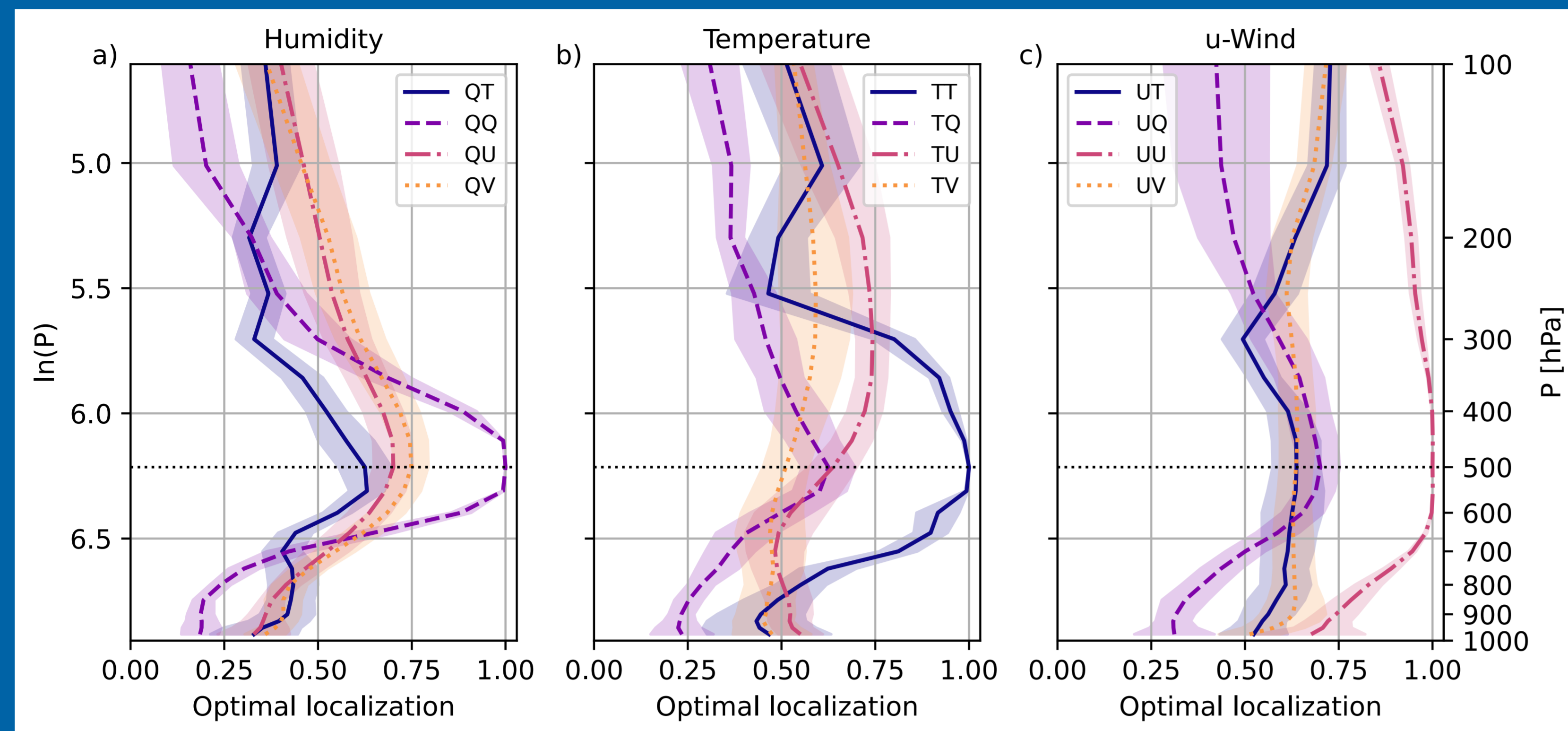


# Guidance on how to improve vertical covariance localization based on a 1000-member ensemble

Philipp Griewank<sup>1</sup>, Tobias Necker<sup>1</sup>, David Hinger<sup>1</sup>, Takemasa Miyoshi<sup>2</sup>, Martin Weissmann<sup>1</sup>

1) Institute für Meteorology und Geophysics, Universität Wien, Vienna, Austria 2) RIKEN Center for Computational Science, Kobe, Japan



The empirical optimal localization (EOL) method allows to derive optimal covariance localization from large ensembles

