Regional Cooperation for
Limited Area Modeling in Central Europe

LACE
nwp central europe

ACCORD
A Consortium for COnvection-scale modelling
Research and Development

Dynamics in LACE - towards hectometric scales

Petra Smolíková (CHMI), Alexandra Craciun (NMA), Mario Hrastinski (DHMZ), Nika Kastelec (ARSO), Jozef Vivoda (ECMWF)

GeoSphere Austria · DHMZ · Czech Hydrometeorological Institute · HungaroMet · IMGW · METEO ROMANIA · SHMÚ · ARSO METEO Slovenia

Outline

- Dynamical core in ACCORD
- SI time scheme
- Orographic terms in linear model (based on work of Jozef Vivoda and Fabrice Voitus)
  - Some equations
  - Idealised tests
  - Real simulations @200m
- Vertical velocity definition in nonlinear model (based on work of Fabrice Voitus)
  - Real simulations @200m

Dynamical core in ACCORD

LACE
nwp central europe

Basic equations

- hydrostatic primitive equation system (HPE) or Euler equations (EE); recently implemented quasi elastic equation system (QE)
- prognostic variables $\vec{v}, T, q_s = \ln(\pi_s)$, in EE with $w, \hat{q} = \ln(\frac{p}{\pi})$

Discretization

- spectral method for horizontal direction
- hybrid vertical coordinate η based on hydrostatic pressure $\pi(\eta) = A(\eta) + B(\eta)\pi_s$; $A(top) = B(top) = 0, A(bottom) = 0, B(bottom) = 1$
- finite differences or finite elements for vertical direction discretization
- semi-implicit or iterative centred implicit scheme for time discretization
- semi-Lagrangian advection

Semi-Implicit time scheme

System evolution

$$\frac{dX}{dt} = \mathcal{M}X$$

Linearization

$$X = \mathbf{X}^* + \mathbf{X}', \quad \mathcal{M} \longrightarrow \mathcal{L}^*$$

Using linear model $\mathcal{L}^*$ we get

$$\frac{dX}{dt} = \mathcal{L}^*\overline{[X]}^t + (\mathcal{M} - \mathcal{L}^*)X$$

and discretize in time to obtain

Semi-implicit scheme

$$\frac{\mathbf{X}^+ - \mathbf{X}^0}{\Delta t} = \mathcal{L}^*\left(\frac{\mathbf{X}^+ + \mathbf{X}^0}{2}\right) + (\mathcal{M} - \mathcal{L}^*)\mathbf{X}^{+\frac{1}{2}}$$

or

Iterative centered implicit scheme

$$\frac{\mathbf{X}^{+(n)} - \mathbf{X}^0}{\Delta t} = \frac{\mathcal{L}^*\mathbf{X}^{+(n)} + \mathcal{L}^*\mathbf{X}^0}{2} + \frac{(\mathcal{M} - \mathcal{L}^*)\mathbf{X}^{+(n-1)} + (\mathcal{M} - \mathcal{L}^*)\mathbf{X}^0}{2}$$

We know that both can be second order accurate in time when some care is taken (averaging along semi-Lagrangian trajectory).

Full model

LACE nwp central europe

Temperature

$$\frac{dT}{dt} = \frac{\kappa T}{\kappa - 1}(D + d)$$

Horizontal momentum

$$\frac{d\vec{v}}{dt} = -RT\frac{\nabla\pi}{\pi} - \nabla\phi - RT\nabla\hat{q} - \frac{1}{m}\frac{\partial(p-\pi)}{\partial\eta}\nabla\phi$$

Vertical momentum

$$\frac{dw}{dt} = \frac{g}{m}\frac{\partial(p-\pi)}{\partial\eta}$$

Pressure departure

$$\frac{d\hat{q}}{dt} = \frac{1}{\kappa - 1}(D + d) - \frac{1}{\pi}\frac{d\pi}{dt}$$

Surface pressure

$$\frac{dq_s}{dt} = -\frac{1}{\pi_s}\int_0^1 \nabla\cdot(m\vec{v})d\eta$$

Diagnostic relations

$$\begin{aligned}
\frac{d\pi}{dt} &= \vec{v}\cdot\nabla\pi - \int_0^\eta \nabla\cdot(m\vec{v})d\eta' \\
\phi &= \phi_s - \int_\eta^1 \frac{mRT}{p}d\eta' \\
d &= -\frac{p}{mRT}\left(\nabla\phi\frac{\partial\vec{v}}{\partial\eta} - g\frac{\partial w}{\partial\eta}\right)
\end{aligned}$$

Definitions

$$\begin{aligned}
D &= \nabla\cdot\vec{v} \\
\kappa &= \frac{c_p}{R} \\
m &= \frac{\partial\pi}{\partial\eta}
\end{aligned}$$

Full model

LACE
nwp central europe

Temperature

$$\frac{dT}{dt} = \frac{\kappa T}{\kappa - 1}(D + d)$$

Horizontal momentum

$$\frac{d\vec{v}}{dt} = -RT\frac{\nabla\pi}{\pi} - \nabla\phi - RT\nabla\hat{q} - \frac{1}{m}\frac{\partial(p-\pi)}{\partial\eta}\nabla\phi$$

Vertical momentum

$$\frac{dw}{dt} = \frac{g}{m}\frac{\partial(p-\pi)}{\partial\eta}$$

Pressure departure

$$\frac{d\hat{q}}{dt} = \frac{1}{\kappa - 1}(D + d) - \frac{1}{\pi}\frac{d\pi}{dt}$$

Surface pressure

$$\frac{dq_s}{dt} = -\frac{1}{\pi_s}\int_0^1 \nabla \cdot (m\vec{v})d\eta$$

Diagnostic relations

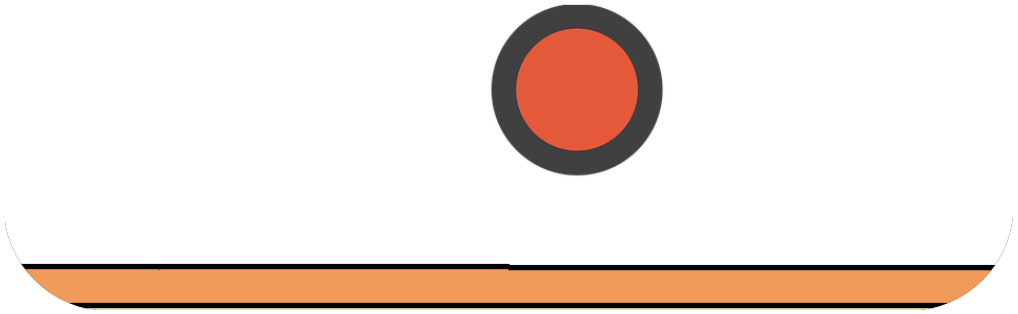
$$\begin{aligned}
\frac{d\pi}{dt} &= \vec{v}\cdot\nabla\pi - \int_0^\eta \nabla\cdot(m\vec{v})d\eta' \\
\phi &= \phi_s - \int_\eta^1 \frac{mRT}{p}d\eta' \\
d &= -\frac{p}{mRT}\left(\nabla\phi\frac{\partial\vec{v}}{\partial\eta} - g\frac{\partial w}{\partial\eta}\right)
\end{aligned}$$

