Physical Oceanography - UNAM, Mexico
Lecture 3: The Wind-Driven Oceanic Circulation

Robin Waldman

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A first taste...

Many large-scale circulation features are wind-forced !

Outline

The Ekman currents and Sverdrup balance

The western intensification of gyres

The Southern Ocean circulation

The Tropical circulation

Outline

Ekman currents

Introduction :

- ▶ First quantitative theory relating the winds and ocean circulation.
- ▶ Can be deduced by applying a dimensional analysis to the horizontal momentum equations within the surface layer. The resulting balance is geostrophic plus Ekman :
  - ▶ geostrophic : Coriolis and pressure force
  - ▶ Ekman : Coriolis and vertical turbulent momentum fluxes modelled as diffusivities.

Ekman currents

Ekman's hypotheses :

- ▶ The ocean is infinitely large and wide, so that interactions with topography can be neglected ;
- ▶ It has reached a steady state, so that the Eulerian derivative $\frac{\partial \mathbf{u_h}}{\partial t} = \mathbf{0}$;
- ▶ It is homogeneous horizontally, so that $(\mathbf{u_h}.\boldsymbol{\nabla})\mathbf{u_h} = 0$, $\boldsymbol{\nabla_h}.(\kappa_{hu}\boldsymbol{\nabla_h})\mathbf{u_h} = 0$ and by continuity $w = 0$ hence $w\frac{\partial \mathbf{u_h}}{\partial z} = 0$;
- ▶ Its density is constant, which has the same consequence as the Boussinesq hypotheses for the horizontal momentum equations ;
- ▶ The vertical eddy diffusivity κ_{zu} is constant.

Ekman currents

Ekman balance :

Ekman currents

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that is :

Ekman currents

Ekman balance :

$$f\mathbf{k} \times \mathbf{u_E} = \kappa_{zu}\frac{\partial^2 \mathbf{u_E}}{\partial z^2}$$

that is :

$$u_E = \frac{\kappa_{zu}}{f}\frac{\partial^2 v_E}{\partial z^2}$$

$$v_E = -\frac{\kappa_{zu}}{f}\frac{\partial^2 u_E}{\partial z^2}$$

Ekman transports

Computation of vertically-integrated Ekman transports :

▶ At the surface, by continuity of the turbulent vertical fluxes, and as we saw for NEMO model in Chapter 2, we have :

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$$\boldsymbol{\tau} = \rho_0 \kappa_{zu} \frac{\partial \mathbf{u_h}}{\partial z} \bigg|_0$$

  with $\boldsymbol{\tau}$ the surface wind stress.

- ▶ Contrary to Ekman, we assume that at the bottom of the Ekman layer h_E, vertical turbulent fluxes cancel out : $\boldsymbol{\tau_b} = \kappa_{zu} \frac{\partial \mathbf{u_h}}{\partial z} \big|_{-h_E} = 0$.

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- ▶ Vertical integration over the Ekman layer :

$$U_E = \int_{-h_E}^{0} u_E dz = \left[\frac{\kappa_{zu}}{f} \frac{\partial v_E}{\partial z} \right]_{-h_E}^{0} = +\frac{\tau_y}{\rho_0 f}$$

$$V_E = \int_{-h_E}^{0} v_E dz = \left[-\frac{\kappa_{zu}}{f} \frac{\partial u_E}{\partial z} \right]_{-h_E}^{0} = -\frac{\tau_x}{\rho_0 f}$$

Ekman transports

This is one of the most useful relations in physical oceanography :

- ▶ It predicts that the wind-driven Ekman transports are orthogonal to surface winds, to their right in the Northern Hemisphere and to their left in the Southern Hemisphere.

- ▶ This explains the location of the main upwelling regions, which are either due to offshore Ekman transports at the coast (e.g. the California upwelling system) or to divergent Ekman transports (e.g. the Equatorial upwelling).

- ▶ It also predicts that for a given wind, the Ekman transports will be stronger at low latitudes. This explains the particularly strong meridional heat transport by the ocean at low latitudes.

- ▶ At the Equator the Coriolis acceleration cancels out and this relation does not hold anymore.

Ekman transports

Exercise : upwelling rate of the California upwelling system.

We assume an along-coast Northerly wind of $v_{10m} = -10m/s$ and that the Ekman theory holds at $L = 100km$ off the coast. Deduce the Ekman volumic transport T_E at that distance across a section of width $W = 100km$ and depth $-h_E$, and the average upwelling rate at the basis of the Ekman layer $w(-h_E)$ within this coastal box. We assume $C_D \sim 2 \times 10^{-3}$, $\rho_a \sim 1kg/m^3$, $\rho_0 \sim 1000kg/m^3$, $f_0 \sim 10^{-4}s^{-1}$.

Ekman transports

Solution : the Ekman volumic transport is

$T_E = U_E W = -\frac{C_D \rho_a v_{10m}^2 W}{\rho_0 f_0} = -0.2 Sv$, and hence by continuity (with no normal flow at the coast) the average upwelling rate at the basis of the Ekman layer is $w(-h_E) = -\frac{T_E}{WL} = +2 \times 10^{-5} m/s \simeq 2m/day$. We deduce that :

- *Any equatorward wind along a North-South coast generates an offshore Ekman transport which drives Ekman upwelling.*
- *Although the upwelling magnitude seems modest, due to the strong near-surface stratification, it generates intense cold and fresh anomalies in those regions.*
- *It is the case of all Eastern Boundary subtropical regions.*

Ekman pumping

In the absence of a coast, Ekman transports can generate vertical motion by "Ekman pumping" (upwelling) or "Ekman suction" (downwelling). We integrate the continuity equation within the Ekman layer :

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$$\int_{-h_E}^{0} \frac{\partial w_E}{\partial z} dz = -\int_{-h_E}^{0} \left(\frac{\partial u_E}{\partial x} + \frac{\partial v_E}{\partial y}\right) dz$$

$$\implies 0 - w_E(-h_E) = -\int_{-h_E}^{0} \left[\frac{\partial}{\partial x}\left(\frac{\kappa_{zu}}{f}\frac{\partial^2 v_E}{\partial z^2}\right) - \frac{\partial}{\partial y}\left(\frac{\kappa_{zu}}{f}\frac{\partial^2 u_E}{\partial z^2}\right)\right] dz$$

$$\implies w_E(-h_E) = \left[\frac{\partial}{\partial x}\left(\frac{\kappa_{zu}}{f}\frac{\partial v_E}{\partial z}\right) - \frac{\partial}{\partial y}\left(\frac{\kappa_{zu}}{f}\frac{\partial u_E}{\partial z}\right)\right]_{-h_E}^{0}$$

$$= \frac{1}{\rho_0}\left[\frac{\partial}{\partial x}\left(\frac{\tau_y}{f}\right) - \frac{\partial}{\partial y}\left(\frac{\tau_x}{f}\right)\right]$$

$$w_E(-h_E) = \frac{Curl(\boldsymbol{\tau}/f)}{\rho_0}$$

with $Curl(\mathbf{a}) = \left[\boldsymbol{\nabla} \times \mathbf{a}\right]_z$ the vertical vorticity operator.

Ekman pumping
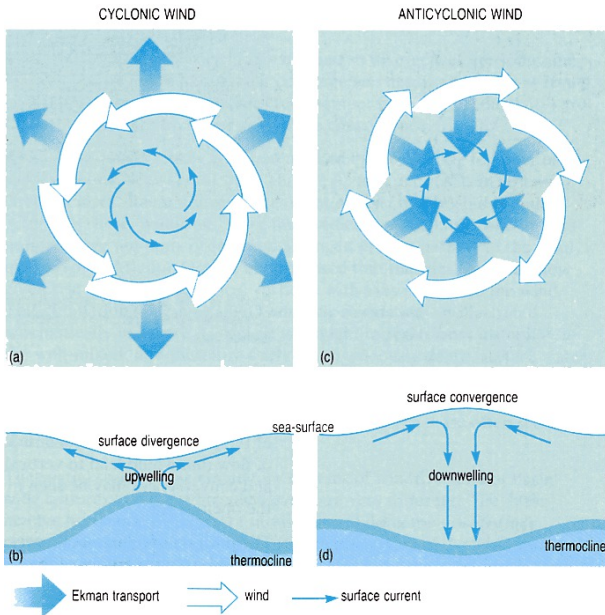
Ekman pumping

Interpretations :

- ▶ Because Ekman transports are orthogonal to surface winds, any positive (negative) vorticity of those winds induces a divergence (convergence) of Ekman transports, which by continuity causes upwelling (downwelling) at the basis of the Ekman layer.

- ▶ The beta effect can also generate vertical motions even for constant winds, but except at the planetary scale and near the Equator it has a minor role.

- ▶ This explains the deep thermocline at subtropical latitudes between the tropical Easterlies and the mid-latitude Westerlies (negative vorticity of winds hence downwelling) and the shallow one at subpolar latitudes where Westerlies weaken (positive vorticity hence upwelling).

Ekman pumping

Exercise : average upwelling rate in the subtropical North Atlantic.

We assume that the Easterly wind blows at $u1_{10m} = -5m/s$ at $\phi_1 = 10°N$ and that the Westerly wind blows at $u2_{10m} = +10m/s$ at $\phi_2 = 40°N$. Deduce the average downwelling rate $w_E(-h_E)$ between those latitudes. We assume $C_D \sim 2 \times 10^{-3}$, $\rho_a \sim 1kg/m^3$, $\rho_0 \sim 1000kg/m^3$, $f_1 \sim 2 \times 10^{-5}s^{-1}$, $f_2 \sim 10^{-4}s^{-1}$, $\Delta y = 3000km$.

Ekman pumping

Solution : $w_E(-h_E) = \frac{1}{\rho_0} \frac{-\partial \tau_x / f}{\partial y} = \frac{C_d \rho_a}{\rho_0 \Delta y} \left(\frac{-u2_{10m}^2}{f_2} - \frac{u1_{10m}^2}{f_1} \right) \simeq$ $-2 \times 10^{-6} m/s \simeq -1m/5days$. This Ekman suction is one order of magnitude lower than the Ekman pumping estimated in coastal upwelling systems. But at the climatological timescale, it is sufficient to induce the deep subtropical thermocline.

