ALADIN PhD:
"Evaluation of assimilation cycles in a mesoscale limited area model"

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1. Formalism : a brief reminder

(cf. ALADIN Newsletter 25)

The larger scales of the ARPEGE analysis (x^{AA}) are introduced in the ALADIN 3D-VAR as a new source of information. The subsequent information vector is :

$$Z=\begin{pmatrix} x^b \\ y \\ H_1(x^{AA}) \end{pmatrix},$$

where x^b is the background state, *y* is the observation vector, and the projections H_1, H_2 are defined as : H_1 : ARPEGE → ALADIN *low res.*; H_2 : ALADIN *full res.* → ALADIN *low res.*

The cross-covariances between the 3 error vectors : ε^b, ε^o, and $\varepsilon^k=H_1(x^{AA})-H_2(x^t)$ are summed up in the following matrix :

$$W=\begin{pmatrix} B & 0 & E(\varepsilon^b\varepsilon^{k\,\mathrm{T}}) \\ 0 & R & 0 \\ E(\varepsilon^k\varepsilon^{b\,\mathrm{T}}) & 0 & V \end{pmatrix},$$

assuming that the observation errors are correlated neither with the background errors nor with the "large scale" errors.

If the non-diagonal terms of the *W* matrix are negligible, the cost function is simply modified with an extra-term :

$$J(x)=(x^b-x)^{\mathrm{T}}B^{-1}(x^b-x)+(y-H(x))^{\mathrm{T}}R^{-1}(y-H(x))+(H_1(x^{AA})-H_2(x))^{\mathrm{T}}V^{-1}(H_1(x^{AA})-H_2(x)),$$

or in its incremental formulation:

$$J(\delta x)=\delta x^{\mathrm{T}}B^{-1}\delta x+(d^o-\boldsymbol{H}\delta x)^{\mathrm{T}}R^{-1}(d^o-\boldsymbol{H}\delta x)+(d^k-\boldsymbol{H_2}\delta x)^{\mathrm{T}}V^{-1}(d^k-\boldsymbol{H_2}\delta x),$$

where $\delta x=x-x^b$, $d^o=y-H(x^b)$ and $d^k=H_1(x^{AA})-H_2(x^b)$.

First, the *B* and *V* covariances, and the cross-covariances $E(\varepsilon^k\varepsilon^{b\mathrm{T}})$ (hereafter named E_{kb}) are evaluated in a low-resolution spectral space, and some horizontal diagnoses are plotted. Then the first results are shown.

2. Evaluation of the statistics in ARPEGE-ALADIN

The statistics are evaluated thanks to an ensemble method. We rely on the ensembles generated in ARPEGE by Margarida Belo-Pereira (Gaussian perturbation of the observations using their own σ^o) and the subsequent ensembles generated in ALADIN by Simona Stefanescu.

2.1 Spectral

The nominal ALADIN-France truncation is 149 both for zonal and meridional wavenumbers. 12 is chosen to be the truncation of the low-resolution spectral space. Caution : all formulas hereafter correspond to a "square" domain (NSMAX=NMSMAX).

2.1.1 Vertical profiles of standard deviation

For vertical level *l*, the standard deviation σ_l is a definite positive quantity which gathers the contributions from the horizontal wavenumbers :

$$\sigma_l=\sqrt{\sum_{m,n}Q_{l,l}(m,n)}$$

where $Q_{l,l}(m,n)$ is the auto-covariance for vertical level l and wavenumber pair *(m, n).*

The vertical profiles for the "full-resolution" ARPEGE analysis standard-deviations (dotted lines on Fig. 1) are greater than the "low-resolution" ARPEGE analysis standard-deviations (σ_l^k, solid lines), which is a direct consequence of the definite-positiveness of the standard deviation. The contributions of the smaller scales seem to be more important in the troposphere than in the stratosphere.

The ("full-resolution") ALADIN background standard-deviations (σ_l^b, dashed lines) are larger than the σ_l^k for vorticity and divergence, but the σ_l^b and σ_l^k profiles have the same shape for temperature and specific humidity.