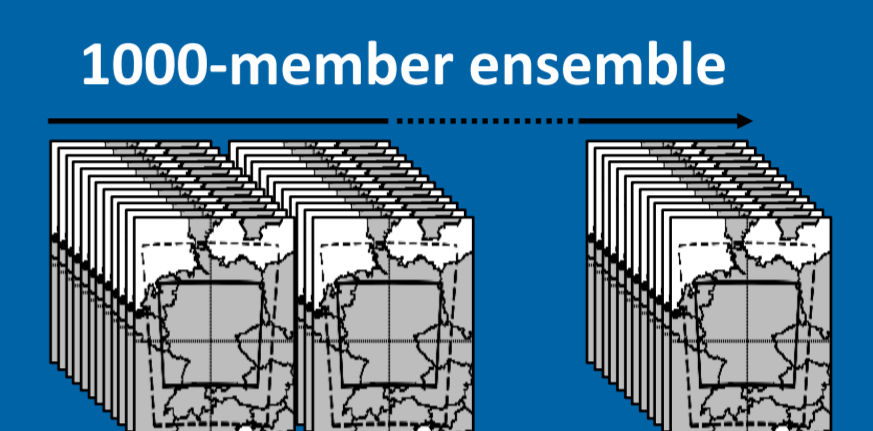
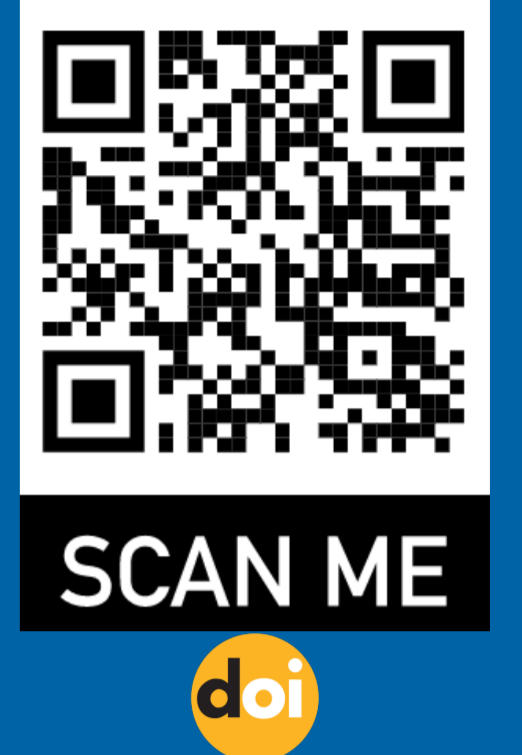


Figure caption: Empirical Optimal Localization (EOL) for vertical sample correlations of 40-member ensembles from reference level 500hPa: (a) humidity; (b) temperature; (c) u-wind. Mean and standard deviation over 10 forecasts from 29 May to 2 June 2016.

## Research questions

1. How to optimally localize vertical error correlations from small ensembles?
2. How much can different localization approaches reduce the sampling error?
3. How can we achieve positive semi definiteness for localization?

## Take home messages

1. Vertical correlation length scales strongly vary within the troposphere
2. A variable-dependent domain-uniform EOL localization reduced the sampling error by 27% (Gaspari-Cohn only 11%)
3. The NCM-method can help to achieve a SPSPD localization matrix

