Including the coupling model state in LAM variational assimilation without a J_k term

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Variational cost function with J_k term added

To account for the coupling model state, an extra penalty term is added

$$\begin{aligned} \min_x J(x) &= J_b + J_o + J_k \\ &= \frac{1}{2}(x - x_b)^{\mathrm{T}}\mathbf{B}^{-1}(x - x_b) + \frac{1}{2}(y - \mathbf{H}x)^{\mathrm{T}}\mathbf{R}^{-1}(y - \mathbf{H}x) + \frac{1}{2}(x - x_c)^{\mathrm{T}}\mathbf{C}^{-1}(x - x_c) \end{aligned}$$

where

- x_b is the background state, **B** the background error covariance matrix
- y is the observations vector, **R** its error covariance matrix, **H** observation operator
- x_c is the coupling model state, **C** the coupling model error covariance matrix

Can the two model “background” states be combined into one?

Yes, if we define

$$\widetilde{x_b} = \mathbf{C}(\mathbf{B}+\mathbf{C})^{-1}x_b + \mathbf{B}(\mathbf{B}+\mathbf{C})^{-1}x_c$$

and

$$\widetilde{\mathbf{B}} = \mathbf{B}(\mathbf{B}+\mathbf{C})^{-1}\mathbf{C} = \mathbf{C}(\mathbf{B}+\mathbf{C})^{-1}\mathbf{B}, \qquad \widetilde{\mathbf{B}}^{-1} = \mathbf{B}^{-1} + \mathbf{C}^{-1}$$

then

$$J(x) = \frac{1}{2}(x-\widetilde{x_b})^{\mathrm{T}}\widetilde{\mathbf{B}}^{-1}(x-\widetilde{x_b}) + \frac{1}{2}(y-\mathbf{H}x)^{\mathrm{T}}\mathbf{R}^{-1}(y-\mathbf{H}x)$$

meaning that (at least theoretically) the minimization can be performed without the extra J_k term explicitly present, provided

- we pre-mix x_b and x_c into a new background term $\widetilde{x_b}$
- we use a modified background error covariance matrix $\widetilde{\mathbf{B}}$

Not necessarily any simpler for a completely general $\mathbf{C}$.

Covariance matrix structure, assumptions

The covariance matrices are block diagonal, with one NLEV $\times$ NLEV block for each 1D effective wavenumber k^*, e.g.:

$$\mathbf{B} = \begin{bmatrix} B_0 & 0 & \cdots & 0 \\ 0 & B_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & B_{K-1} \end{bmatrix}, \qquad \mathbf{C} = \begin{bmatrix} C_0 & 0 & \cdots & 0 \\ 0 & C_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & C_{K-1} \end{bmatrix}$$

We now assume that the C_{k^*} blocks are similar to the B_{k^*} blocks, except for a wavenumber dependent covariance scaling:

$$C_{k^*} = \rho(k^*)^2 B_{k^*}, \quad \rho = \frac{\sigma_c}{\sigma_b}$$

Statistical balances between variables are assumed to be the same.

Spectral mixing

With this assumed relation between the covariance matrices, we get a fairly simple spectral mixing formula (recall $\widetilde{x_b} = \mathbf{C}(\mathbf{B}+\mathbf{C})^{-1}x_b + \mathbf{B}(\mathbf{B}+\mathbf{C})^{-1}x_c$):

$$\widetilde{x_b}(m,n,l) = \frac{\rho^2}{1+\rho^2}(k^*)x_b(m,n,l) + \frac{1}{1+\rho^2}(k^*)x_c(m,n,l)$$

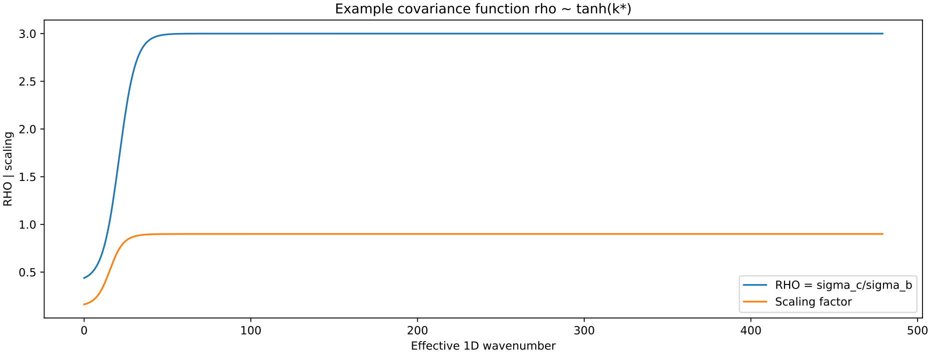
for wavenumbers m, n and level l, and where $k^* = f(m,n)$ (elliptic truncation). For the covariance mixing (recall $\widetilde{\mathbf{B}} = \mathbf{C}(\mathbf{B}+\mathbf{C})^{-1}\mathbf{B}$):

$$\widetilde{\mathbf{B}} = \mathrm{diag}\left[\frac{\rho^2}{1+\rho^2}(k^*)\right]\mathbf{B},$$

giving a really simple imlementation in the code.