Definitions

$$\begin{aligned}
D &= \nabla\cdot\vec{v} \\
\kappa &= \frac{c_p}{R} \\
m &= \frac{\partial\pi}{\partial\eta}
\end{aligned}$$

Current state:

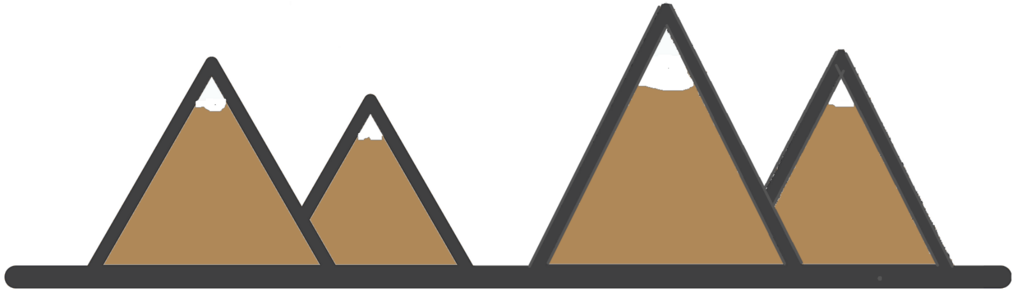
- stationary
- resting
- hydrostatically balanced (π_s^*)
- dry
- isothermal (T^*)
- with constant orography ($\nabla\phi^* = 0$)



Basic state

New:

- stationary
- resting
- hydrostatically balanced (π_s^*)
- dry
- isothermal (T^*)
- with constant orographic slope (in absolute value, $|\nabla\phi^*| \neq 0$)



Linear model

LACE
nwp central europe

Temperature

$$\frac{\partial T}{\partial t} = \frac{\kappa T^*}{\kappa - 1}(D + d)$$

Horizontal momentum

$$\frac{\partial \vec{v}}{\partial t} = -RT^*\frac{\nabla\pi}{\pi^*} - \nabla\phi - RT^*\nabla\hat{q} - \frac{1}{m^*}\frac{\partial \pi^*\hat{q}}{\partial \eta}\nabla\phi^*$$

Vertical momentum

$$\frac{\partial w}{\partial t} = \frac{g}{m^*}\frac{\partial \pi^*\hat{q}}{\partial \eta}$$

Pressure departure

$$\frac{\partial \hat{q}}{\partial t} = \frac{1}{\kappa - 1}(D + d) + \frac{1}{\pi^*}\int_0^\eta m^* D d\eta'$$

Surface pressure

$$\frac{\partial q_s}{\partial t} = -\frac{1}{\pi_s^*}\int_0^1 m^* D d\eta$$

Diagnostic relations

$$\nabla\phi = \nabla\phi_s - \int_\eta^1 \nabla\left(\frac{mRT}{p}\right) d\eta'$$

$$\nabla\phi^* = g\Lambda^* S^*(\eta)$$

$$d = -\frac{p}{mRT}\left(\nabla\phi\frac{\partial \vec{v}}{\partial \eta} - g\frac{\partial w}{\partial \eta}\right)$$

Definitions

$$\Lambda^* = \frac{1}{g}||\nabla\phi_s||^*$$

$$S^*(\eta) = \frac{B(\eta)\pi_s^*}{\pi^*(\eta)}$$

$$m^* = \frac{\partial \pi^*}{\partial \eta}$$

Linearized slope in SI

Modified vertical divergence

$$d = \frac{p}{mRT}\left(\nabla\phi\,\frac{\partial\vec{v}}{\partial\eta} - g\frac{\partial w}{\partial\eta}\right)$$

Time evolution in linear model

$$\frac{\partial\vec{v}}{\partial t} = \mathbb{A} - \Lambda^* S^*(\eta)\mathbb{B}$$

$$\frac{\partial w}{\partial t} = \mathbb{B}$$

$$\frac{\partial d}{\partial t} = \frac{1}{RT^*}\left[\nabla\phi^*\ \partial^*\left(\frac{\partial\vec{v}}{\partial t}\right) - g\partial^*\left(\frac{\partial w}{\partial t}\right)\right]$$

$$= \frac{1}{RT^*}\left[g\Lambda^* S^*(\eta)\partial^*\mathbb{A} - g\Lambda^{*2} S^*(\eta)\left(S^*\partial^*\mathbb{B} + \mathbb{B}\partial^* S^*\right) - g\partial^*\mathbb{B}\right]$$

where

$$\partial^* X = \frac{\pi^*}{m^*}\frac{\partial X}{\partial\eta}$$

Linearized slope in SI

We omit the first order terms in Λ^* and then $\frac{\partial \vec{v}}{\partial t}$ is unchanged and all operators of the RHS of $\frac{\partial d}{\partial t}$ apply on $\hat{q}$.

Time evolution in linear model

$$\frac{\partial \vec{v}}{\partial t} = \mathbb{A} - \cancel{\Lambda^* S^* (\eta) \mathbb{B}}$$

$$\frac{\partial d}{\partial t} = \frac{1}{RT^*}\left[\cancel{g\Lambda^* S^* (\eta) \partial^* \mathbb{A}} - g{\Lambda^*}^2 S^* (\eta) \left(S^* \partial^* \mathbb{B} + \mathbb{B} \partial^* S^*\right) - g\partial^* \mathbb{B}\right]$$

Finally, since $\mathbb{B} = g\left(\partial^* + 1\right)\hat{q}$

Time evolution in linear model

$$\frac{\partial \vec{v}}{\partial t} = \mathbb{A} \rightsquigarrow \frac{\partial D}{\partial t} = \frac{\partial (\nabla \cdot \vec{v})}{\partial t} = \nabla \cdot \mathbb{A}$$

$$\frac{\partial d}{\partial t} = \mathcal{L}^*_{new}\hat{q}$$

Linearized slope in SI

LACE nwp central europe

We can define

New vertical Laplacian operator

$$\mathcal{L}^*_{new} = \alpha\, \partial^* \left(\partial^* + 1\right) + \beta \left(\partial^* + 1\right)$$

$$\alpha = 1 + \Lambda^{*2} S^{*2} \left(\eta\right)$$
$$\beta = \Lambda^{*2} S^* \left(\eta\right) \partial^* S^* \left(\eta\right)$$

$$\Lambda^* = 0 : \mathcal{L}^*_{new} \to \mathcal{L}^*_v = \partial^* \left(\partial^* + 1\right)$$
$$S^*\left(\eta\right) = 0 : \mathcal{L}^*_{new} \to \mathcal{L}^*_v$$

How to discretize the proposed solution?