Ekman pumping

Exercise : average Equatorial upwelling.

We assume that the Easterly wind blows at $u_{10m} = -5m/s$ all along the Equatorial band, and that the Ekman relation is valid at $\phi_1 = 5°N$ and $\phi_2 = 5°S$. Deduce the average poleward Ekman transport over a basin of width $W = 6000km$, and the average upwelling rate in the Equatorial band between both latitudes. Here we assume that $f_0 = \pm 10^{-5} s^{-1}$ at $5°$ of latitude.

Ekman pumping

Solution : the problem is symmetric at both latitudes with only f_0 changing sign. We have : $T_E(\phi_1) = U_E(\phi_1)W = +\frac{C_D \rho_a u_{10m}^2 W}{\rho_0 f_0} = +20 Sv$, hence $T_E(\phi_2) = -20 Sv$. We deduce by continuity : $w(-h_E) = \frac{T_E(\phi_1) - T_E(\phi_2)}{WL} \simeq 6 \times 10^{-6} m/s \simeq 1 m/day$. This upwelling rate is much larger than what was found for the subtropical gyre. Indeed, as was mentioned earlier, Ekman transports are much more intense in the Tropics because of the reduced value of f_0. This Equatorial divergence of Ekman transports largely explains the cold and fresh anomaly of the surface Equatorial ocean.

Ekman spiral

Although of lesser importance for oceanography than the vertically-integrated Ekman transports, the oceanic Ekman spiral allows to predict :

- The rotation of Ekman currents with depth.
- The relation between the wind stress and the Ekman layer depth.

Ekman spiral

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&= \frac{\kappa_{zu}}{f}\Big(\frac{\partial^2 v_E}{\partial z^2} - \iota\frac{\partial^2 u_E}{\partial z^2}\Big) \\
&= -\frac{\kappa_{zu}}{f}\iota\frac{\partial^2 \bar{v}_E}{\partial z^2}
\end{aligned}
$$

that is :

$$
\frac{\partial^2 \bar{v}_E}{\partial z^2} = \frac{\iota f}{\kappa_{zu}}\bar{v}_E
$$

It can be simplified by using the polar form :
$\iota = e^{i\pi/2} = (e^{i\pi/4})^2 = (\frac{1+\iota}{\sqrt{2}})^2$. Hence :

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\frac{\partial^2 \bar{v}_E}{\partial z^2} = \Big(\frac{1+\iota}{h_E}\Big)^2 \bar{v}_E
$$

with $h_E = \sqrt{2\kappa_{zu}/f}$ the Ekman depth.

Ekman spiral

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$$\bar{v_E} = \alpha \exp((1+\iota)z/h_E) + \beta \exp(-(1+\iota)z/h_E),$$

with α et β two complex integration constants. The boundary conditions are :

- bounded velocities at the ocean bottom : $\bar{v_E} < \infty$ when $z \to -\infty$, hence $\beta = 0$
- surface turbulent fluxes are equal to the wind stress, which we assume constant and zonal : $\tau_0 = \tau_x \mathbf{i} = \rho_0 \kappa_{zu} \frac{\partial \mathbf{u_E}}{\partial z}$.

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The surface boundary condition yields :

- $\rho_0 \kappa_{zu} \frac{\Re(\alpha) + \Im(\alpha)}{h_E} = 0 \Longrightarrow \Im(\alpha) = -\Re(\alpha)$,
- $\rho_0 \kappa_{zu} \frac{\Re(\alpha) - \Im(\alpha)}{h_E} = \tau_x \Longrightarrow \Re(\alpha) = \frac{\tau_x h_E}{2\rho_0 \kappa_{zu}}$,

Ekman spiral

Hence

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Hence

$$
\begin{aligned}
\bar{v}_E &= (1-\iota)\frac{\tau_x h_E}{2\rho_0 \kappa_{zu}} \exp\left((1+\iota)\frac{z}{h_E}\right) \\
&= \frac{\tau_x h_E}{\sqrt{2}\rho_0 \kappa_{zu}} \exp\left(\iota\left(\frac{z}{h_E} - \frac{\pi}{4}\right)\right) \exp\left(\frac{z}{h_E}\right)
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$$

that is

$$
\begin{aligned}
u_E &= V_0 \cos\left(\frac{z}{h_E} - \frac{\pi}{4}\right) \exp\left(\frac{z}{h_E}\right) \\
v_E &= V_0 \sin\left(\frac{z}{h_E} - \frac{\pi}{4}\right) \exp\left(\frac{z}{h_E}\right)
\end{aligned}
$$

with $V_0 = (\tau_x h_E)/(\sqrt{2}\rho_0 \kappa_{zu}) = \tau_x/(\rho_0\sqrt{\kappa_{zu}f})$.
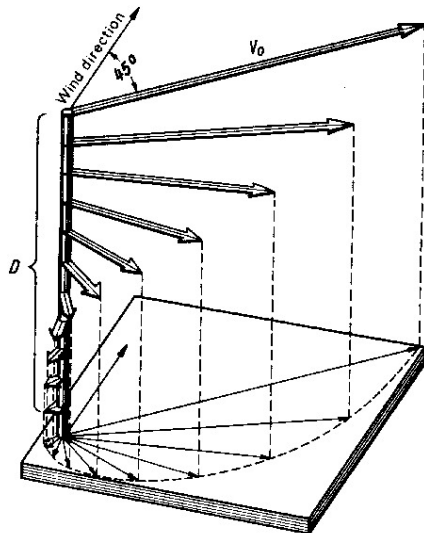
Ekman spiral

Figure 2 – Ekman spiral.

Ekman spiral

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▶ Surface currents are rotated by $-\frac{\pi}{4} = -45°$ to surface winds. Their magnitude is :

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- ▶ Surface currents are rotated by $-\frac{\pi}{4} = -45°$ to surface winds. Their magnitude is :

$$V_0 = \frac{\rho_a C_D}{\rho \sqrt{\kappa_{zu} f}} ||\mathbf{u_{10m}}||^2 \sim 0.02 ||\mathbf{u_{10m}}||$$

  that is typically $20cm/s$ for $||\mathbf{u_{10m}}|| = 10m/s$. Thus they do not explain the most intense surface currents.

- ▶ They spiral to the right with depth in the Northern Hemisphere and weaken exponentially.

Barotropic vorticity equation

It is the core equation for the analysis of gyre circulation, including the most simple, Sverdrup theory. Principle :

1. Derive an equation for the barotropic, that is the vertically-integrated, vorticity. A leading-order term in the barotropic vorticity equation is the so-called "beta effect", which corresponds to the meridional advection of planetary vorticity βV. Any meridional motion of water masses induces relative vorticity due to the varying Coriolis acceleration with latitude $\frac{df}{dy} = \beta$.

2. Isolating this so-called "beta term", we get an equation for the vertically-integrated meridional transport V :

$$V = RHS/\beta$$

with RHS the right hand side of the barotropic vorticity equation.

Barotropic vorticity equation

3 A barotropic streamfunction Ψ_{BT} can be constructed from the vertically-integrated horizontal transport (U, V) defined by :

$$\frac{\partial \Psi_{BT}}{\partial x} = V$$

$$\frac{\partial \Psi_{BT}}{\partial y} = -U$$

Hence the gyre circulation can be comprehensively reconstructed from the zonal integration of the barotropic vorticity equation :

$$\Longrightarrow \frac{\partial \Psi_{BT}}{\partial x} = RHS/\beta$$

$$\Longrightarrow \Psi_{BT}(x,y) = +\frac{1}{\beta} \int_{x_r}^{x} RHS(x',y)dx'$$

with x_r a reference longitude where the streamfunction is assumed null. x_r is usually chosen at the eastern boundary $(x < x_r)$, hence :

$$\Psi_{BT}(x,y) = -\frac{1}{\beta} \int_{x}^{x_r} RHS(x',y)dx'$$

Barotropic vorticity equation

1 Let us first derive the full barotropic vorticity equation from the Boussinesq momentum equations, in order to identify all the terms that can potentially induce a gyre circulation. We recall the momentum equations derived in Chapter 2 :

$$
\frac{\partial u}{\partial t} + (\mathbf{u}.\boldsymbol{\nabla})u - fv = -\frac{1}{\rho_0}\frac{\partial P}{\partial x} + \boldsymbol{\nabla_h}.(\kappa_{hu}\boldsymbol{\nabla_h})u + \frac{\partial}{\partial z}(\kappa_{zu}\frac{\partial u}{\partial z}) \tag{1}
$$
$$
\frac{\partial v}{\partial t} + (\mathbf{u}.\boldsymbol{\nabla})v + fu = -\frac{1}{\rho_0}\frac{\partial P}{\partial y} + \boldsymbol{\nabla_h}.(\kappa_{hu}\boldsymbol{\nabla_h})v + \frac{\partial}{\partial z}(\kappa_{zu}\frac{\partial v}{\partial z}) \tag{2}
$$

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$$\frac{\partial \zeta_{BT}}{\partial t} + Curl\left[\int_{-h}^{\eta}(\mathbf{u}.\boldsymbol{\nabla})\mathbf{u_h}dz\right] + \beta V = -\frac{1}{\rho_0}Curl\left[\int_{-h}^{\eta}\boldsymbol{\nabla_h}Pdz\right]$$

$$+ Curl\left[\int_{-h}^{\eta}\boldsymbol{\nabla_h}.(\kappa_{hu}\boldsymbol{\nabla_h})\mathbf{u_h}dz\right] + \frac{Curl(\boldsymbol{\tau})}{\rho_0}$$

$$\Longleftrightarrow \frac{\partial \zeta_{BT}}{\partial t} + Curl(\mathbf{A}) + \beta V = \frac{1}{\rho_0}J(P_b, h) + Curl(\mathbf{D_h}) + \frac{1}{\rho_0}Curl(\boldsymbol{\tau})$$

with $\zeta_{BT} = \frac{\partial V}{\partial x} - \frac{\partial U}{\partial y}$ the barotropic vorticity, $J(a, b) = \frac{\partial a}{\partial x}\frac{\partial b}{\partial y} - \frac{\partial a}{\partial y}\frac{\partial b}{\partial x}$ the Jacobian operator, P_b the bottom pressure, $\mathbf{A}$ and $\mathbf{D_h}$ advection and horizontal turbulent diffusion.