## Empirical Optimal Localization (EOL) method

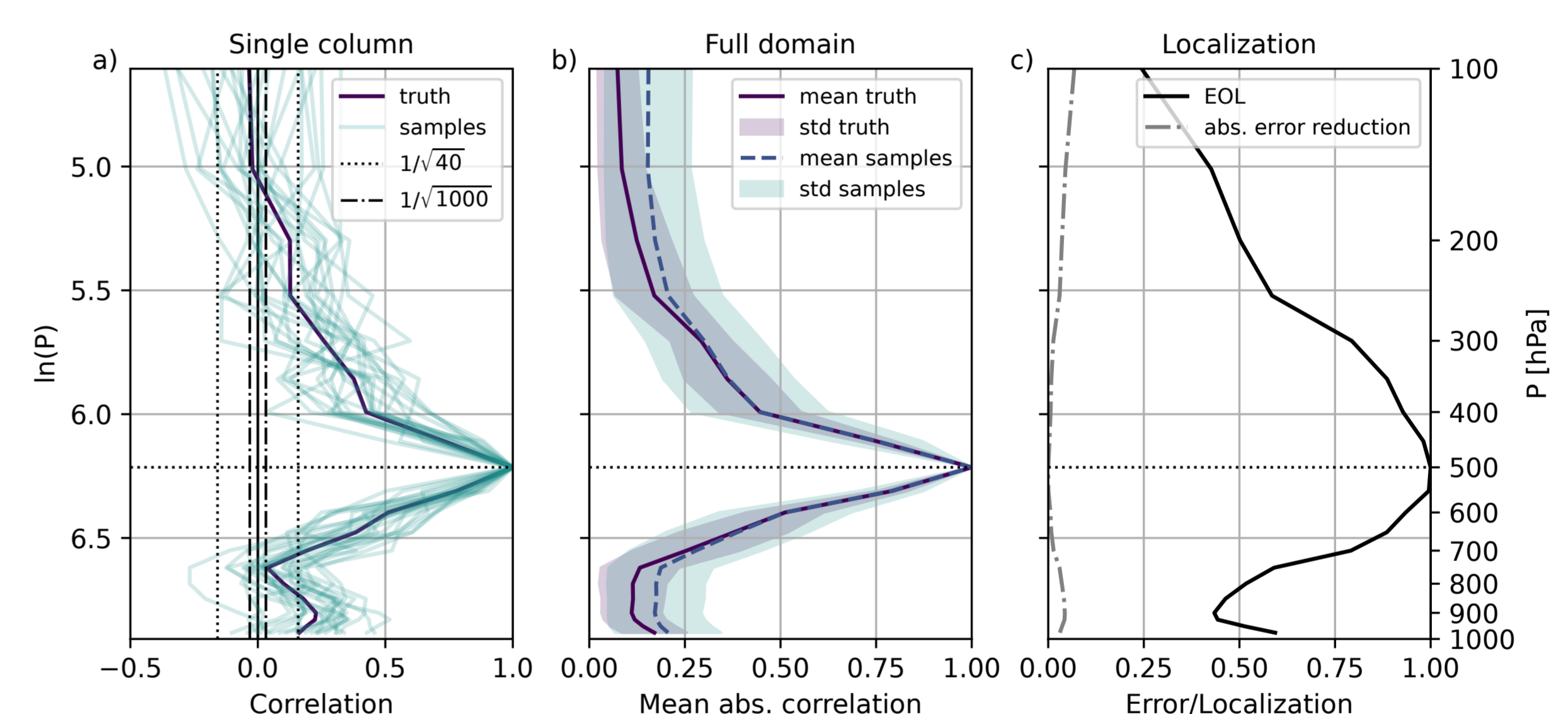
- Minimize cost function  $J$  to derive empirical optimal localization -  $\alpha$

$$J(\alpha, t, z, p, A) = \sqrt{\sum_{s=1}^S \sum_{k=1}^K (\alpha r_{s,k}^{40} - r_k^{1000})^2}$$

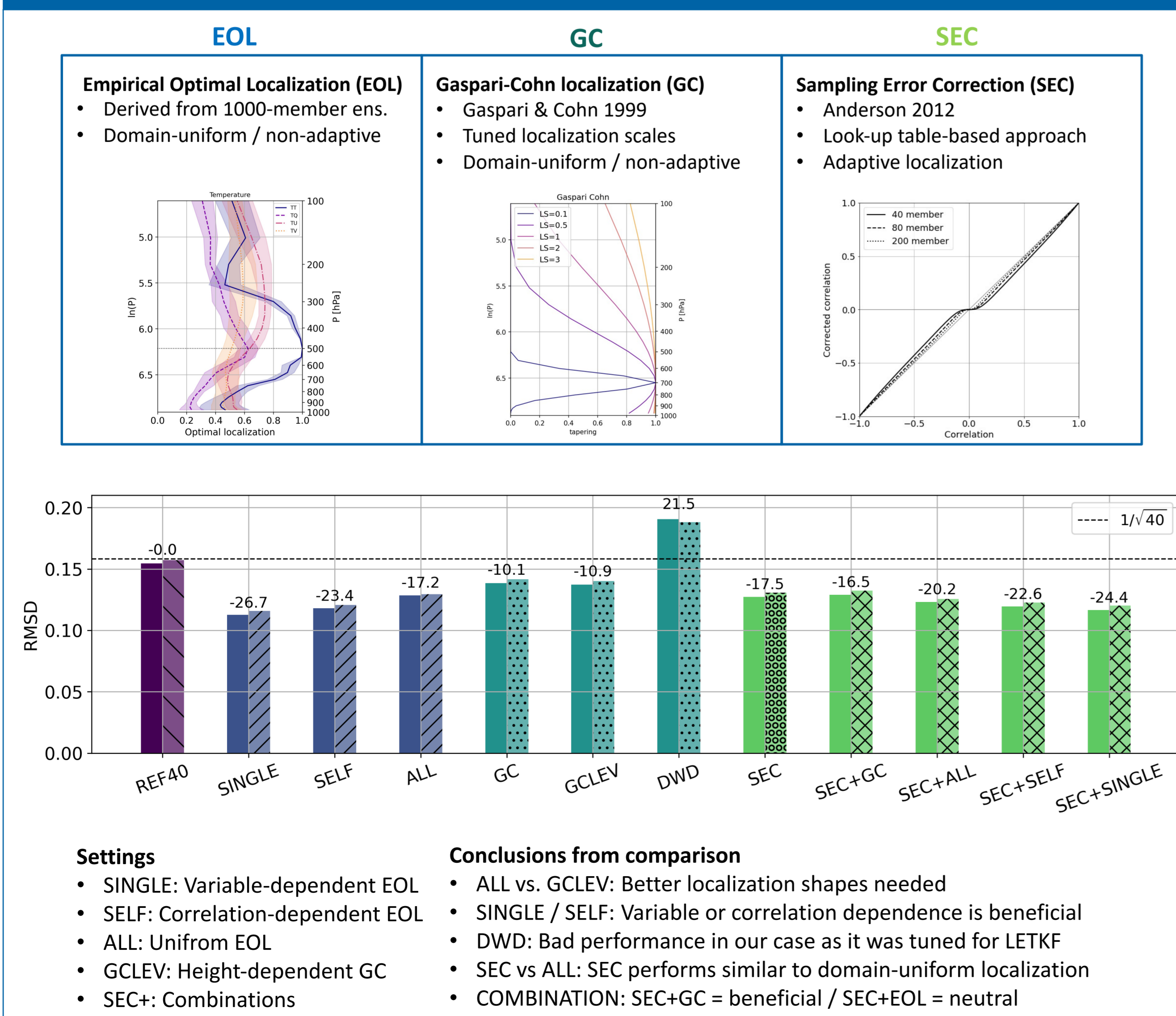
- „  $\alpha$  minimizes the root mean square difference between subsamples and 1000-m correlation“