New discretized vertical Laplacian operator

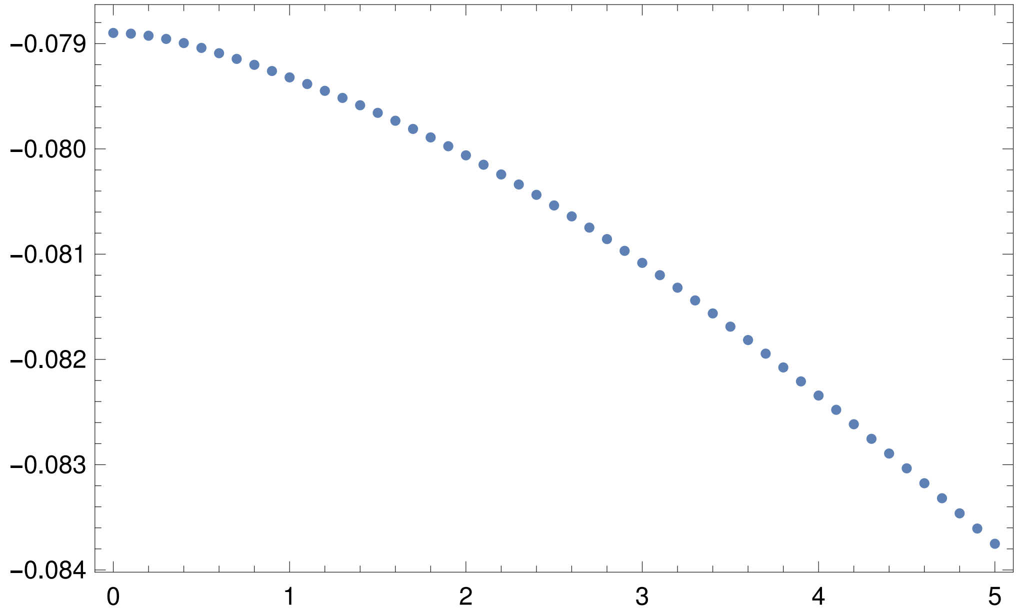
$$[\partial^*(\partial^* + 1)X]_l = \dots$$
$$[(\partial^* + 1)X]_l = \dots$$
$$S^*(\eta_l) = \dots$$
$$\partial^* S^*(\eta_l) = \dots$$

How to set boundary conditions?

Linearized slope in SI

Does $\mathcal{L}^*_{new}$ have only real and negative eigenvalues?

For an example of 87 vertical levels used in Czech operations we are safe.

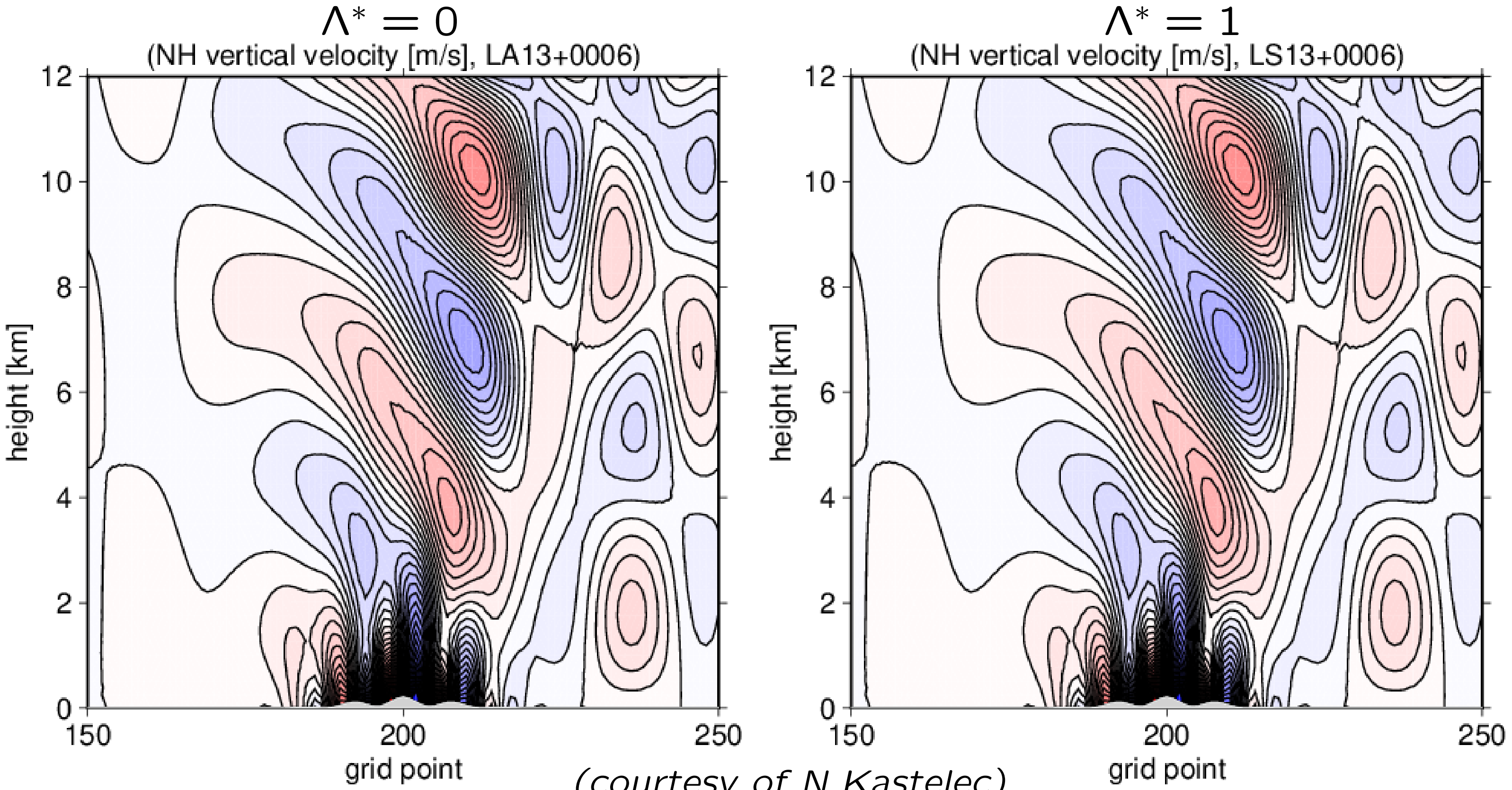


(courtesy of N.Kastelec)