Barotropic vorticity equation

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$$V = \frac{1}{\beta} \left[- Curl(\mathbf{A}) + \frac{1}{\rho_0} J(P_b, h) + Curl(\mathbf{D_h}) + \frac{1}{\rho_0} Curl(\boldsymbol{\tau}) \right]$$

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3 and integrating zonally :

$$\implies \Psi_{BT}(x,y) = -\frac{1}{\beta} \int_x^{x_r} \left[-Curl(\mathbf{A}(x',y)) + \frac{1}{\rho_0} J(P_b(x',y), h(x',y)) \right.$$
$$\left. + Curl(\mathbf{D_h}(x',y)) + \frac{1}{\rho_0} Curl(\tau(x',y)) \right] dx'$$

Barotropic vorticity equation

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- The vorticity of momentum advection $-Curl(\mathbf{A})$;
- The interaction of bottom pressure with bathymetry $\frac{1}{\rho_0} J(P_b, h)$ (the so-called bottom pressure torque) ;
- The vorticity of lateral dissipation $Curl(\mathbf{D_h})$;
- The wind stress (and/or bottom stress) curl : $\frac{1}{\rho_0} Curl(\boldsymbol{\tau})$.

Barotropic vorticity equation

Most of the work has been done regarding the gyre circulation!

- ▶ It describes all the physical contributions that can drive or modulate it.
- ▶ It is solvable numerically, provided the forcings to its right hand side are known. It means that the gyre circulation can be reconstructed from observed estimates of those forcings, and that in a numerical model the physical drivers of any gyre can be analyzed.
- ▶ Note that $\zeta_{BT} = \Delta_h \Psi_{BT}$, so that a cyclonic gyre will have a positive curvature of the barotropic streamfunction, that is a negative streamfunction, and reversely for an anticyclonic gyre.

Sverdrup balance

The hypotheses of Sverdrup's theory are very similar to those of Ekman, with three notable exceptions :

- ▶ No horizontal homogeneity is assumed, precisely because it aims at predicting the horizontal structure of the gyre. However, advection and lateral turbulent diffusion of momentum are assumed negligible (linear and inviscid hypotheses) ;

- ▶ The hypothesis of constant κ_{zu} is withdrawn, because it is unnecessary to predict vertically-integrated transports.

- ▶ Because the momentum equations are integrated down to the bottom, two assumptions are necessary to neglect the role of bathymetry : no bottom friction $\boldsymbol{\tau_b} = \mathbf{0}$ and flat bottom $\boldsymbol{\nabla_h} h = \mathbf{0}$.

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$$-f\mathbf{k} \times \mathbf{u_h} = -\frac{1}{\rho_0}\boldsymbol{\nabla_h}P + \frac{\partial}{\partial z}(\kappa_{zu}\frac{\partial \mathbf{u_h}}{\partial z})$$

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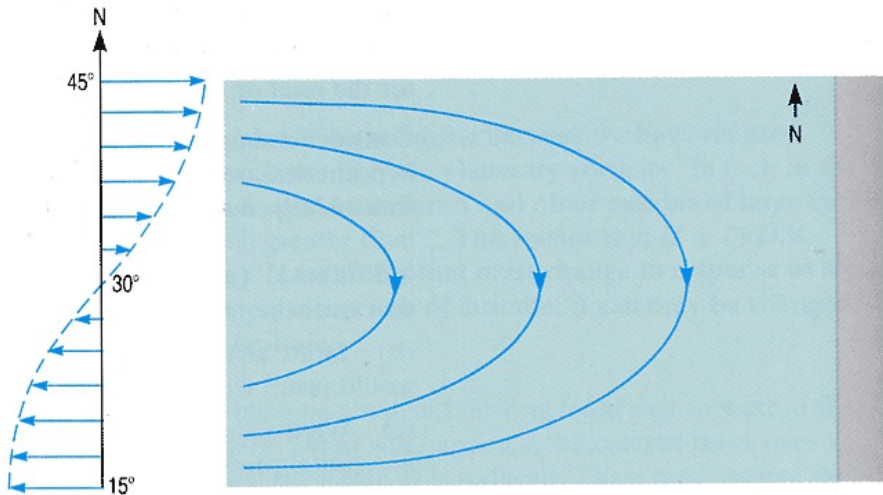
Figure 3 – Zonal wind stress used in Sverdrup's model (left) and streamlines when integrating from the eastern boundary.

Sverdrup balance

Great success of Sverdrup theory : it correctly predicts the location of the main subtropical and subpolar gyres.

▶ Over a given basin, the sign of the average wind stress curl at a given latitude gives the correct direction for the meridional flow in the interior ocean.

▶ At subtropical Northern latitudes, the negative curl is consistent with agerage southward flow in the interior, that is positive values of Ψ_{BT}.

▶ At subpolar Northern latitudes, the positive curl is consistent with agerage northward flow in the interior, that is negative values of Ψ_{BT}.

It also gives reasonable quantitative predictions for the interior transport of subtropical gyres.

Sverdrup balance

Exercise : estimation of the Sverdrup transport of North Atlantic and Pacific subtropical and subpolar gyres.

We assume that the Atlantic basin has a width $W \simeq 6000km$ with purely zonal winds of $u_{10m}(10°N) = -5m/s$, $u_{10m}(40°N) = +10m/s$ and $u_{10m}(75°N) = 0m/s$ varying linearly between those latitudes. Deduce from the Sverdrup relation the integral meridional transport at $30°N$ and $60°N$. Do the same in the Pacific, assuming the same wind profile and $W \simeq 8000km$. We assume $C_D \sim 2 \times 10^{-3}$, $\rho_a \sim 1kg/m^3$, $\rho_0 \sim 1000kg/m^3$, $\beta(30°N) \simeq 2 \times 10^{-11}m^{-1}s^{-1}$, $\beta(60°N) \simeq 10^{-11}m^{-1}s^{-1}$.

Sverdrup balance

The zonal integration of the Sverdrup relation yields :
$$\psi_{BT}(30°N) = \frac{-1}{\rho_0 \beta(30°N)} \int_{x_W}^{x_E} Curl(\tau)dx = \frac{W}{\rho_0 \beta(30°N)} \frac{\partial \tau_x}{\partial y} =$$
$$\frac{Cd\rho_a W}{\rho_0 \beta(30°N)} \frac{u_{10m}^2(30°N) + u_{10m}^2(60°N)}{L} \sim +20Sv \text{ and similarly we get}$$
$$\psi_{BT}(60°N) \sim -40Sv.$$

- ▶ *Once again, a gyre of negative vorticity will have positive values of its streamfunction, and vice versa for a cyclonic gyre.*
- ▶ *Although we get the right sign and order of magnitude for the basin-integrated gyre circulation, we largely underestimate it, which is a typical bias of Sverdrup theory.*
- ▶ *In the Pacific, the higher basin width by 25% mechanically induces a stronger basin-integrated Sverdrup transport by 25%.*

Sverdrup balance

a. Interior geostrophic flow from Sverdrup theory
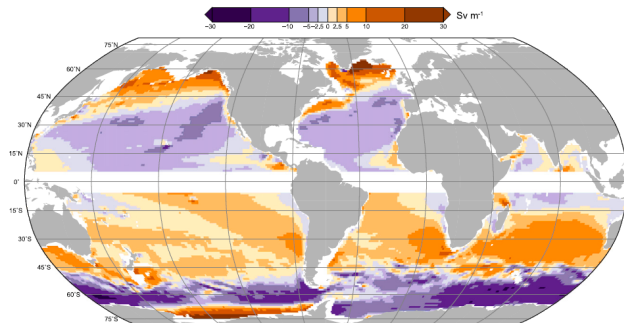
b. Relative difference to observed flow
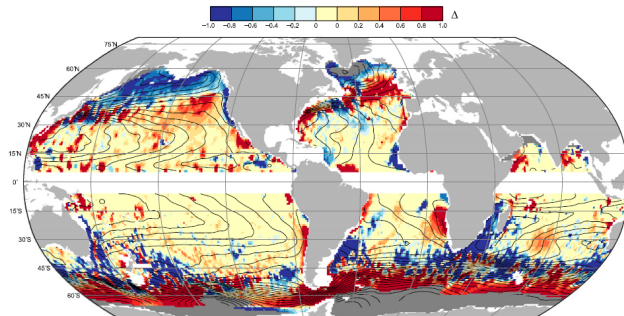
Sverdrup balance

Due to its extreme simplifications, the Sverdrup theory poses a series of issues :

- ▶ It is known to fail in representing the western side of gyres and subpolar gyres, which highly limits its domain of validity.
- ▶ Also, it tends to largely underestimate (typically by a factor ~ 3) the magnitude of both subtropical and subpolar gyre transports.
- ▶ Most importantly, it does not predict any return flow for the gyres, which is however required by continuity. In particular, it does not predict whether the return flow must occur in the eastern or western boundary : the integration of the Sverdrup relation from either the eastern or western boundary gives identical results in the interior.

Interpretation in terms of Ekman and geostrophic flow

The Sverdrup balance = vorticity balance of an Ekman + gestrophic ocean

1 In the Ekman layer the vertically-integrated vorticity equation writes as :

Interpretation in terms of Ekman and geostrophic flow

> The Sverdrup balance = vorticity balance of an Ekman + gestrophic ocean

1. In the Ekman layer the vertically-integrated vorticity equation writes as :

$$\beta V_E + f \int_{-h_E}^{\eta} \boldsymbol{\nabla}_{\boldsymbol{h}}.\mathbf{u_h} dz = \frac{1}{\rho_0} Curl(\boldsymbol{\tau})$$

$$\Longrightarrow \beta V_E - f(w(0) - w(-h_E)) = \frac{1}{\rho_0} Curl(\boldsymbol{\tau})$$

$$\Longrightarrow \beta V_E + fw(-h_E) = \frac{1}{\rho_0} Curl(\boldsymbol{\tau})$$

One additional term : the planetary vortex stretching $fw(-h_E)$. Related to the conservation of planetary angular momentum : any positive stretching of a water column induces positive vorticity, and vice versa for negative stretching. It acts as a vorticity coupling between the Ekman layer and the interior ocean.