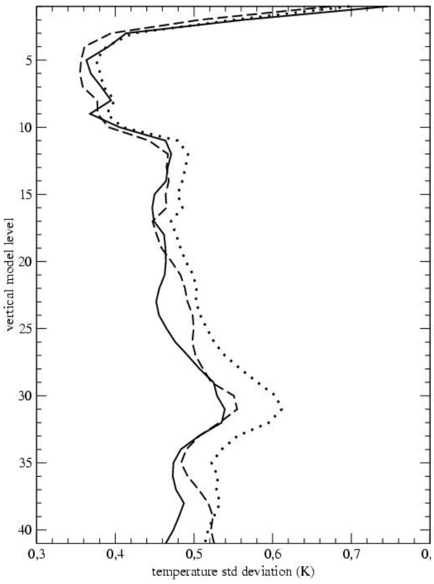


Figure 1 : Vertical profiles of standard deviation for temperature (K).:

dashed : ALADIN background error
dotted : ARPEGE analysis error nominal ALADIN
solid : ARPEGE analysis error low resolution ALADIN

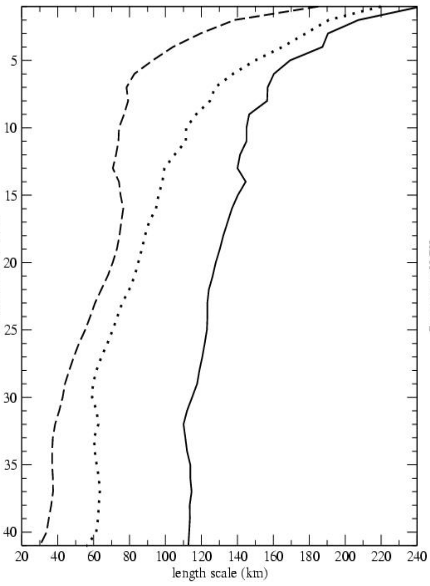


Figure 2 : Vertical profiles of length-scale for temperature (K).:

dashed : ALADIN background error
dotted : ARPEGE analysis error nominal ALADIN
solid : ARPEGE analysis error low resolution ALADIN

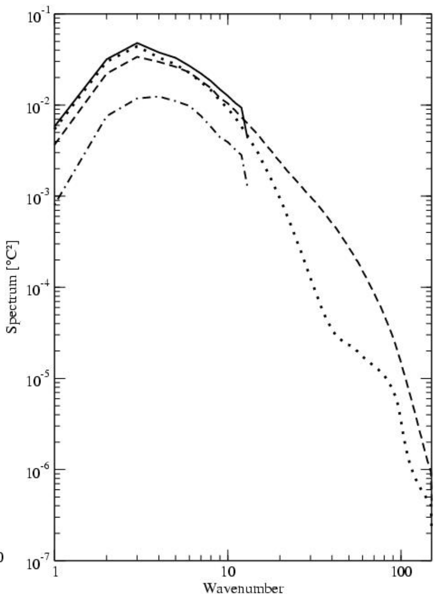


Figure 3 : Horizontal variance spectra for temperature (K) on model level 22:

dashed : ALADIN background error
dotted : ARPEGE analysis error nominal ALADIN
solid : ARPEGE analysis error low resolution ALADIN
dot-dashed : *Ekb*

2.1.2 Vertical profiles of length-scale

For a vertical level l, the length-scale can be defined as :

$$L_l = F \sqrt{\frac{\sum_{m,n} Q_{l,l}(m,n)}{\sum_{m,n} (m^2+n^2)\, Q_{l,l}(m,n)}},$$

where F is a scaling factor. The larger the truncation, the smaller L_l.

The change of truncation (nominal to low resolutions) clearly implies an increase of the length-scale (dotted to solid lines, on Fig. 2). The length-scales of ALADIN background errors are a bit smaller than those of the "full-resolution" ARPEGE analysis errors in the troposphere for all variables, and also in the stratosphere for temperature.

2.1.3 Horizontal variance spectra

Assuming horizontal homogeneity and isotropy, the B and V matrices are diagonal for each variable and vertical level. With the same hypotheses, we have :

$$E_{kb} = \begin{pmatrix} \square & & 0 & 0 & \cdots & 0 \\ & \ddots & & \vdots & & \vdots \\ 0 & & \square & 0 & \cdots & 0 \end{pmatrix}.$$

The first block is a $q \times q$ diagonal block and the second block is a $q \times (n-q)$ null block, where n is the nominal truncation and q the low-resolution truncation.

The spectra for the full and low resolution ARPEGE analysis errors overlap quite well for the first wavenumbers (dotted and solid lines respectively, on Fig. 3). A strong decrease can be observed for wavenumber 12 in the low-resolution case, which is similar to an "end of spectrum" in full resolution.