Max value of eigen values depending on Λ

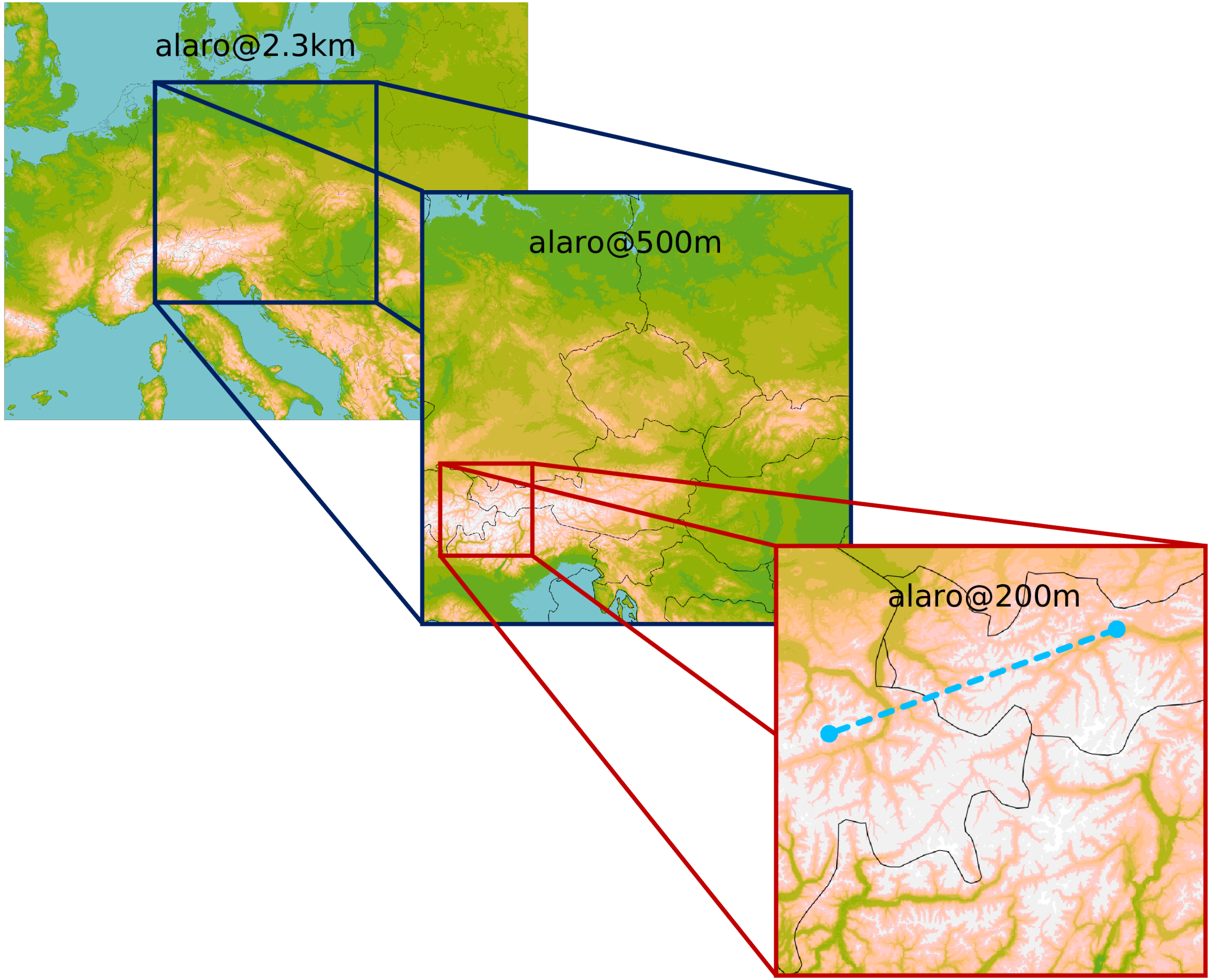
Then we can eliminate the discretized equations up to horizontal divergence D and solve the Helmholtz equation for D.

Vertical velocity for the Schär mountain case depending on Λ^*. ($\Delta x = 500\,m, \Delta z = 250\,m$, mountain height $h = 250\,m, T_0 = 288K, u_0 = 10\,m/s, \Delta t = 32\,s$)



(courtesy of N.Kastelec)

Real simulations @200m



We show results of 200m simulation over Alps.

Real simulations @200m

LACE
nwp central europe

The basic algorithmic choices for ALARO configurations @200m are:

Dynamical core

- semi-Lagrangian advection scheme with 4 iterations for trajectory calculation
- PC time scheme with one iteration, cheap variant (SL trajectories are not recalculated in corrector)
- modified vertical divergence d4 for vertical motion, transformation to vertical velocity w in the non-linear model
- reference values of the linear model: SITR=300K, SITRA=100K, SIPR=900hPa
- no decentering
- semi-Lagrangian horizontal diffusion applied on all model variables + TKE,TTE, hydrometeors
- linear truncation for all spectral fields except orography; quadratic truncation of orography

GeoSphere Austria DHMZ Czech Hydrometeorological Institute HungaroMet IMGW METEO ROMANIA SHMU ARSO METEO Slovenia

Real simulations @200m

LACE
nwp central europe

ALARO physics

- radiation scheme ACRANEB2
- turbulence and shalow convection scheme TOUCANS, model 2
- scale aware deep convection and microphysics scheme 3MT