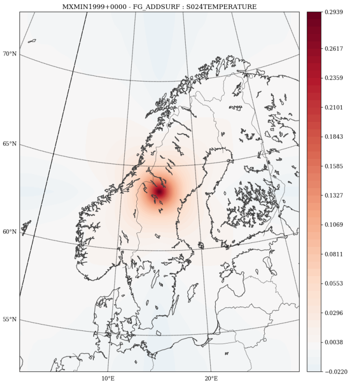
Covariance matrix structure, continued

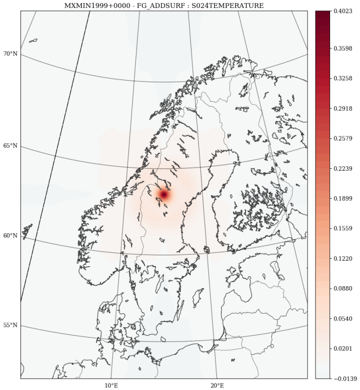
We assume the coupling model to have smaller errors in the large scales, but the LAM model to be better on the small scales, thus $\rho(0) < 1$ and $\rho(K-1) > 1$.



Effect on assimilation (I)

One single temperature observation +1K at 500 hPa, L24, $\rho(0) = 0.4, \rho(K-1) = 3$.

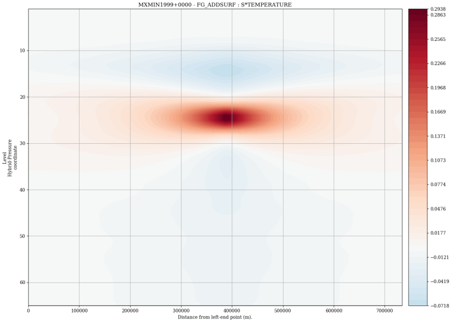


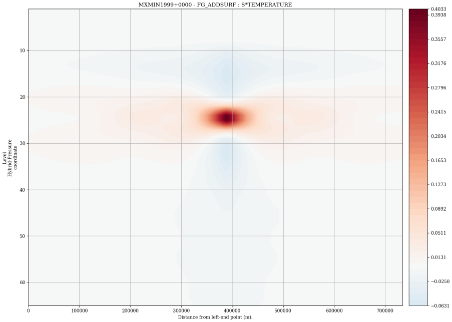


Left with original $\mathbf{B}$, right with spectrally mixed $\widetilde{\mathbf{B}}$.

Effect on assimilation (II)

One single temperature observation +1K at 500 hPa, vertical cross-section





Left with original **B**, right with spectrally mixed $\widetilde{\mathbf{B}}$. Note how smaller assumed errors in large scales lead to assimilation increments dominated by smaller scales.

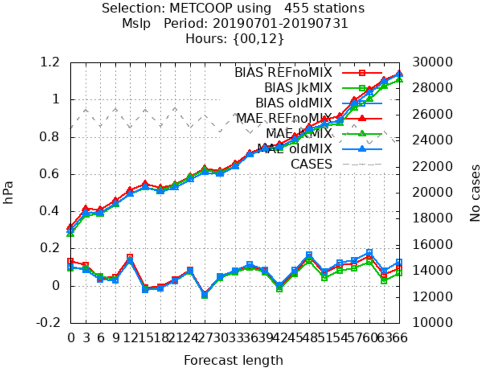
Experiments

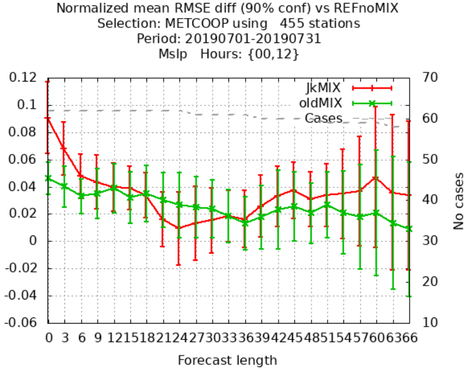
3 experiments were run with harmonie-43h2.1(+) on a summer (July 2019) and winter (February 2020) period, using $\rho(0) = 0.4, \rho(K-1) = 3$, $\rho = 1$ at $\approx$ 75 km.

