The use of Kalman filter
for improving the short range forecast of 2m temperature

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Introduction

Numerical model forecasts are often biased mostly for boundary layer parameters. Its main reason is that model orography differs from the real geographical one. Inaccuracy of the parameterisation of subgrid scale phenomena may be a source of systematic errors as well. To have more realistic forecasts post-processing procedures are used that may be based on either dynamical or statistical considerations. Statistical approach is used to adjust model outputs to reality. Our purpose is to make the 2m temperature model forecast of ALADIN/HU model more accurate. To this end we use the Kalman filter technique because in this method error statistics are intrinsically refreshed. Thus the algorithm rapidly adjusts to weather changes. Forecasts corrected by Kalman filter are performed recursively by combining them with observations. Our Kalman filter model is the adaptation of the program developed by Jozef Vivoda.

Theory of Kalman filter

Here we introduce a simplified version Kalman filter procedure (Homleid, 1996). Let x_t be the state of the unknown discrete stochastic process at time t. The state at time t is associated with that of t-1 through the system equation :

$$x_t = F_t \cdot x_{t-1} + w_t \cdot \qquad (1.1)$$

where F_t is the transition matrix of the system and w_t represents the random change between time t-1 and t, i.e. w_t is the error of forecast by F_t. F_t is linear. x_t is associated with the observation y_t through the observation equation :

$$y_t = H_t \cdot x_t + v_t \cdot \qquad (1.2)$$

where H_t is the observation matrix and vector v_t stands for the random component of observation error.

Observation matrix H_t and transition matrix F_t change in time but they are supposed to be known. The terms v_t and w_t represent white noise processes with a Gaussian distribution of expected value 0 all along time t and they are assumed to be independent in time. Thus for every time $t \neq s$, $E(w_s\, w_t^T) = 0$ and $E(v_s\, v_t^T) = 0$, where E denotes the expected value. Additionally v_t and w_t are independent noises : $E(v_s\, w_t^T) = 0$ for every s and t. Let W_t denote the covariance matrix of a w_t, i.e. $W_t = E(w_t\, w_t^T)$ and similarly for the covariance matrix of v_t, $V_t = E(v_t\, v_t^T)$.

The Kalman filter provides the recursive estimation of states x_t by taking into account every observation until time t. They are denoted by $y_1, y_2, \ldots, y_t$. To compute the state the estimation of error covariances is used and it is denoted by P_t. To make the estimation from t-1 to t two steps are performed. They are summarised below :

$x_{t,\,t-1}$	is the estimation of x_t at time t by taking into account observations until time t-1
$P_{t,\,t-1}$	is the estimation of the error covariance matrix of $x_{t,\,t-1}$
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Let us suppose that the observations until time t-1 are known and we have computed the optimal estimation $x_{t-1,\,t-1}$ as well. If we take into account only the observations until time t-1 then the optimum estimation for time t is :

$$x_{t,t-1} = F_t \cdot x_{t-1,t-1} \quad . \tag{1.3}$$

The error covariance matrix of state estimation $x_{t,t-1}$ can be computed from :

$$P_{t,t-1} = F_t \cdot P_{t-1,t-1} \cdot F_t^T + W_t \tag{1.4}$$

When the y_t observation for time t is known, the estimation of x_t can be updated from the linear combination of $x_{t,t-1}$ and from the forecast error $(y_t - H_t \cdot x_{t,t-1})$:

$$x_{t,t} = x_{t,t-1} + K_t \cdot (y_t - H_t \cdot x_{t,t-1}) \quad . \tag{1.5}$$

In (1.5) K_t denotes the Kalman gain matrix :

$$K_t = P_{t,t-1} \cdot H_t^T \cdot (H_t \cdot P_{t,t-1} \cdot H_t^T + V_t)^{-1} \tag{1.6}$$

The estimation of error covariance matrix can be performed by :

$$P_{t,t} = (1 - K_t \cdot H_t) \cdot P_{t,t-1} \tag{1.7}$$

The system of equations described above is sufficient for performing the Kalman filter algorithm.

To be able to apply this model for improving temperature forecasts one has to estimate the statistical parameters and the matrices characterising the system. Thus one has to determine the state transition matrix F_t and the observation matrix H_t and to estimate the covariance matrices of the noises of the system and the observations W_t and V_t. Additionally one has to estimate the initial states, i.e. $x_{0,0}$ and $P_{0,0}$.

The Kalman filter (KF) in practice

On the basis of Homleid's description Jozef Vivoda developed the model of Kalman filter in Slovakia to improve ALADIN products by surface observation (Vivoda, 1998). Our KF model is its adaptation. We perform the corrected forecasts for the time range 00 to 24 hours with a 1 hour time-steps. The covariance matrix of system noises has been computed from temporal and spatial correlations. Following the way proposed by Homleid to compute the ratio *W/V* we obtained the value 0.06. The system matrix was the identity. For each city at the n-th time-step H_t is n-dimensional, its k-th component being 1 if at time-step k there was an observation and *0* if not. An advantage of Kalman filter is that the state is rapidly converging to its theoretical value independently from its initial value.

