then it extends to the other boundary through the E-zone, affecting the whole domain. It has been noticed that, the wider the E-zone is, the larger its influence over the integration area becomes.



It seems that the use of the E-zone extrapolation as an alternative to direct Fourier transform causes unbalanced waves (such as typical gravity waves) which spread out to the whole physical domain through the boundaries. This E-zone effect was previously pointed out by Haugen and Machenhauer (1993) for initialization issue. The Radnoti scheme (1995) avoids this difficulty by setting the E-zone perturbation values to zero. We must be careful, because what we do in the E-zone reveals of crucial importance.

An absorbing boundary zone has been added in the E-zone, to damp spurious waves. It works effectively well in reducing, but doesn't restore stability for large time-steps. Another technique was proposed by J-F Geleyn (during Bratislava's meeting), but is not implemented yet. It consists in applying an appropriate mapping factor in the E-zone in such a way that the spurious waves supported there are significantly vanished when arriving at boundaries.

5. Concluding remarks and futur work

The proposed boundary scheme is unfortunately not able to control the growth of gravity wave disturbances at boundaries, Experiments show that the E-zone extrapolation is guilty. Therefore, the extension procedure has to be rebuilt in order to fulfil simultaneously stability and periodicity condition. Moreover, since efficient and robust boundary schemes are highly required in NWP system, some implicit boundary treatment should be investigated. Eventually, other coupling methods should be explored, provided that we succeed to solve the periodicity issue.

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