Interpretation in terms of Ekman and geostrophic flow

2 Indeed, in the latter we have the geostrophic vorticity balance :

2 Indeed, in the latter we have the geostrophic vorticity balance :

$$
\begin{aligned}
\beta V_g + f \int_{-h}^{-h_E} \boldsymbol{\nabla_h}.\mathbf{u_h} dz &= 0 \\
\implies \beta V_g - f(w(-h_E) - w(-h)) &= 0 \\
\implies \beta V_g &= fw(-h_E)
\end{aligned}
$$

Hence wind stress imparts vorticity into the Ekman layer, most of which is transmitted to the geostrophic interior ocean by Ekman pumping. It can only be equilibrated in the geostrophic interior by the beta effect, which ultimately equilibrates the wind stress.

Interpretation in terms of Ekman and geostrophic flow

The sum of both balances yields the Sverdrup relation :

Interpretation in terms of Ekman and geostrophic flow

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$$\beta V_E + \beta V_g = \beta V = \frac{1}{\rho_0} Curl(\boldsymbol{\tau})$$

which is equivalent to the wind stress decomposition :

Interpretation in terms of Ekman and geostrophic flow

The sum of both balances yields the Sverdrup relation :

$$\beta V_E + \beta V_g = \beta V = \frac{1}{\rho_0} Curl(\boldsymbol{\tau})$$

which is equivalent to the wind stress decomposition :

$$-\frac{\beta}{\rho_0 f} \tau_x + \frac{f}{\rho_0} Curl(\boldsymbol{\tau}/f) = \frac{1}{\rho_0} Curl(\boldsymbol{\tau})$$

▶ Because the first term on the left hand side is relatively smaller, the Sverdrup transport is mostly a response of the geostrophic interior to the wind stress curl.

▶ Most importantly, because $f/\beta \sim 10^7 m$, $v_g/w(-h_E) \sim f/(\beta H) \sim 2000$, so that an Ekman pumping as low as $1m/2days$ will induce meridional velocities of $1cm/s$ in the interior ocean.

Ekman pumping is important not only for near-surface dynamics, but also for the circulation of the interior ocean.

Interpretation in terms of Ekman and geostrophic flow
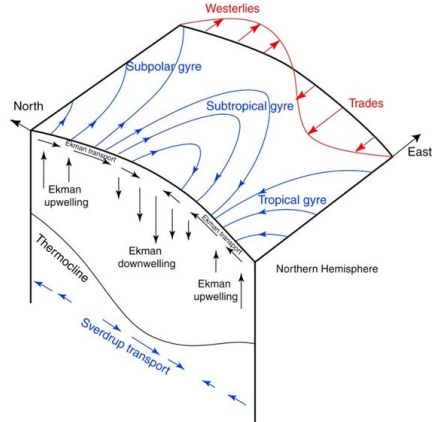
Figure 5 – Schematic link between wind stress, surface Ekman transport, interior geostrophic transport and gyre circulation (Talley et al 2012).

The wind stress curl induces a convergence of Ekman transports (Ekman suction) near surface, which activates an interior southward transport that sets up the Sverdrup balance.

Outline

Bottom friction : Stommel model

First model to predict the western return flow of the gyre circulation.

- Additional force : bottom friction force modelled as a linear drag on barotropic vorticity : $\tau_b = -r\zeta_{BT} = -r\Delta_h\Psi_{BT}$.
- The barotropic vorticity equation becomes :

Bottom friction : Stommel model

First model to predict the western return flow of the gyre circulation.

- ▶ Additional force : bottom friction force modelled as a linear drag on barotropic vorticity : $\tau_b = -r\zeta_{BT} = -r\Delta_h\Psi_{BT}$.

- ▶ The barotropic vorticity equation becomes :

$$\beta V = \frac{1}{\rho_0} Curl(\boldsymbol{\tau}) - r\Delta_h\Psi_{BT}$$

- ▶ Analytical solution for Ψ_{BT} that closes the gyre circulation !

Bottom friction : Stommel model
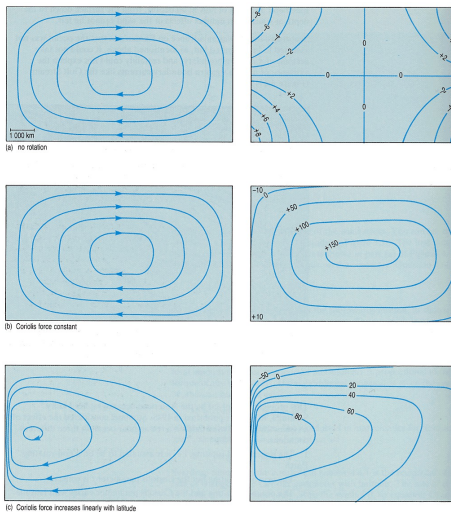
Figure 6 – Results from Stommel's model : streamlines (left) sea surface height (right) in three idealized cases : no Earth rotation ($f = 0$, top), f plane ($f = f_0$, middle) and beta plane ($f = f_0 + \beta y$, bottom).

Bottom friction : Stommel model

Interpretation :

- ▶ No rotation : anticyclonic gyre forced by anticyclonic wind stress, without assymetry.
- ▶ f plane : same circulation because no Coriolis in the barotropic vorticity balance.
- ▶ beta plane : zonal assymetry :
  - ▶ To the east, the cyclonic beta effect partially compensates the anticyclonic wind stress, so that less bottom friction is needed for the flow to reach a vorticity balance. As friction is proportional to vorticity, this means that the flow must slow down.
  - ▶ To the west, both the beta effect and the wind stress impart anticyclonic vorticity. Hence the bottom friction must be enhanced to balance both terms. This is done through the intensification of the northward flow.
  - ▶ As the basin is closed, by continuity the northward and southward transports must be equal, which implies a narrow western boundary northward flow and a wide interior southward flow.

Bottom friction : Stommel model

Interest :

- It explains the gyre zonal assymetry.
- Indeed, because of the beta effect, an enhanced cyclonic vorticity source is needed at the western boundary, and a weakened one in the interior ocean.

Limitations :

Bottom friction : Stommel model

Interest :

- It explains the gyre zonal assymetry.
- Indeed, because of the beta effect, an enhanced cyclonic vorticity source is needed at the western boundary, and a weakened one in the interior ocean.

Limitations :

- Bottom velocities would need to be unrealistically large for this bottom friction to have a significant role in the vorticity balance.
- The predicted western boundary current is far too narrow compared to observations.

Lateral friction : Munk model

Same approach as Stommel, but lateral dissipation instead of bottom friction. The barotropic vorticity equation becomes :

$$\beta V = Curl\left[\int_{-h}^{\eta} \boldsymbol{\nabla_h}.(\kappa_{hu}\boldsymbol{\nabla_h})\mathbf{u_h}dz\right] + \frac{1}{\rho_0}Curl(\boldsymbol{\tau})$$

He assumed the eddy diffusivity coefficient to be constant : $\kappa_{hu} = A$, so that the lateral diffusion term writes as :

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He assumed the eddy diffusivity coefficient to be constant : $\kappa_{hu} = A$, so that the lateral diffusion term writes as :

$$Curl\left[\int_{-h}^{\eta} \boldsymbol{\nabla_h}.(\kappa_{hu}\boldsymbol{\nabla_h})\mathbf{u_h}dz\right] = A\,Curl(\Delta_h U_h)$$
$$= A\,\Delta_h\zeta_{BH}$$

Hence the barotropic vorticity balance becomes :

Lateral friction : Munk model

Hence the barotropic vorticity balance becomes :

$$\beta V = A\Delta_h \zeta_{BH} + \frac{1}{\rho_0} Curl(\boldsymbol{\tau})$$

- ▶ Horizontal velocities are assumed null at the border (no-slip boundary condition), so that lateral diffusivity plays the same role as Stommel's bottom friction : it slows down the gyre circulation.
- ▶ A notable difference is that it acts preferentially along the borders, whereas bottom friction is also active in the interior.

Lateral friction : Munk model
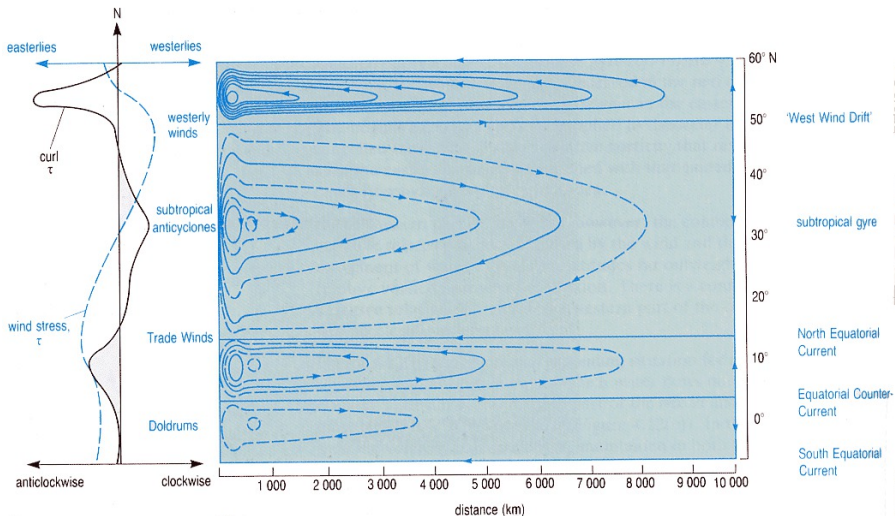
Figure 7 – Meridional wind and wind stress curl in Munk's model (left) and barotropic streamfunction (right) in an idealized North Atlantic basin.

Lateral friction : Munk model

- ▶ Results are very similar to Stommel's model, with only one major improvement : transports cancel at the borders. He also applied a more realistic meridional wind profile.
- ▶ However, like Stommel's model, his lateral dissipation requires unrealistic horizontal velocities at the western boundary to balance the vorticity equation. Hence it also predicts a too narrow western boundary current.

Less intuitive terms of the vorticity balance intervene in the western intensification of gyres !