Correction of 2m temperature forecasts of ALADIN/HU

To evaluate a numerical model one computes forecast errors. We have computed mean errors (ME) mean square errors (MSE). ME gives information on the bias or systematic errors of the model and MSE characterizes the magnitude of errors.

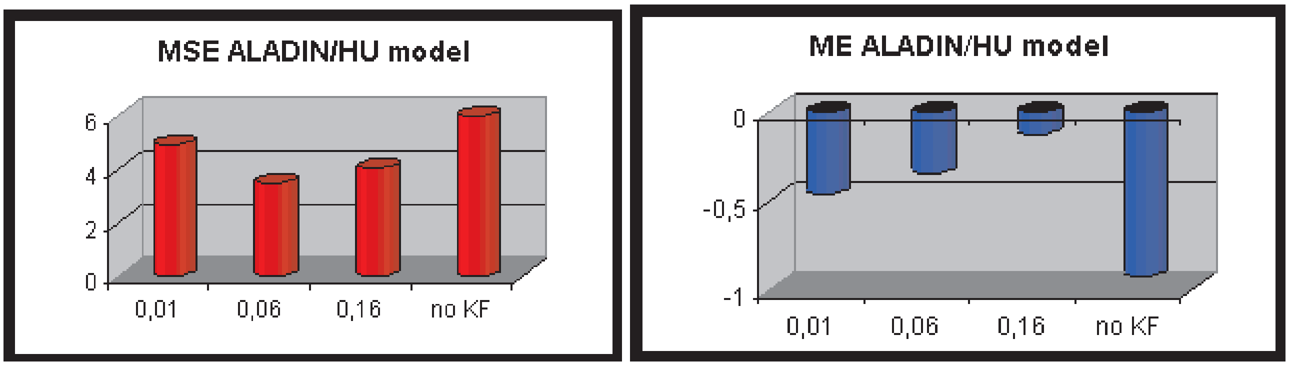

Figure 1. ME and MSE errors for Hungary in the period 2001. 01. 01-04. 20 for 3 different noise ratios of KF and for no KF

In case of ALADIN/HU model there are two stations giving extremely incorrect forecasts. One of them is Kékestetõ (situated in the North-Eastern part of the country being the highest point of Hungary) that is seen by ALADIN/HU model on a height of 357 m while its actual altitude is 1015

m. This difference of 658 m appears as a very significant error in the forecast of temperature, we can speak about a systematic overestimation of 3-4 K. Beside Kékestetõ the meteorological station in Budapest, Kitaibel street (Headquarter of the Hungarian Meteorological Service, situated in the downtown of Budapest) is remarkable with the mean error of 2 K. In case of this meteorological station the error may be caused by the positive temperature anomaly of the urban heat island. In other locations (Poroszló, Jósvafõ and Miskolc for instance) the problem is in the temperature of the surrounding grid points, as in case of these towns the nearest grid points have different geographical characteristics. In case of towns in the Great Hungarian Plain and the Little Plain (Northwestern Hungary) the forecast of ALADIN/HU model is relatively correct as the relief is here the same as the orography used in the model; the difference of altitude does not exceed 10 m in case of these towns, e.g. it is 6 m in case of Gyõr, while the mean error is less than 0,5 centigrade.

Based on the above facts we can say that in Hungary the systematic error originates mainly from the differences between the relief used in the numerical model and the actual orography.

If mean error is *W/V*=0,01 in towns where the error of the numerical model was moderate or smaller than the average, the Kalman filter improved the forecast. So in case of towns of the Great Hungarian Plain and on West Hungarian areas an improvement of 0,5 centigrade is characteristic on the average. In case of this noise ratio the temperature forecasts of meteorological stations Kékestetõ and Budapest, Kitaibel street are improved very slightly. It is no wonder as according to tests based on constant data series the small noise ratio means that the filter takes into account rather the products of the numerical model than the observed data. Concerning squared errors improvement cannot be really experienced. In some towns this value decreased very slightly but considering all the 32 stations we can state that significant improvement has not followed. (At the same time it is true that we have not even made the situation worse.) If the Kalman filter is run with greater value - *W/V*=0,16 than the empirical noise ratio, we can see significant change in every respect. The degree of improvement is the highest in case of the two stations with extremely high mean error. The mean errors improved from 3 K to 0,8 K in case of Kékestetõ station while from –3 K to -1,6 K in Budapest, Kitaibel street. Applying this noise ratio we can see that instead of the systematic underestimation of temperature in numerical models the Kalman filter overestimates temperature in nearly one third of towns. Its reason is that in case of such noise ratios the filter relies rather on measurements but with much larger amplitudes as we could see in the test with constant data series. Thus in case of Pécs where the model gave one of the most correct forecasts, the overestimation was nearly 0,3 K what can be attributed to the large amplitudes of the model. Certainly it can be seen as well that both the mean and the squared error decreased significantly.