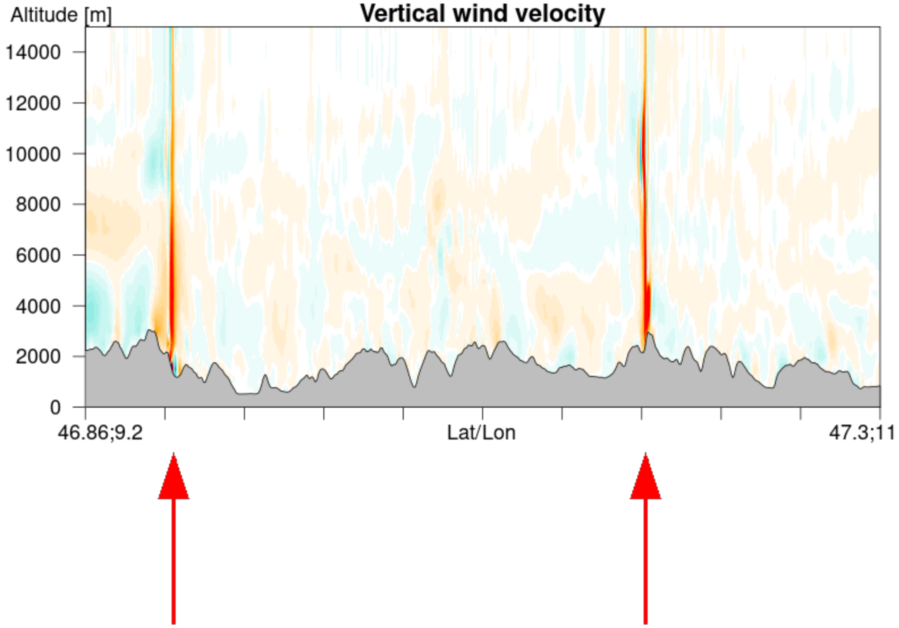
Initialization

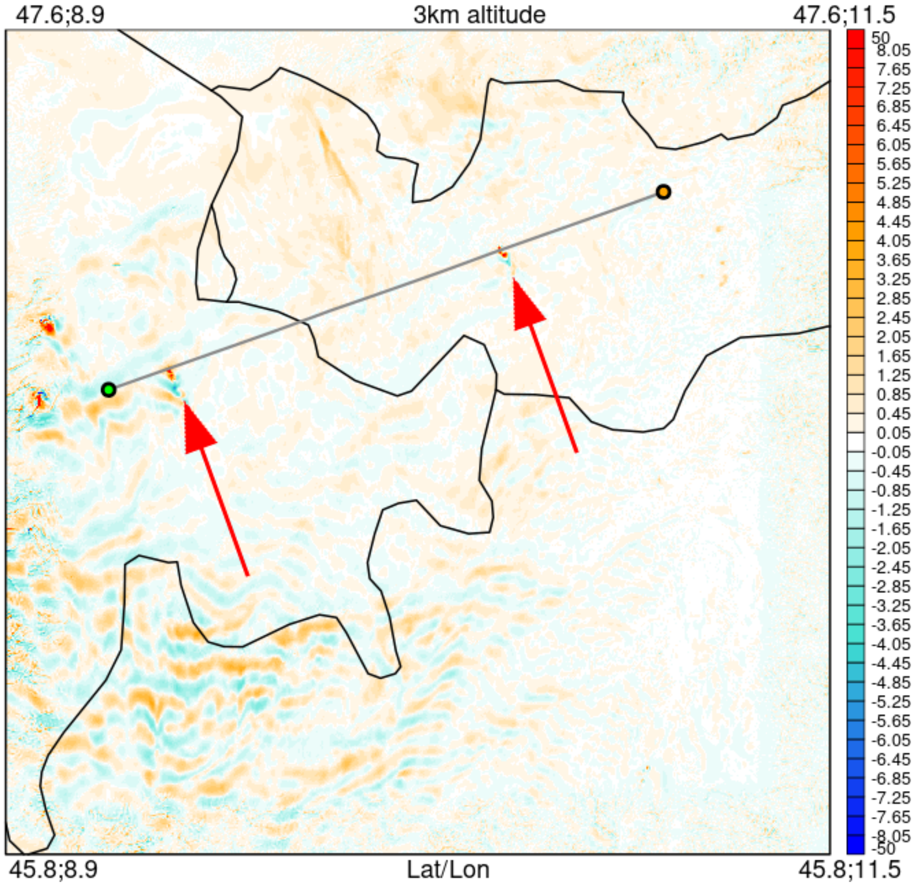
- initialization with 3DVAR + surface DA (canari) for 2.325km run; dynamical adaptation + DFI for 500m and 200m runs

Particular choices for ALARO@200m:

- cubic truncation of orography
- SITRA=50K
- no 3MT (deep convection), only STRAPRO (stratiform precipitation)

Vertical velocity for the alpine case 19 August 2022 0UTC + 24hours.

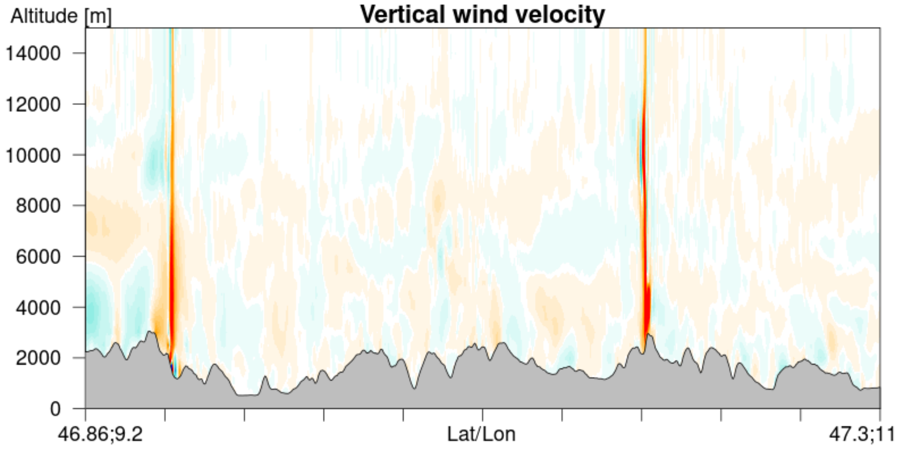




Real simulations

Vertical velocity depending on Λ^* for the alpine case 19 August 2022 0UTC + 24hours.

$\Lambda^* = 0$



With additional iterations of the SI scheme, the integration crashes.