E_{kb} is roughly 5 times smaller than B or V. At first order, we will assume E_{kb} is negligible. But, later, the (σ^k) $\left(\text{i.e. } \sigma_l^k \text{ for all } l\right)$ should be retuned to take these cross-covariances into account.

2.2 Gridpoint

The variances $\sigma_l^{k\,2}$ and $\sigma_l^{b\,2}$ have been plotted for various variables and model levels l. They have the same horizontal inhomogeneities, which are also of the same order over the ALADIN France domain as the ARPEGE analysis error variances (in ARPEGE geometry). [not shown]

An average over a 45-days period (02-03 2002) of the innovation, $d^k = H_1(x^{AA}) - H_2(x^b)$, is plotted on Fig. 4a for temperature on model level 29. The innovation is stronger over the Atlantic Ocean, over the North-Western corner of the domain and over the Alps. One can split the innovation into to contributions : $d^k = \left(H_1(x^{AA}) - H_1(x^{BB})\right) + \left(H_1(x^{BB}) - H_2(x^b)\right)$. Here x^{BB} is the ARPEGE background, that is to say the innovation is the sum of the ARPEGE analysis increment and of the difference between the ARPEGE and ALADIN forecasts, both put on the ALADIN low-resolution geometry. When having a look at the average of $H_1(x^{AA}) - H_1(x^{BB})$ over the period (Fig. 4b), it arises that the ARPEGE analysis increment is the main contribution to the innovation.

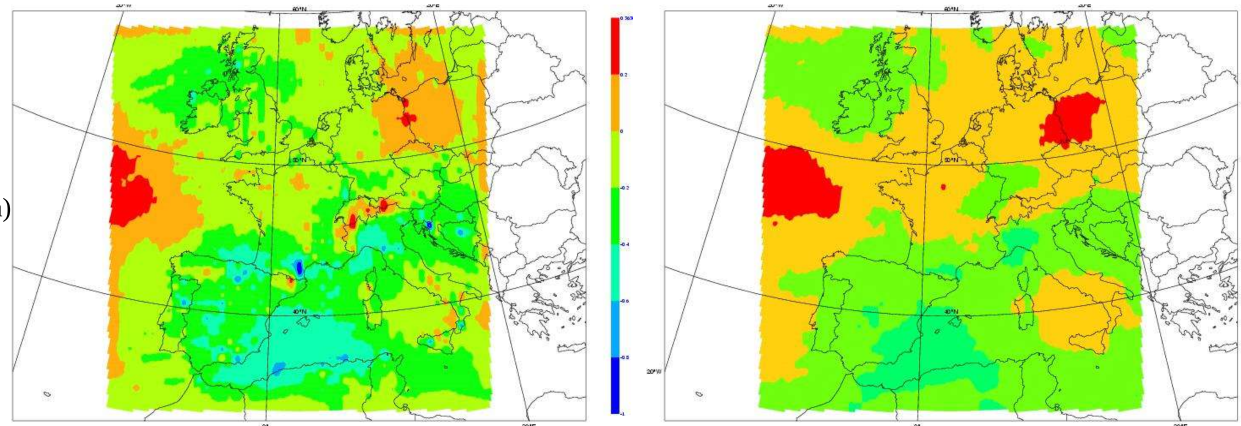
a)  b)

Figure 4: Temperature on model level 29
a) average of the d^k innovation, b) average of the ARPEGE analysis increment

3. First Results

3.1 Technical Implementation

The new cost-function J_k, defined as $J_k(x) = (H_1(x^{AA}) - H_2(x))^{\mathrm{T}} V^{-1} (H_1(x^{AA}) - H_2(x))$, has been first implemented in the ARPEGE-ALADIN cycle 28 environment. The cost-function is activated through a new namelist (NEMJK) and key LEJK. The truncation of the low-resolution spectral space is 12. No particular tuning of the statistics is performed upstream. The weight of the cost function can be tuned thanks to the real parameter ALPHAK.

The first results, shown hereafter, have been produced with the operational cycle 26T1. In this particular test, ARPEGE and ALADIN were fed with the *same* observations, which is not really suitable with the formulation.

