The "how ?" and "why ?"
of the discretized governing equations
in the proposed new physics-dynamics interface

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One might ask: "Why do we need such a set of discretized governing equations?". The answer to this question is the easy part of this note and is twofold: (1) to ensure consistency between the different models which the new interface should host and (2) to be able to have a meaningful cross-model comparison and useful DDH-type diagnostic tools. The last point is of course logic, the first point might be a bit less clear. But for instance, in the case of the AROME/ALARO-10 prototypes, there appears to be a lack of enthalpy transport by precipitation and of local enthalpy formal conservation, for which tests in ALADIN have shown a potentially significant impact on the precipitation forecasts. I hope this explains the "why?".

To answer the "how?" part I won't give you an equation-related derivation. Instead, I will explain the reasoning behind these equations. For those really interested in the equations themselves and not having read them, please contact bart.catry@ugent.be or jean-francois.geleyn@chmi.cz.

The starting point is a small manuscript, ARPEGE/ALADIN oriented and dating back to 1983, that derives conservative forms of the thermodynamic equation in case of three water phases (water vapour, liquid water and ice) in the cases \( \delta m=0 \) and \( \delta m=1 \). In the latter case however there wasn't a true conservative form (the tendency of enthalpy could not be written fully as the divergence of a flux). As the so-called AROME equations were derived in a barycentric framework and use more water-phases (rain water, snow, graupel, hail), it seemed useful to redo the exercise of many years ago.

This exercise was indeed redone with two partial limitations: for a mass-type vertical coordinate (like for AROME and unlike for Meso-NH) and without yet considering the dissipation terms linking dynamics and thermodynamics, especially in the compressible case.

As hail and graupel have the same thermodynamical properties as snow they can be incorporated into snow for our purposes. Another assumption is that all processes should go through the vapour phase which is of course physically not the case but thermodynamically it is fully correct. The allowed phase-changes are shown in the figure. Furthermore we used the proposal of Martina Tudor that in the barycentric case and in case of \( \delta m=0 \) only dry air moves to compensate for the mass fall associated with precipitation. Using these assumptions we were able to derive a set of conservation laws for the different mass species and also to find back a conservative form of the thermodynamic equation similar to the one with only three water phases but with additional fluxes (phase-changes) of course.

In the case of \( \delta m=1 \) we don't have any compensation by dry air anymore but due to the barycentric behaviour, new compensating fluxes appear in the conservation laws of the mass species. Fortunately, these additional fluxes are the reason why we also find in this case a conservative form of the thermodynamic equation. This thermodynamic equation was furthermore independently derived starting from the basic entropy equation related to phase-changes and precipitation.

Moreover, from dynamical point of view, because in the barycentric case there are no fluxes which can be considered as source terms, the continuity equation can be simplified which has consequences for the vertical velocities, which now depend only on the surface fluxes of
evaporation and precipitation.

Finally, the addition/removal of heat due to phase-changes should in the non-hydrostatic compressible case not only lead to a temperature change but also to an associated pressure change. Using basic principles (the state law and the relation \( C_p = C_v + R \)) we were able to derive the associated pressure change, which not only depends on the diabatic heat source but also on the change in air composition.

So we have a barycentric set of equations (the only one deemed by the AROME team fit to accommodate compatibility with both AROME and ARPEGE/ALADIN dynamical cores) where the following issues are treated:

1. multi-phase choice;
2. enthalpy conservation;
3. choice between \( \delta m = 0 \) and \( \delta m = 1 \);
4. optional projection of the heat source on temperature and pressure in the compressible case.

Furthermore, no additional simplifying hypotheses were needed on top of those already used in the derivation of the AROME equations. The latter condition plus obeying the four above-mentioned constraints were indeed the "boundary conditions" of our work, set on the basis of known open questions, for lack of a purely AROME-based definition.

Work has now started on how to implement the mathematical and/or physical consequences of the obtained set of equations with respect both to ALADIN (extension and simplification of the concept) and to AROME (projection onto a new dynamical core of what was originally thought only for the Meso-NH one with its short time-steps). This should lead to something fully prepared for ALARO and for the HIRLAM likely demand, but this goes beyond the scope of the present note.