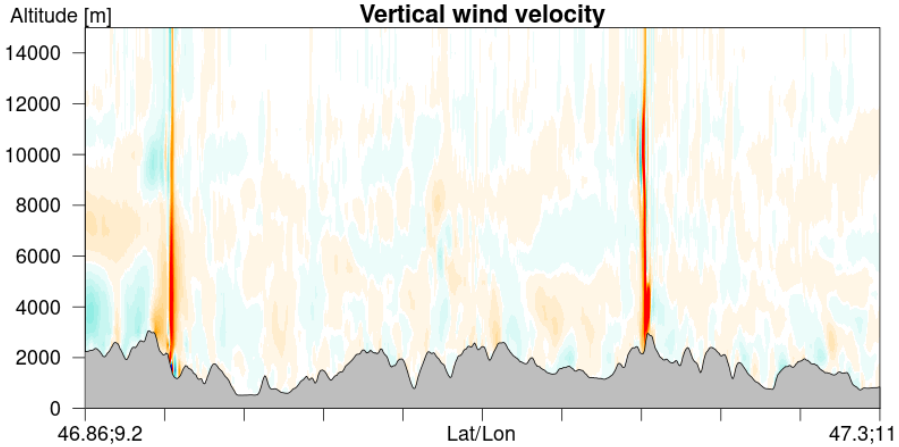
Real simulations

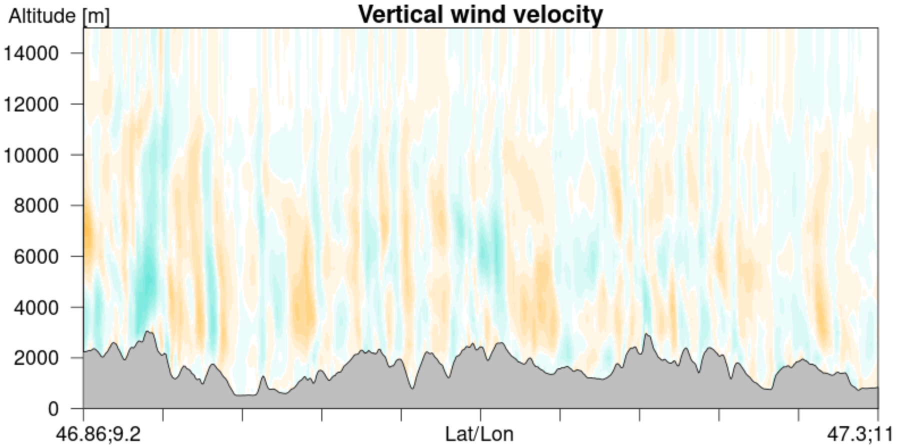
LACE nwp central europe

Vertical velocity depending on Λ^* for the alpine case 19 August 2022 0UTC + 24hours.

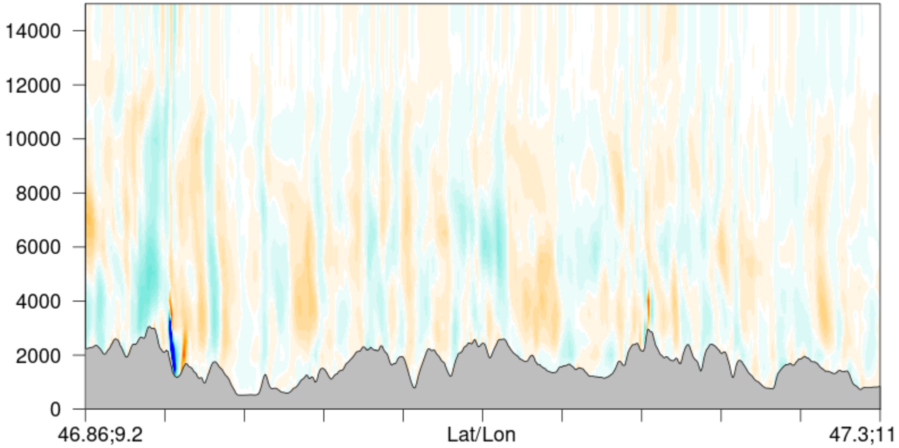
$\Lambda^* = 0$

$\Lambda^* = 1$





PC+1



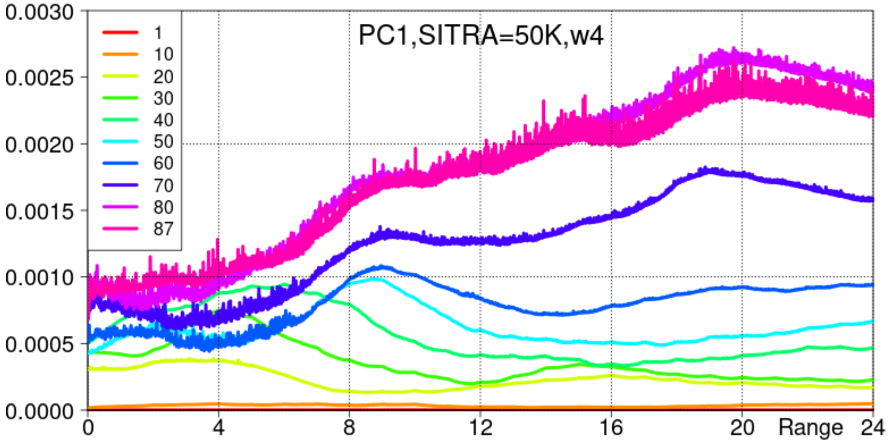
PC+2

Real simulations

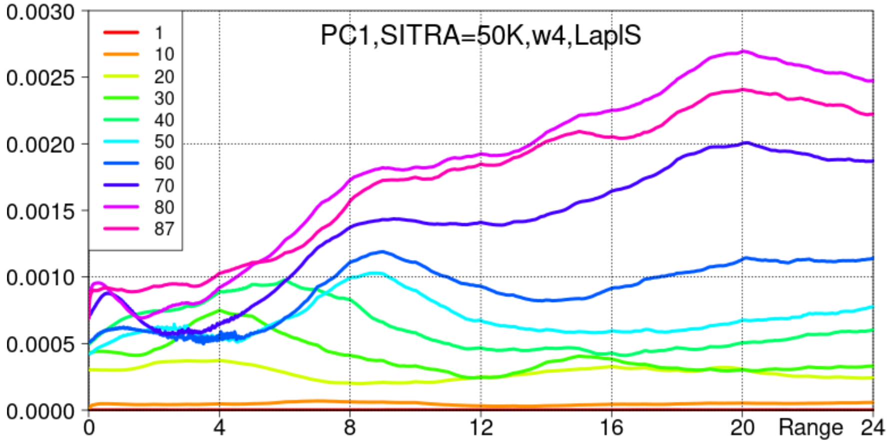
LACE
nwp central europe

Averaged spectral norms of vertical divergence for the alpine case 19 August 2022 0UTC + 24hours.