$$\alpha = \frac{\sum_{s=1}^S \sum_{k=1}^K r_{s,k}^{40} r_k^{1000}}{\sum_{s=1}^S \sum_{k=1}^K (r_{s,k}^{40})^2}$$

$J$ : Cost function  
 $\alpha$ : localization  
 $r$ : correlation  
 $t$ : time index  
 $z$ : ref. Level  
 $p$ : pressure level  
 $A$ : Variable pair  
 $S$ : Subsamples  
 $K$ : Grid points

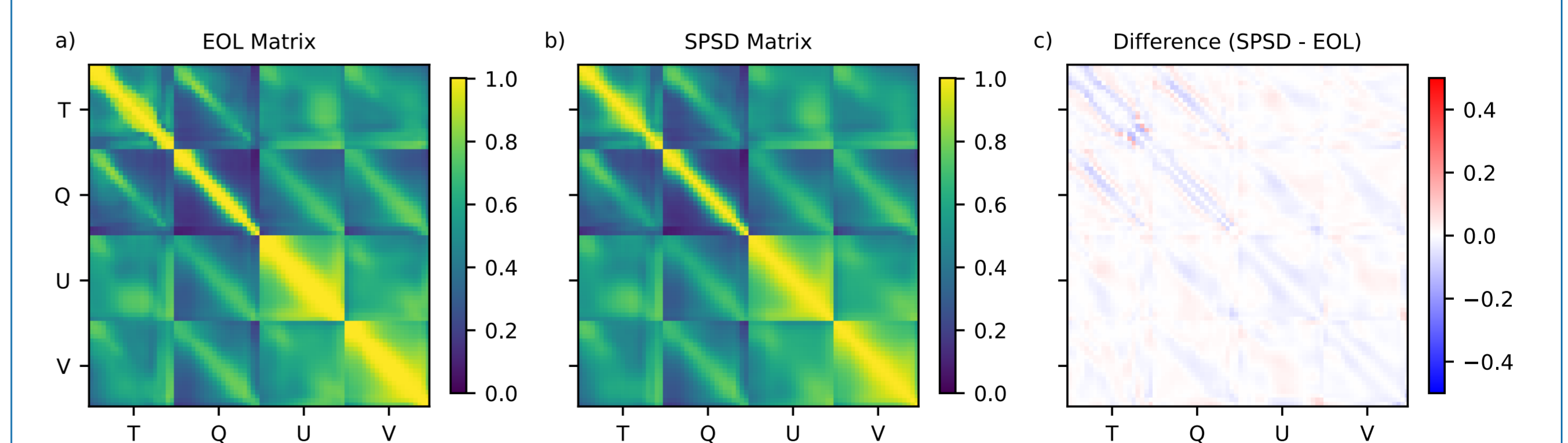


## Error reduction: Comparison of different localization approaches



## Ensuring a symmetric positive semidefinite covariance matrix

- Constructing a localization matrix based on the EOL does not guarantee a symmetric positive semidefinite (SPSPD) localized covariance matrix.
- The Nearest-Correlation-Matrix (NCM) algorithm (Higham, 2002) allows to achieve positive semi-definiteness, which resulted in only minor changes in the EOL



## Reference

Necker, T., Hinger, D., Griewank, P. J., Miyoshi, T., and Weissmann, M. 2023: Guidance on how to improve vertical covariance localization based on a 1000-member ensemble, Nonlin. Processes Geophys., 30, 13–29, <https://doi.org/10.5194/npg-30-13-2023>.