Topographic torques

The role of topography appears explicitely in the barotropic vorticity equation as the so-called bottom pressure torque term.

In the barotropic case, its physical effect is to attach the flow to topographic contours, or more precisely to geostrophic f/h contours.
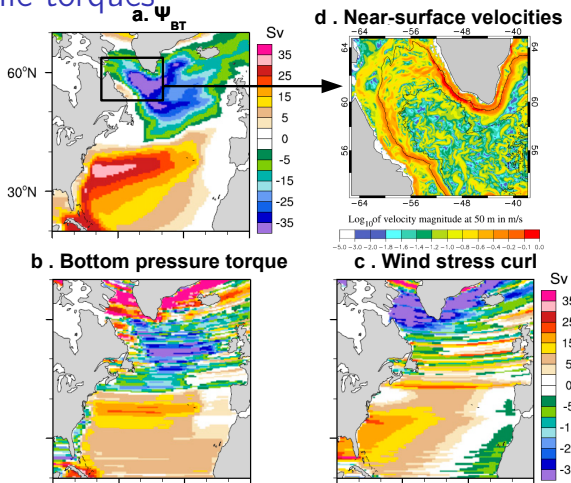
Topographic torques

Figure 8 – a) Barotropic streamfunction in an ocean climate model and contributions of b) the bottom pressure torque and c) the wind stress curl (Yeager 2015). d) Near-surface currents superimposed to the $1000\,m$ and $3000\,m$ isobaths (black contours) in the Labrador Sea from an high-resolution ocean model (Saenko et al 2014).

Topographic torques

- ► High-latitude circulation is relatively barotropic due to the deep-reaching mixed layers and the beta effect becomes weak. Thus currents follow closely topography.
- ► The bottom pressure torque is at least as important as the wind stress curl in the barotropic vorticity balance.

Bottom pressure torque as a torque by solid Earth :

Topographic torques

Bottom pressure torque as a torque by solid Earth :

$$J(P_b, h) = 0 \iff P_b = P(h)$$

the bottom pressure torque is null if and only if bottom pressure is constant along isobaths : in this case the force exerted by solid Earth on the ocean bottom has no torque !

Topographic torques

Bottom pressure torque as a bottom geostrophic vortex stretching :
geostrophic vorticity balance with varying bathymetry $-h(x,y)$ for a
barotropic ocean ($\rho = \rho_0$) :

Topographic torques

Bottom pressure torque as a bottom geostrophic vortex stretching :
geostrophic vorticity balance with varying bathymetry $-h(x,y)$ for a
barotropic ocean ($\rho = \rho_0$) :

$$
\begin{aligned}
\beta V &= \frac{1}{\rho_0} J(P_b, h) \\
&= \frac{1}{\rho_0} \left(\frac{\partial P_b}{\partial x} \frac{\partial h}{\partial y} - \frac{\partial P_b}{\partial y} \frac{\partial h}{\partial x} \right) \\
&= f \left(v_g(-h) \frac{\partial h}{\partial y} + u_g(-h) \partial y \frac{\partial h}{\partial x} \right) \\
&= f \mathbf{u_g}(-h) . \boldsymbol{\nabla} h \\
&= f \frac{dh}{dt} \\
&= -f w(-h)
\end{aligned}
$$

When bottom pressure does not follow isobaths, geostrophic velocities
push the fluid up or down the bathymetry, which generates a bottom
geostrophic vortex stretching !

Topographic torques

Now remembering that $\beta V = h\frac{df}{dt}$ for a barotropic fluid, we deduce :

$$
\begin{aligned}
h\frac{df}{dt} - f\frac{dh}{dt} &= 0 \\
\implies \frac{1}{h}\frac{df}{dt} - \frac{f}{h^2}\frac{dh}{dt} &= 0 \\
\iff \frac{df/h}{dt} &= 0
\end{aligned}
$$

This expresses the Lagrangian conservation of potential vorticity, which in this simple case is f/h.
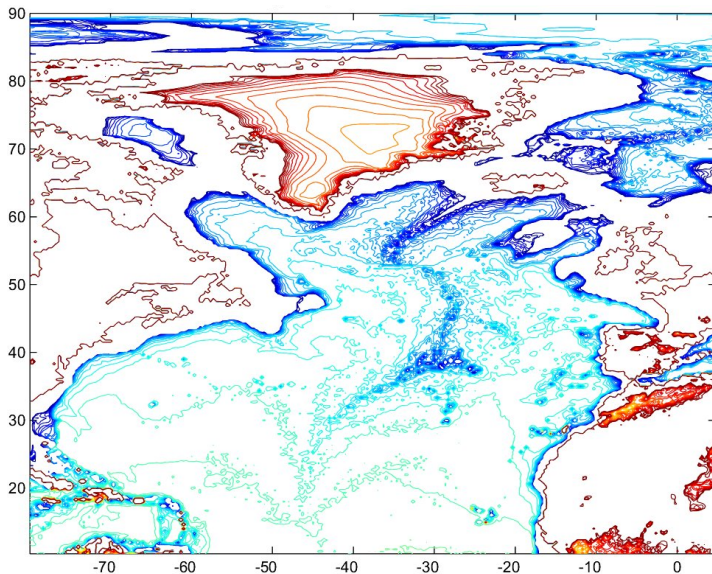
Topographic torques

Figure 9 – North Atlantic geostrophic f/h contours (Peter Rhines's lecture).

Topographic torques

Important consequence for the gyre circulation :

- ▶ Bathymetric changes along the flow are sufficient to equilibrate the beta effect induced by any meridional transport. More specifically, a northward flow can be equilibrated if it moves down the bathymetry, and vice versa for a southward flow.

- ▶ Thus a geostrophic circulation can be closed if geostrophic contours f/h are closed, which is the case for much of the North Atlantic subpolar gyre.

- ▶ As f contours are purely zonal, a strong meridional deformation of f/h contours means that the topographic slope largely dominates over the beta effect in the barotropic vorticity balance.

Topographic torques

Adding the wind stress forcing back to the barotropic vorticity balance we get the so-called topographic Sverdrup balance :

Topographic torques

Adding the wind stress forcing back to the barotropic vorticity balance we get the so-called topographic Sverdrup balance :

$$\beta V = f\mathbf{u_g}(-h).\boldsymbol{\nabla}h + \frac{1}{\rho_0}Curl(\boldsymbol{\tau})$$

There is no need of any energy sink such as bottom friction or lateral dissipation to explain the western boundary return flow of gyres !

- ▶ It is enough for the western boundary current to flow down the topography as it goes north.
- ▶ Recent results show that western boundary currents are relatively conservative : the bottom pressure torque is a better candidate than bottom friction and lateral dissipation to permit the meridional mass flux in the western boundary.

Topographic torques

The ocean is generally baroclinic :

- ► The three-dimensional density structure matters, because the bottom pressure corresponds to the weight of the overlying ocean column.
- ► Hence so-called "buoyancy-driven" gyres can exist, which are not driven by the wind stress curl but by buoyancy fluxes (heat and water fluxes). This is for instance the case of cyclonic gyres driven by buoyancy loss (cooling or evaporation) in semi-enclosed seas (e.g. the Nordic Seas, Baltic Sea, Mediterranean Sea, Labrador Sea).

Topographic torques
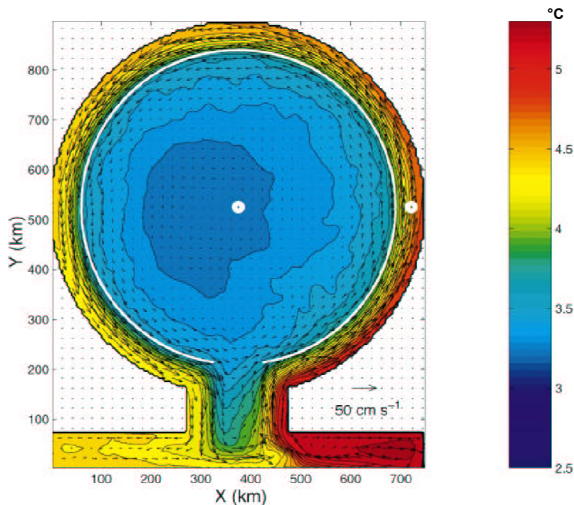
Figure 10 – Mean surface temperature and velocity of an idealized buoyancy-driven cyclonic gyre driven by no wind stress and a surface heat loss of $Q_{tot} = -200\,W/m^2$ (Spall 2003).

Recirculation gyres

The only term that has not been analyzed so far : non-linear vorticity advection :

$$Curl(\mathbf{A}) = Curl\left[\int_{-h}^{\eta}(\mathbf{u}.\boldsymbol{\nabla})\mathbf{u_h}dz\right]$$

▶ Redistribution of vorticity by the ocean in motion : null net effect but explains the presence of intense western recirculation gyres.

▶ Nonlinear advection because, contrary to the planetary vorticity advection $\beta V = \int_{-h}^{\eta}\frac{df}{dt}dz$, it is nonlinear in velocities. Significant values only where intense velocities.

Recirculation gyres

a. Non-advective Ψ_{BT}

b. Advective Ψ_{BT}

c. Advective : vorticity balance

$\zeta + f$ $\zeta + f$

B = beta effect
W = wind stress
F = lateral friction (= dissipation)
N = nonlinear advection
o, +, * = transitional regimes
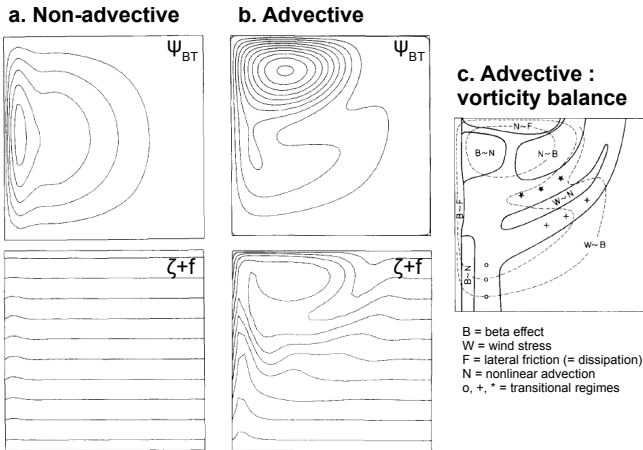
Figure 11 – Barotropic circulation Ψ_{BT} and potential vorticity contours $\zeta + f$ in an idealized subtropical gyre model a) with weak advection ($\mathbf{A} \propto \mathbf{D_h}/5$) and b) with intense advection ($\mathbf{A} \propto \mathbf{D_h}$) and c) vorticity balance per main dynamical region in the advective case (Boning 1986). Author refers to N as non-linear advection $\mathbf{A}$ and F as lateral friction, which is identical to lateral dissipation $\mathbf{D_h}$.