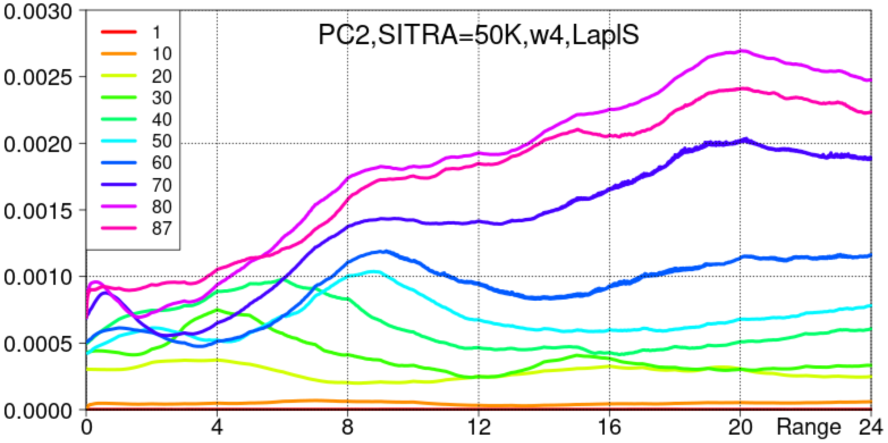
$\Lambda^* = 0$



$\Lambda^* = 1$



PC+1



PC+2

Conclusions

- We must continue our efforts.
- We plan to test various possible definitions of vertical function $S^*(\eta)$.
- We plan to test various possible discretizations including boundary conditions.
- We plan to make further idealised tests and real simulations.
- If the time scheme allows a source of a noise, further time iterations may not help to stabilize the scheme. To the contrary, the scheme with further iterations may show even less stability.

Vertical motion variable

Fabrice Voitus proposed a modification of the vertical velocity defined in the model to simplify the bottom boundary condition and allow more precise calculation.

https://events.ecmwf.int/event/167/contributions/1379/attachments/794/1401/AS2020-Voitus.pdf

Current state

$$
\begin{aligned}
w &= \frac{dz}{dt} \\
d &= -g\frac{p}{mRT}\frac{\partial w}{\partial \eta} + X \\
X &= \frac{p}{mRT}\frac{\partial \mathbf{V}}{\partial \eta}\nabla\phi \\
\frac{\partial d}{\partial t} &= -\frac{g^2}{RT^*}\mathcal{L}_v^*\hat{q} \\
\mathcal{L}_v^* &= \partial^*\left(\partial^* + 1\right)
\end{aligned}
$$

Fabrice's definition

$$
\begin{aligned}
W &= w - \mathbf{V}\cdot S(\eta)\frac{1}{g}\nabla\phi_s \\
d &= -g\frac{p}{mRT}\frac{\partial W}{\partial \eta} + X \\
X &= \frac{p}{mRT}\frac{\partial \mathbf{V}}{\partial \eta}\nabla\left(\phi - S(\eta)\phi_s\right) - \frac{p}{mRT}\mathbf{V}\frac{\partial \nabla\phi}{\partial \eta} \\
\frac{\partial d}{\partial t} &= -\frac{g^2}{RT^*}\mathcal{L}_{mod}^*\hat{q} \\
\mathcal{L}_{mod}^* &= \partial^*\left(\partial^* + 1 + \Lambda^{*2}S(\eta)\gamma^*\right)
\end{aligned}
$$

Then discretization of these terms involves interpolations between half levels (where w is represented) and full levels (where V is represented) and is cumbersome ...

Bottom boundary condition

LACE
nwp central europe

Current state

in the non-linear model

$$w_s = \mathbf{V}_s \frac{1}{g} \nabla \phi_s$$

$$\frac{dw_s}{dt} = \frac{d\left(\mathbf{V}_s \frac{1}{g} \nabla \phi_s\right)}{dt}$$

while in the linear model explicit guess of d is calculated consistently and used in the implicit part

Complicated and not exact! It is a source of noise which may grow.

Fabrice's definition

in the linear and non-linear model

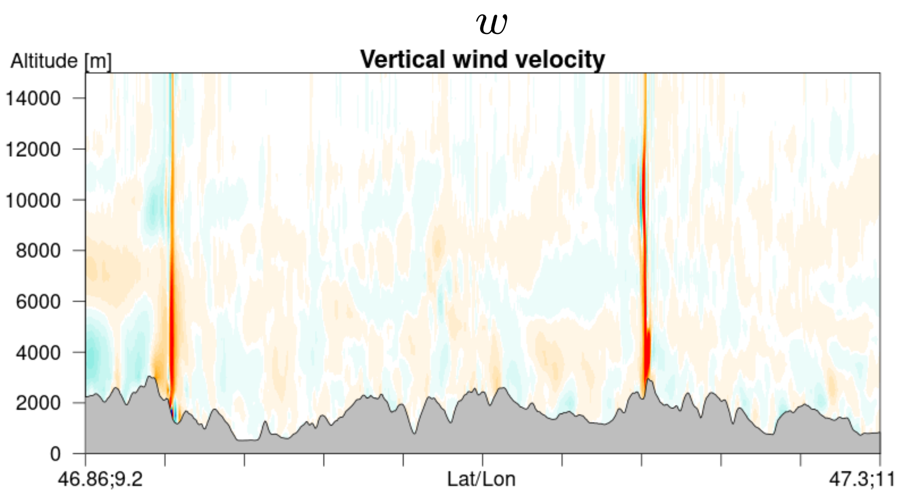
$$W_s = 0$$

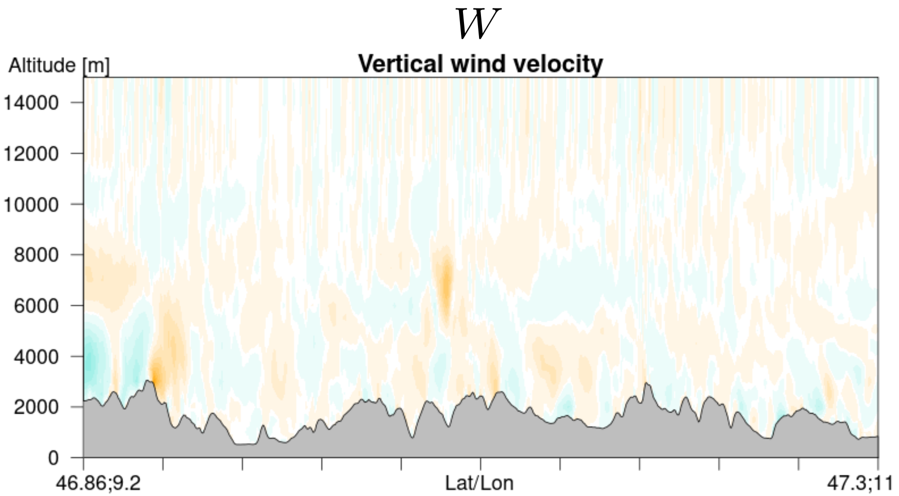
$$\frac{dW_s}{dt} = 0$$

Easily applicable!