3.2 Verification

This early test has been performed on the situation of the day (April, the 19th). The results for the temperature on model level 22 are shown on Fig. 5. Two areas are highlighted :

- Blue rectangle over the Atlantic Ocean :
  The ARPEGE analysis isolines are shifted northwards compared to the ALADIN background (5a). The ALADIN analysis without J_k (5b) remains closer to the background than the analysis with J_k (5c). The analysis is modified as was expected, i.e. towards the ARPEGE analysis, as it is as large-scale shift.
- Blue circle between Sardinia and Sicily :
  There is a small-scale oscillation in the ALADIN background but not in the ARPEGE analysis (5a). This pattern remains in the analysis without J_k (i.e. it is not modified by the observations) and in the analysis with J_k (i.e. it is not modified by the new source of information).

a)

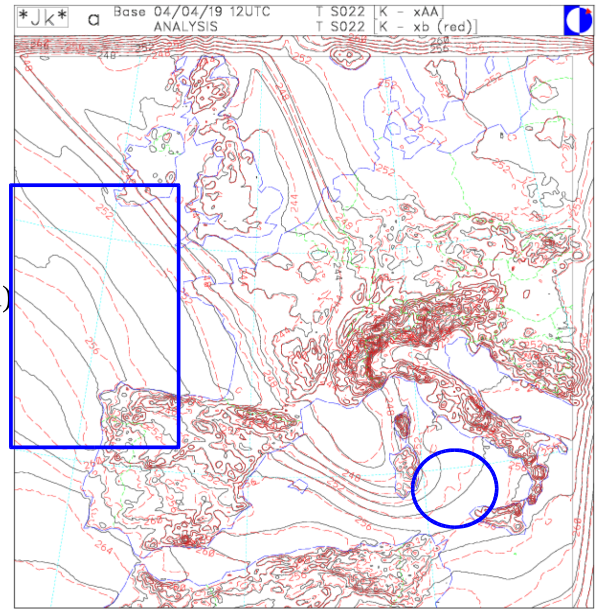
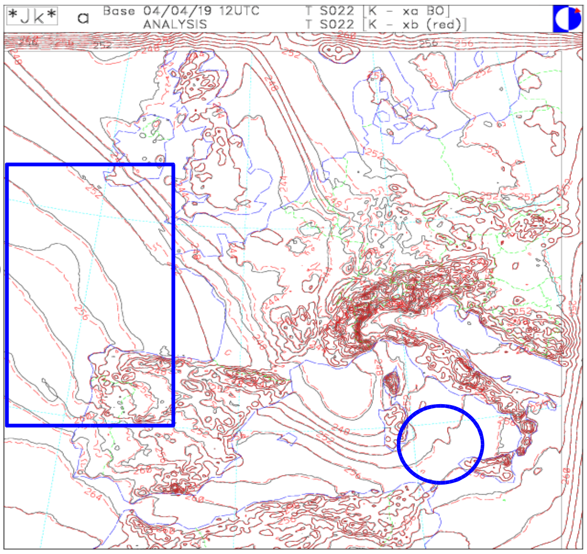
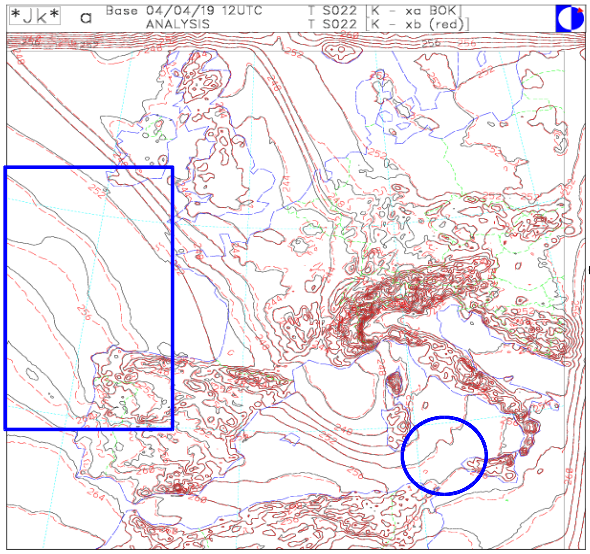


Figure 5: Temperature on model level 22 valid at 12 UTC on 2004/04/19.

ALADIN +06h forecast in red.
a) ARPEGE analysis on ALADIN nominal grid
b) ALADIN analysis without J_k
c) ALADIN analysis with J_k

b)





c)

3.3 Conclusion

This new cost-function introduces some information about the large scales, but it does not modify the meso- and small-scale patterns either present in the background or built by the "classical" analysis.

A "full" evaluation of this new analysis will be performed over 2 periods of 15 days, with score computation and case studies.

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