Recirculation gyres

Vorticity advection can become a dominant term in a so-called "recirculation gyre" ! The barotropic vorticity balance is dominated there by :

Recirculation gyres

Vorticity advection can become a dominant term in a so-called "recirculation gyre" ! The barotropic vorticity balance is dominated there by :

$$
\begin{aligned}
\beta V &= -Curl\left[\int_{-h}^{\eta}(\mathbf{u}.\boldsymbol{\nabla})\mathbf{u_h}dz\right] \\
&= -\mathbf{U}.\boldsymbol{\nabla}\zeta_{BT}
\end{aligned}
$$

meaning that the advection of planetary vorticity compensates for the advection of relative vorticity.

- ▶ Mechanism : convergence of anticyclonic vorticity to the east of the boundary current, and reversely to the west. In response of the beta effect, southward flow to the east, and a northward flow to the west.
- ▶ With this mechanism, the gyre circulation can be highly intensified without any additional source of energy.

Recirculation gyres

Lagrangian interpretation : conservation of potential vorticity, which writes in this flat bottom barotropic case as :

Recirculation gyres

Lagrangian interpretation : conservation of potential vorticity, which writes in this flat bottom barotropic case as :

$$\frac{d(hf + \zeta_{BT})}{dt} = 0$$

- ▶ Neglecting the forcing (wind stress) and dissipation (lateral diffusion), the flow must follow geostrophic contours which have constant $hf + \zeta_{BT}$.

- ▶ In most of the domain those contours are blocked by boundaries because they are mostly zonal and dominated by f, and as a consequence water masses cannot circulate freely.

- ▶ Within the recirculation gyre, because of an intense ζ_{BT}, the potential vorticity is homogenized and water parcels can freely undergo an intense circulation.

Recirculation gyres
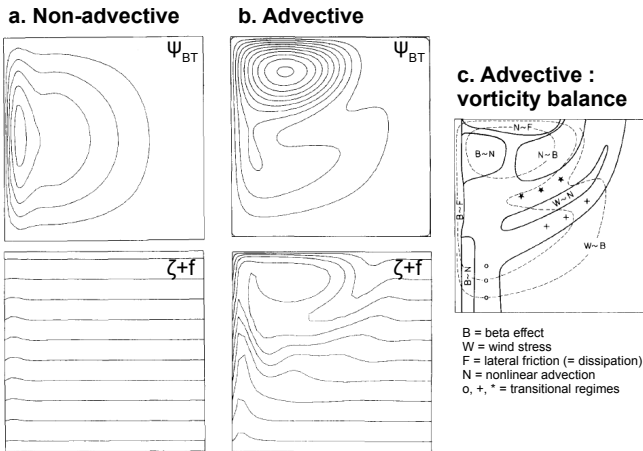
Figure 12 – Barotropic circulation Ψ_{BT} and potential vorticity contours $\zeta + f$ in an idealized subtropical gyre model a) with weak advection ($\mathbf{A} \propto \mathbf{D_h}/5$) and b) with intense advection ($\mathbf{A} \propto \mathbf{D_h}$) and c) vorticity balance per main dynamical region in the advective case (Boning 1986). Author refers to N as non-linear advection $\mathbf{A}$ and F as lateral friction, which is identical to lateral dissipation $\mathbf{D_h}$.

Recirculation gyres
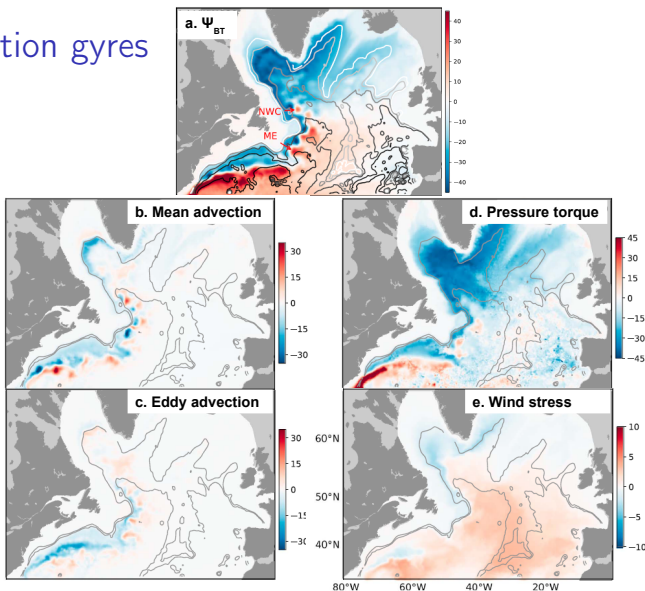
Figure 13 – a) Barotropic circulation Ψ_{BT} in an eddy-resolving model and contributions of b) mean advection, c) eddy advection, d) bottom pressure torque and e) wind stress curl (Wang et al 2017).

Recirculation gyres

- High-resolution ocean models resolve those recirculation and have intensified gyres, which is not the case for low-resolution models.
- The respective contributions of transient eddies and the mean flow can be separated by decomposing the vorticity advection term :

Recirculation gyres

▶ High-resolution ocean models resolve those recirculation and have intensified gyres, which is not the case for low-resolution models.

▶ The respective contributions of transient eddies and the mean flow can be separated by decomposing the vorticity advection term :

$$\begin{aligned} \beta V &= -\mathbf{U}.\boldsymbol{\nabla}\zeta_{BT} \\ &= -\overline{\mathbf{U}}.\boldsymbol{\nabla}\overline{\zeta_{BT}} - \boldsymbol{\nabla}.\overline{\mathbf{U}'\zeta'_{BT}} \end{aligned}$$

with the classical Reynolds decomposition $X = \overline{X} + X'$, both resolved (with X either $\mathbf{U}$ or ζ_{BT}). Results show :

  ▶ Both mean and eddy advection are important, although once again the bottom pressure torque appears as a key ingredient.
  ▶ Both the subtropical and subpolar gyres are concerned by this intensification, mostly along their western boundary.

Eastern shadow zones

Limitations of the steady barotropic framework :

- ▶ It is steady : it does not tell
  - ▶ how the time-dependent circulation sets up gyres (Rossby wave dynamics)
  - ▶ how the most energetic transient eddy dynamics interacts with it (dynamical instabilities and energy cascades)
- ▶ It is barotropic : no information on
  - ▶ vertical overturning circulations in the meridional or zonal plane
  - ▶ the vertical distribution of the gyre transport and the ventilation of the interior ocean

We now try to identify whether some layers in the water column might be unventilated. For that, we abandon the barotropic vorticity formalism and introduce some depth dependence.

Eastern shadow zones
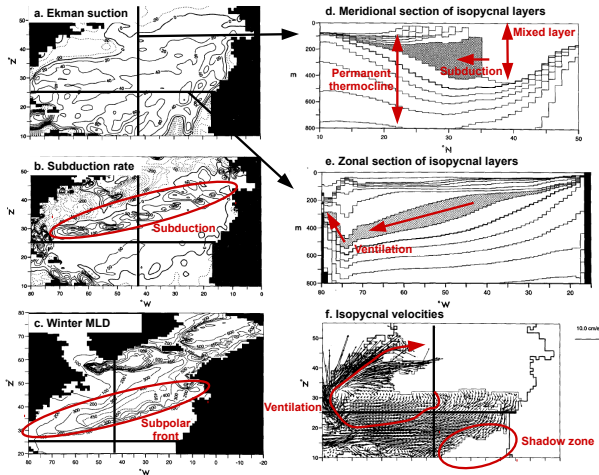
Figure 14 – a) Ekman suction rate ($m/year$), b) subduction rate ($m/year$), c) March mixed layer depth (MLD, m), d) meridional section at $42°W$ and e) zonal section at $35°N$ of isopycnal layer depth (density every $0.15kg/m^3$, $26.70kg/m^3$ layer shaded) and f) isopycnal velocities in the $26.55kg/m^3$ layer.

Eastern shadow zones

- Due to Ekman pumping, the trajectory of water masses around the gyres is not horizontal but slanted. Given by the isopycnal slope because in the interior ocean, water parcels are mostly adiabatic.
- The mixed layer deepens with latitude : hence water masses exit the mixed layer (they are subducted) to the north and then they flow southwest while sinking, isolated from surface.
- Such water masses are named mode waters. They can remain for decades below the mixed layer before resurfacing along the western boundary. They ventilate the interior of subtropical gyres.

Mode waters do not reach the southeastern corner !

Eastern shadow zones
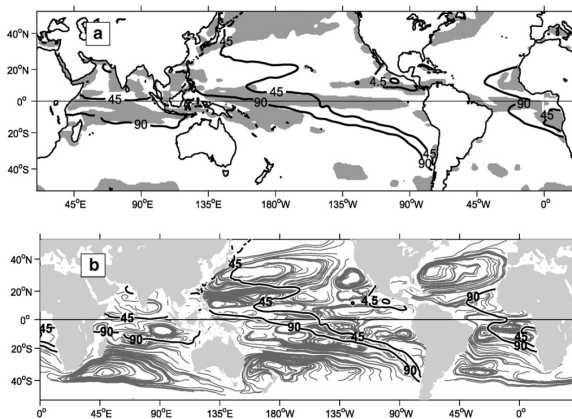
Figure 15 – Observed O_2 minimum (black contours) in relation to a) main Ekman upwelling regions (shaded grey) and b) mode water trajectories (grey contours) (Karstensen et al 2008).

- Eastern subtropical O_2 minimum.
- Associated Ekman pumping : intense biological activity.
- Dynamical isolation from the subtropical gyre.