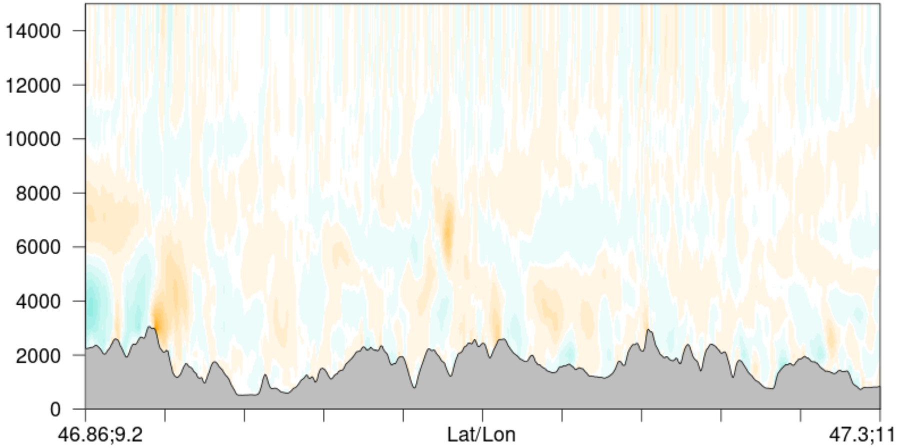
Real simulations

Vertical velocity for the alpine case 19 August 2022 0UTC + 24hours.





PC+1

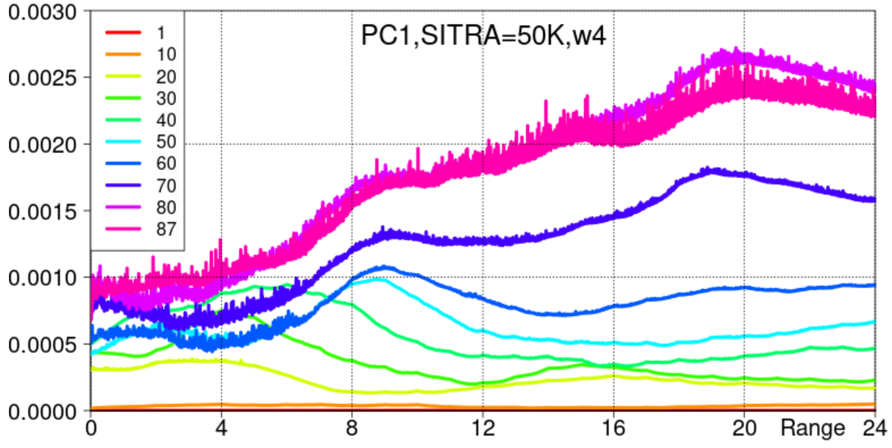


PC+2

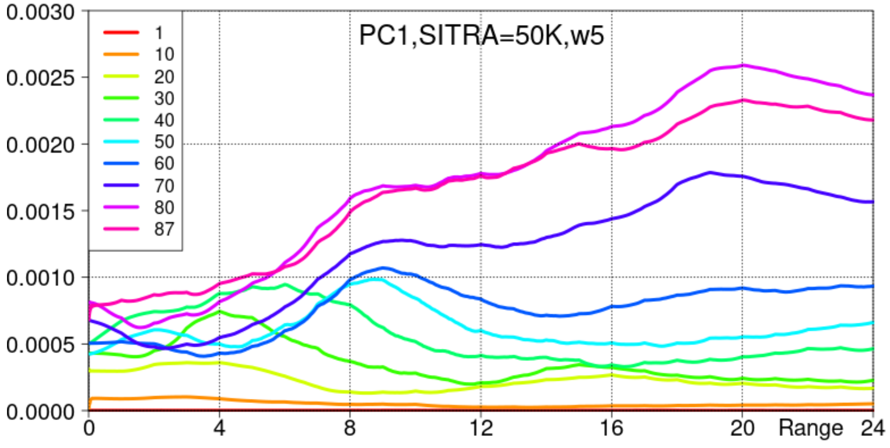
Real simulations

Averaged spectral norms of vertical divergence for the alpine case of 19 August 2022 0UTC + 24hours.

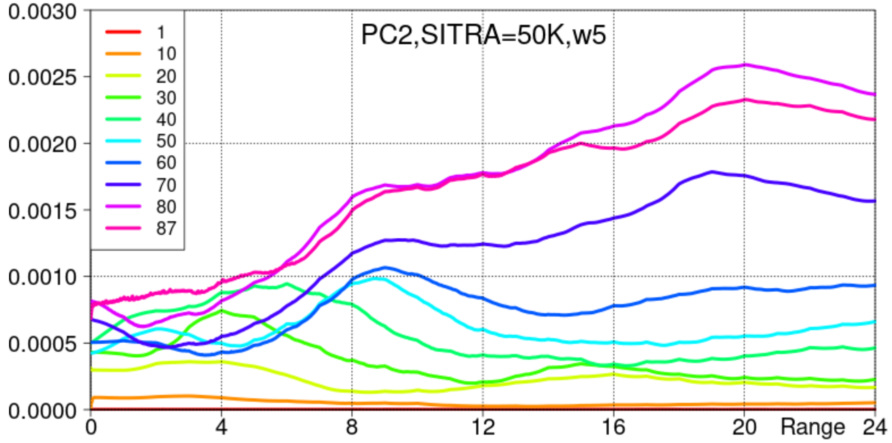
w



W



PC+1



PC+2

Conclusions and advertisements

- The modification is available in cycle CY49t1 under namelist option NVDVAR=5, thanks to Fabrice Voitus and Karim Yessad.
- The new formulations may help to further reduce the non-linear residual of the ICI time scheme and to get rid of the noise coming from steep orography, especially in high resolutions.

Tack för din uppmärksamhet!