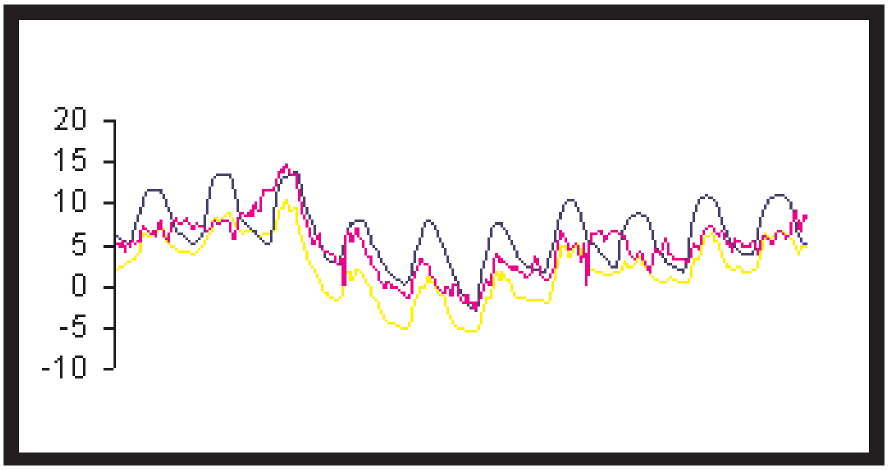

Figure 2. Kékestetõ, values of 2 m temperature (° Celsius) between 11-20.04.2001, synoptic data, ALADIN model, forecast corrected by Kalman filter in case of noise ratio of 0,06

The Kalman filter model has to be run on long period with a noise ratio that relies rather on synoptic values but takes also the results of numerical models properly into account. With the noise ratios applied till now we achieved a conservative, slowly changing, inactive model (value of W/V=0,01) and a more rapid filter with great amplitudes (value of W/V=0,16). The intermediate value is a stable model that follows the changes slower than that of a W/V=0,16 noise ratio but with smaller standard deviation and relies rather on measurements at estimations compared with the filter with W/V=0,01 noise ratio. This intermediate solution is the value of W/V=0,06 (see Figure 2). Comparing the products of the numerical models with the results of Kalman filter model of three different noise ratios it is obvious that the squared errors are the smallest as a result of this procedure. The squared error of only one station is greater than in case of the most correct filter (W/V=0,16) that is of the meteorological station of Budapest, Kitaibel street. This extremely great error is not only the result of positive temperature anomalies of large cities - as filter models improved forecasts in every other case - but the lack of data has also significant role in it. If we examine where we got correct forecasts we can see that the mean errors of temperature forecasts in the areas of Great Hungarian Plain, Little Plain and South-Transdanubia remain below 0,1 centigrade. In these areas also the values of squared errors are 2 K or even less. Besides the mentioned problematic station the squared errors remain below 5 K.

Conclusions

Our objective was to improve the temperature products of numerical models by implementing Kalman filter statistical adaptation procedure. The comparison of our results with the products of numerical models can be seen on Figure 1. We can say that the results of ALADIN/HU model could be improved significantly compared with its 2 m temperature forecast. We have tested the filter with data series of nearly four months for forecasts valid until 24 hours. We examined in which cases the filter makes the numerical model predictions worse. This occurred only in 7 percents of forecasts. The weak point of Kalman filter model is that in case of rapid weather changes the state vector has to change sign rapidly, as the numerical atmospheric model is run daily and thus the estimations are filtered daily. In case of rapid weather change the forecast of the filter has to be disregarded on the first day. The other problem is that the data quality control used by us is very indigenous. A more effective method has to be developed as the incorrect measurements may damage the statistics. If we improve the situation in the above mentioned two respects we can achieve the automatic production of temperature forecasts. The Kalman filter model prepared by us is suitable also for correction of 10 m wind, daily maximum and minimum temperature values if we define statistical parameters for each of these meteorological variables. As we could see in the scientific literature and in the present description very good results can be achieved with Kalman filter. The filter would be really perfect if the numerical weather prediction model was run at the same intervals as the time step of the filter. The statistics in the tests are not final because of the short period, it would be worth to prepare again the verification of our Kalman filter model in one year. If we manage to control the synoptic data more effectively, the statistics will certainly show that the Kalman filter eliminates fully the systematic errors of forecasts.

References:

Bucy, R.S., and Kálmán R., 1961: New results in linear filtering and prediction theory. *Transaction ASME Journal of Basic Engineering*. 83: 95-108.

Homleid, M., 1996: Diurnal Correction of Short-Term Surface Temperature Forecast Using the Kalman Filter. *Weather & Forecasting*, 689-707.

Vivoda, J., 1998: Correction of surface temperature forecast using the discrete Kalman filter. *Meteorologický casopis*, 1.2. 1998, 31-37.