Eastern shadow zones

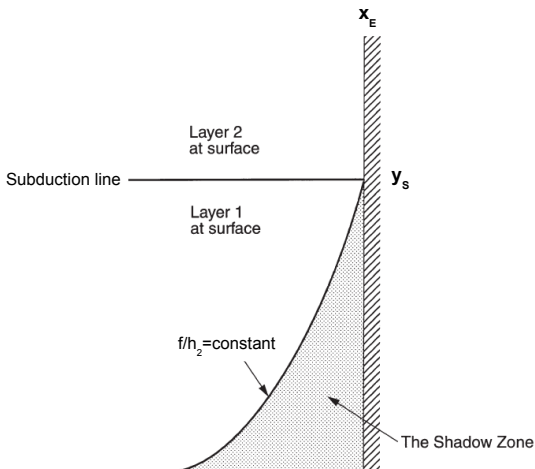Simplest model : the so-called 2.5-layer model.

Figure 16 – Horizontal sketch of the shadow zone in the 2.5 layer geostrophic model (adapted from Vallis 2006).

Eastern shadow zones

Simplest model : the so-called 2.5-layer model.

- ▶ Two upper layers of constant density in motion, and a lower layer at rest.

- ▶ At some latitude (say $y = y_S$), layer 2 is subducted beneath layer 1 and ventilates the interior ocean while conserving its potential vorticity, which is in this geostrophic case f/h_2.

- ▶ Hence any southward motion reduces f and must be accompanied by a shoaling of layer 2.

- ▶ At the eastern boundary x_E, this is forbidden by the no normal flow boundary condition : any variation of h_2 along the boundary would induce a zonal flow u_{g2} towards the continent.

- ▶ Hence once water parcels have subducted at the eastern boundary (at the coordinates (x_E, y_S)), they must move southwestward to satisfy both potential vorticity conservation and no normal flow at the eastern boundary.
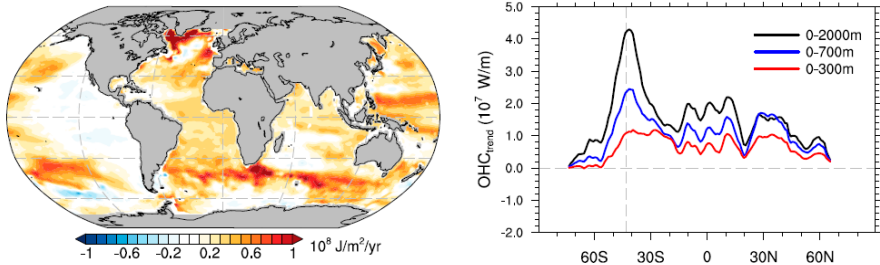
Outline

Figure 17 – 1979–2015 ocean heat content trend map and zonally-integrated as a function of latitude and depth (Shi et al 2018).

The Southern Ocean stands out as the main heat storage region!

Specficities of the Southern Ocean :

- ▶ No gyre circulation, instead a mostly zonal flow, the Antarctic Circumpolar Current.
- ▶ The longest $(24,000km)$ and most intense $(\sim 150Sv)$ oceanic current of the world ocean.
- ▶ The main absorption area of the anthropogenetic heat (Fig.17) and CO_2 anomalies.

Meridional overturning

"Deacon Cell" = wind-driven meridional overturning cell.

- ▶ Surface winds are essentially zonal, so that Ekman transports are northward (to the left in the southern hemisphere).
- ▶ Wind stress curl forms a dipole north and south of the Westerly jet, so that Ekman pumping occurs south, and Ekman suction north of this jet.
- ▶ By continuity, there must be a southward geostrophic branch in the interior ocean !
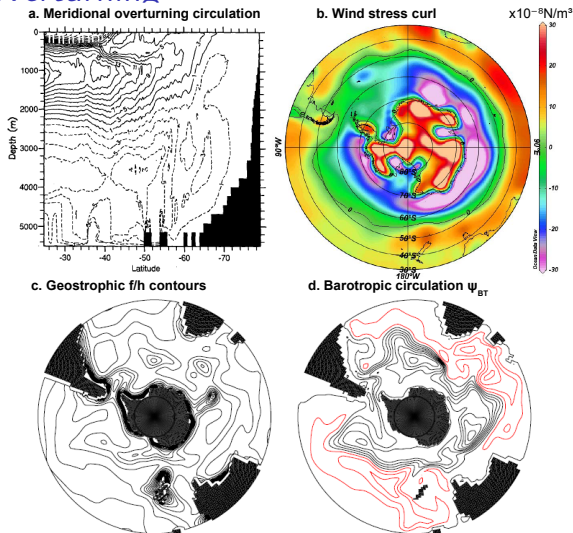
Meridional overturning

Figure 18 – a) Deacon Cell from an idealized model, b) average wind stress curl, c) geostrophic f/h contours and d) barotropic streamfunction Ψ_{BT} from an idealized model (Doos et al 1994, LT's lecture, Olbers et al 2007).

Meridional overturning

Vorticity balance separately for the surface Ekman layer and the interior geostrophic ocean.

- In the Ekman layer :

Meridional overturning

Vorticity balance separately for the surface Ekman layer and the interior geostrophic ocean.

- ► In the Ekman layer :

$$\beta V_E + fw(-h_E) = \frac{1}{\rho_0} Curl(\boldsymbol{\tau})$$

  The wind vorticity input is partly equilibrated in the Ekman layer, and partly transmitted to the underlying geostrophic interior by vortex stretching.

- ► In the interior geostrophic layer :

Meridional overturning

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  The wind vorticity input is partly equilibrated in the Ekman layer, and partly transmitted to the underlying geostrophic interior by vortex stretching.

- In the interior geostrophic layer :

$$\beta V_g = fw(-h_E) + J(P_b, h)$$

  beta effect and bottom pressure torque balance this vorticity input.

Meridional overturning

The fundamental role of topography :

- ▶ Over a zonal integral, by continuity

Meridional overturning

The fundamental role of topography :

- ▶ Over a zonal integral, by continuity

$$< V_E >= - < V_g >$$

  with $< X >= \frac{1}{L_{ACC}} \int_{x_{R-}}^{x_{R+}} X dx$ the zonal mean of a quantity X over the whole circumpolar zonal ring of length L_{ACC} (with x_R an arbitrary reference longitude).

- ▶ Hence the zonally-integrated vorticity balance writes as :

Meridional overturning

The fundamental role of topography :

▶ Over a zonal integral, by continuity

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with $< X >= \frac{1}{L_{ACC}} \int_{x_{R-}}^{x_{R+}} X dx$ the zonal mean of a quantity X over the whole circumpolar zonal ring of length L_{ACC} (with x_R an arbitrary reference longitude).

▶ Hence the zonally-integrated vorticity balance writes as :

$$
\begin{aligned}
< \beta V_E + \beta V_g > &= 0 \\
&= \frac{1}{\rho_0} < Curl(\boldsymbol{\tau}) > + < J(P_b, h) >
\end{aligned}
$$

Without the bottom pressure torque, the interior ocean could not be in geostrophic balance !

Meridional overturning
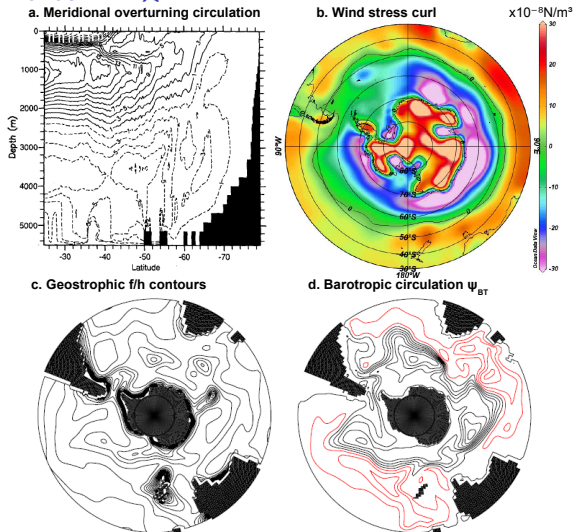

Figure 19 – a) Deacon Cell from an idealized model, b) average wind stress curl, c) geostrophic f/h contours and d) barotropic streamfunction Ψ_{BT} from an idealized model (Doos et al 1994, LT's lecture, Olbers et al 2007).

Meridional overturning

- The fact that the geostrophic interior has to lean on the topography to equilibrate the wind stress curl also explains why the Deacon Cell extends very deep in the water column.

- Wind-driven Ekman flows are typically very shallow ($h_E \sim 50m$), so that the associated vertical motions do not usually extend very deep.

- This coupling mechanism involving the bathymetry in the Deacon Cell is a striking illustration of how some wind-driven flows can ventilate the deep ocean.

Meridional overturning

Why have we not started the analysis of meridional circulation by using the barotropic vorticity equation ?

- ▶ Because due to the absence of any continental barrier, there can be no net meridional flow across the Southern Ocean !
- ▶ The Sverdrup balance cannot hold because there is no western boundary to support a northward return flow and hence ensure continuity. If we assume vorticity advection $Curl(\mathbf{A})$ and turbulent horizontal diffusion $Curl(\mathbf{D_h})$ to be small, the zonal mean barotropic vorticity equation gives :

Meridional overturning

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$$\frac{1}{\rho_0} < J(P_b, h) > + \frac{1}{\rho_0} < Curl(\boldsymbol{\tau}) > = 0$$

  The beta effect is cancelled in the absence of a net meridional flow.

- ▶ Although this equation states nothing about the flow, we see once again the importance of the bottom pressure torque and hence of topography. The strong wind stress curls can be equilibrated by a bottom geostrophic flow across isobaths.

Meridional overturning

However, at a given longitude band, some net meridional transport can still occur :

Meridional overturning

However, at a given longitude band, some net meridional transport can still occur :

$$\beta V = \frac{1}{\rho_0} J(P_b, h) + \frac{1}{\rho_0} Curl(\boldsymbol{\tau})$$

▶ Locally, the wind stress curl and above all the bottom pressure torque can generate a net meridional flow.