- REFnoMIX - No spectral mixing, REDNMC=0.6, REDZONE=150 km
- JkMIX - New method, REDNMC=1.2, REDZONE=100 km
- oldMIX - Old spectral mixing, REDNMC=0.6, REDZONE=150 km

oldMIX is the Harmonie default mixing (LSMIXBC=yes). It creates a modified background $\widehat{x_b}$ as well, but uses an unmodified $\mathbf{B}$ in the assimilation. The spectral mixing formula is also different.

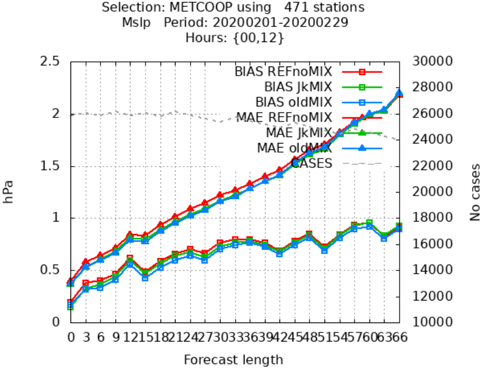
Results: MSLP, summer

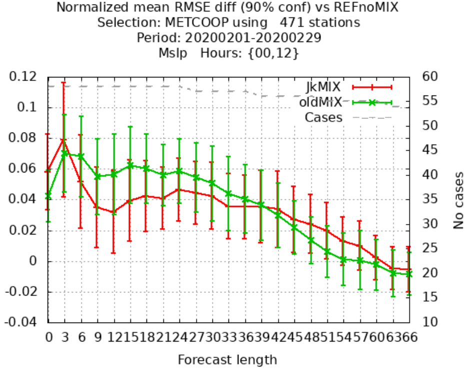




Both types of mixing have positive effect, but there is a dip in JkMIX performance at intermediate forecast lenghts.

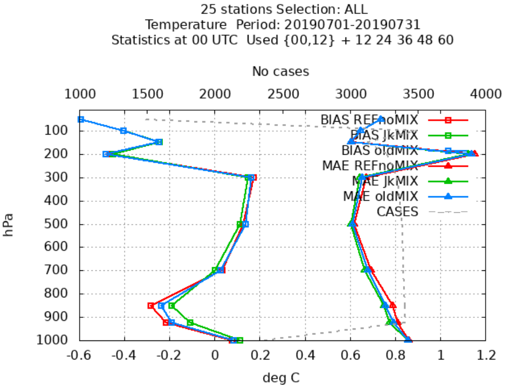
MSLP, winter

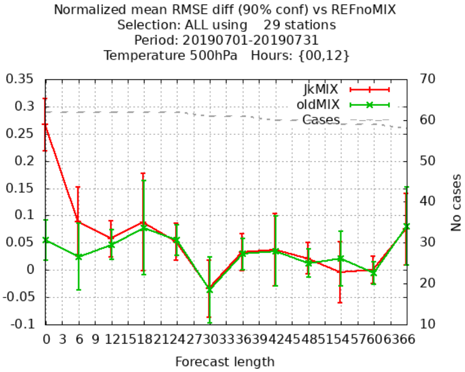




Same type of signal, mixing positive, but less favourable for JkMIX.

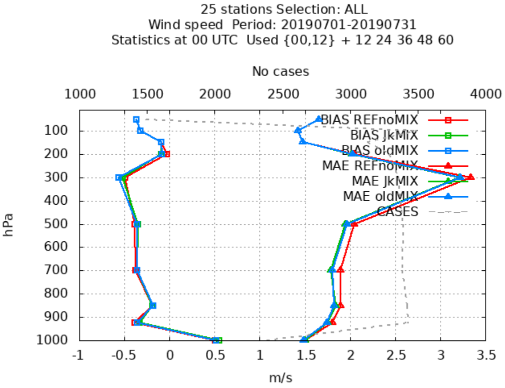
T profile

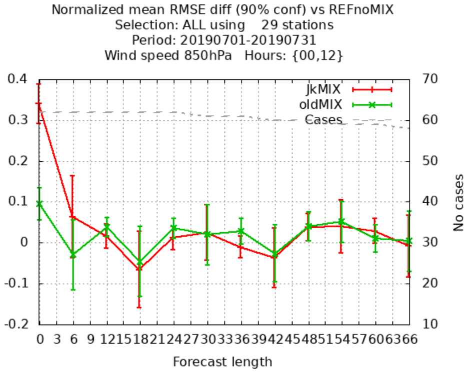




Mixing clearly positive, JkMIX best for short range

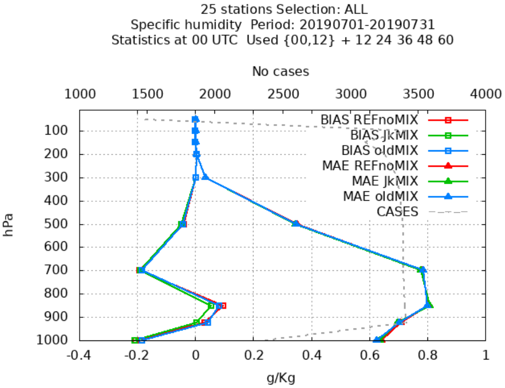
Wind profile

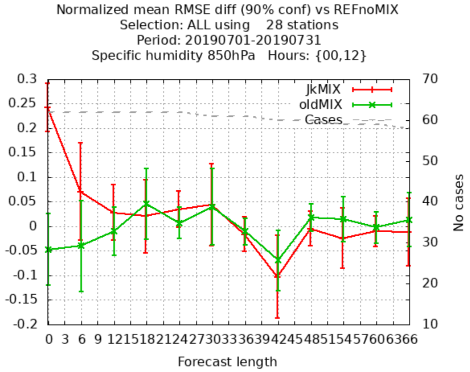




Again JkMIX scores better for the short range

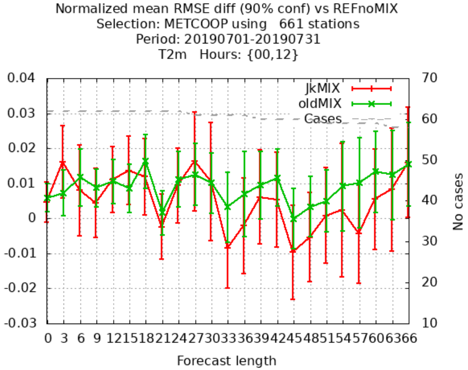
Humidity profile

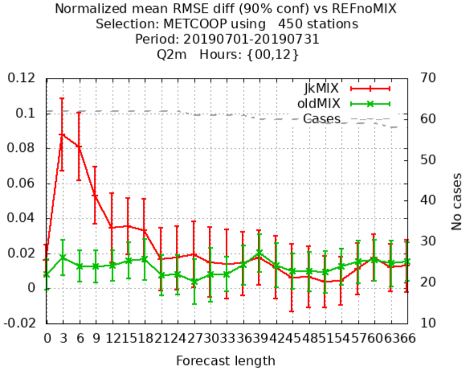




Better short range performance again.

T2m and Q2m





Note the difference, probably because Q is not mixed in oldMIX (for historical reasons).

Summary and some remarks

- A method has been developed for using coupling model information in variational assimilation consistent with the ideas of statistical interpolation, but without explicit appearance of a J_k penalty term.
- The relationship $\rho(k^*)$ between errors in the coupling and own model at different scales needs more attention. It should be computed using a historical archive of coupling files instead of just guessed.
- The good performance in the short range is probably related to the ability of the method to focus on the short scales in the assimilation. Should be highly relevant for nowcasting.
- When generating own **B** statistics, all influence (LSMIXBC, LUNBC) of the coupling model must be switched off.

Quick proof of mixing formula

$$J(x) = \frac{1}{2}(x - x_b)^{\mathrm{T}}\mathbf{B}^{-1}(x - x_b) + \frac{1}{2}(x - x_c)^{\mathrm{T}}\mathbf{C}^{-1}(x - x_c) + J_o$$

$$\begin{aligned}
\nabla J &= \mathbf{B}^{-1}(x - x_b) + \mathbf{C}^{-1}(x - x_c) + \nabla J_o \\
&= \mathbf{B}^{-1}(x - x_b + \widetilde{x_b} - \widetilde{x_b}) + \mathbf{C}^{-1}(x - x_c + \widetilde{x_b} - \widetilde{x_b}) + \nabla J_o \\
&= (\mathbf{B}^{-1} + \mathbf{C}^{-1})(x - \widetilde{x_b}) + \nabla J_o + \mathbf{B}^{-1}(\widetilde{x_b} - x_b) + \mathbf{C}^{-1}(\widetilde{x_b} - x_c) = 0
\end{aligned}$$

Now set $\widetilde{\mathbf{B}}^{-1} = \mathbf{B}^{-1} + \mathbf{C}^{-1}$, the blue part to zero, and solve it for $\widetilde{x_b}$.