▶ This is the source of the so-called "standing meanders" of the Antarctic Circumpolar Current
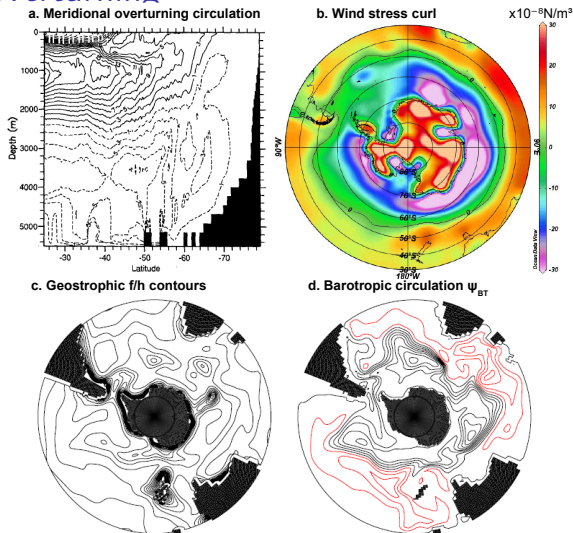
Meridional overturning

Figure 20 – a) Deacon Cell from an idealized model, b) average wind stress curl, c) geostrophic f/h contours and d) barotropic streamfunction Ψ_{BT} from an idealized model (Doos et al 1994, LT's lecture, Olbers et al 2007).

Zonal circulation

How does a zonal wind stress induce a zonal current like the Antarctic Circumpolar Current? This intuitive relation is complicated by the Coriolis acceleration which induces a meridional Ekman response.

- ► Pressure gradient build-up by Ekman transports : wind-driven currents (the Ekman currents) are meridional for a zonal wind, they can drive an intense zonal geostrophic flow through the coupling with tracers.
- ► To illustrate that, let us consider how meridional Ekman transports can intensify a meridional density front.

Zonal circulation

Initial constant density gradient, purely thermal $\frac{\partial \rho}{\partial y}(t=0) = -\alpha_\theta \frac{\Delta \theta}{L}$.

Zonal wind stress $\boldsymbol{\tau} = \tau_x(y)\mathbf{i}$ and no heat source $\dot{\Theta} = 0$. In the Ekman layer, the temperature conservation is :

$$h_E \frac{\partial \theta}{\partial t} = -V_E \frac{\partial \theta}{\partial y}$$

that is, temperature trends are driven by the convergence of the Ekman heat flux. Hence :

$$
\begin{aligned}
h_E \frac{\partial}{\partial t}\left(\frac{\partial \theta}{\partial y}\right) &= -\frac{\partial}{\partial y}\left(V_E \frac{\partial \theta}{\partial y}\right) \\
&= -\frac{\partial V_E}{\partial y}\frac{\partial \theta}{\partial y} \\
&= -w(-h_E)\frac{\partial \theta}{\partial y}
\end{aligned}
$$

neglecting $\frac{\partial^2 \theta}{\partial y^2}$ and $\frac{\partial h_E}{\partial y}$. We deduce :

$$\frac{\partial \theta}{\partial y}(t) = \frac{\partial \theta}{\partial y}(t=0)\exp\left[-\frac{w(-h_E)}{h_E}t\right]$$

Zonal circulation

- ▶ Where Ekman pumping is negative, that is to the north of the Deacon Cell, Ekman transports tend to exponentially increase any initial meridional temperature gradient.

- ▶ There is also a dependency to the Ekman depth h_E because for a given Ekman pumping, the thinner the Ekman depth, the stronger the resulting temperature trend within the mixed layer.

- ▶ Because of the meridional gradient of solar radiation, there is always a meridional temperature gradient in the Southern Ocean, so that this mechanism is always active.

- ▶ This enhanced density gradient mechanically increases zonal geostrophic transports !

Zonal circulation

a. The Antarctic Circumpolar Current

Subantarctic front
Polar front
Southern ACC front

b. Meridional sea level and temperature section

SAF–N
SAF–M
SAF–S
PF–N
PF–M
PF–S
SACCF–N
SACCF–S

c. Zonal density section at 60°S

Pacific Antarctic Ridge
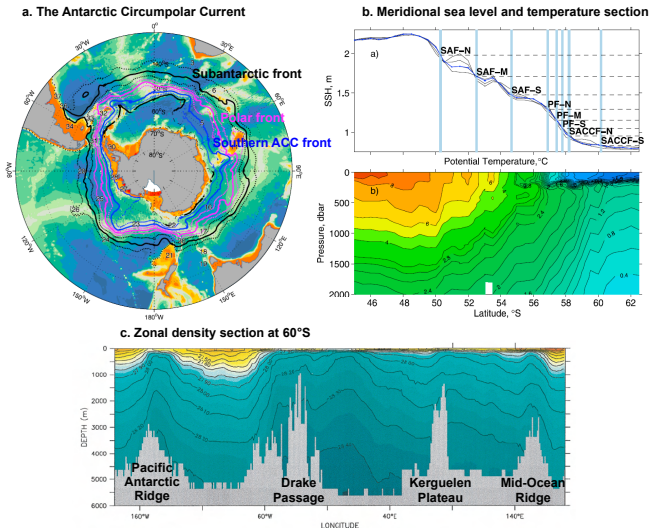Drake Passage
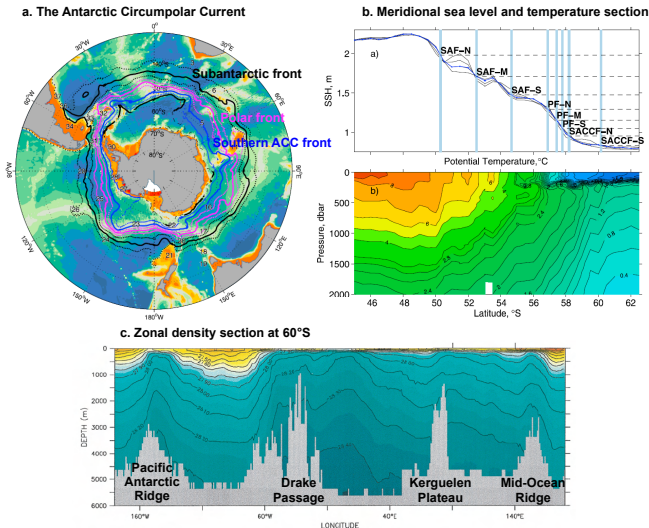Kerguelen Plateau
Mid-Ocean Ridge

Figure 21 – a) Main fronts of the Antarctic Circumpolar Current (ACC), b) their sea level and temperature signatures, and c) zonal potential density section at $60°S$ (Sokolov et al 2009, Olbers et al 2007).

Zonal circulation

The Antarctic Circumpolar Current is entirely determined by the interior geostrophic flow :

$$
\begin{aligned}
U_g &= \int_{-h}^{\eta} u_g \, dz \\
&= \int_{-h}^{\eta} \left[u_g(-h) + \int_{-h}^{z} \frac{\partial u_g}{\partial z}(z') dz' \right] dz \\
&= h \, u_g(-h) + \int_{-h}^{\eta} \left[\int_{z'}^{\eta} \frac{\partial u_g}{\partial z}(z') dz \right] dz' \\
&= h \, u_g(-h) + \int_{-h}^{\eta} z' \frac{\partial u_g}{\partial z}(z') dz'
\end{aligned}
$$

This gives with the thermal wind relation :

$$
\begin{aligned}
U_g &= h \, u_g(-h) + \frac{g}{\rho_0 f} \int_{-h}^{\eta} z \frac{\partial \rho}{\partial y} dz \\
&\simeq \frac{g}{\rho_0 f} \int_{-h}^{\eta} z \frac{\partial \rho}{\partial y} dz
\end{aligned}
$$

Zonal circulation

We have just written the same transport equation as in the Drake Passage exercise. It illustrates the coupling between the dynamics and tracers :

- intense geostrophic velocities are driven by meridional density (mostly temperature) gradients,
- themselves enhanced by the Ekman pumping, although also driven by surface buoyancy (heat and water) fluxes.

Zonal circulation

a. The Antarctic Circumpolar Current

Subantarctic front

Polar front

Southern ACC front

b. Meridional sea level and temperature section

SAF–N
SAF–M
SAF–S
PF–N
PF–M
PF–S
SACCF–N
SACCF–S

SSH, m

Potential Temperature, °C

Pressure, dbar

Latitude, °S

c. Zonal density section at 60°S

DEPTH (m)

Pacific Antarctic Ridge

Drake Passage

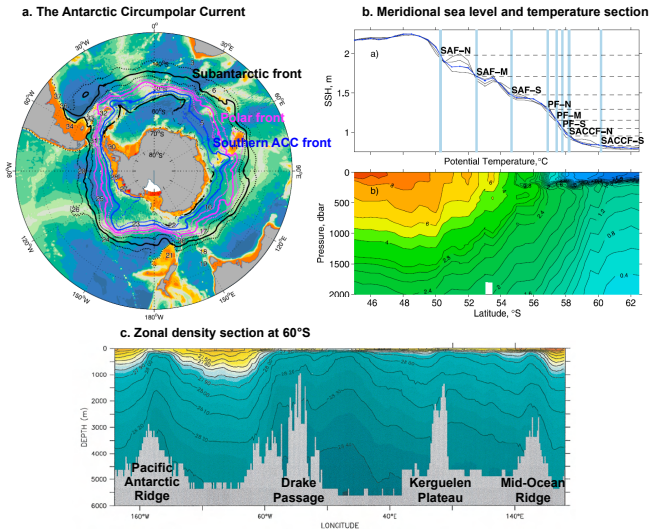Kerguelen Plateau

Mid-Ocean Ridge

LONGITUDE

Figure 22 – a) Main fronts of the Antarctic Circumpolar Current (ACC), b) their sea level and temperature signatures, and c) zonal potential density section at $60°S$ (Sokolov et al 2009, Olbers et al 2007).

Zonal circulation

Structure of the Antarctic Circumpolar Current :

- ▶ Surface-intensified just like meridional density gradients.
- ▶ Constituted of several fronts which delimitate sub-currents at different latitudes.
- ▶ Its vertical extent is among the largest for a surface-intensified current : matters for both surface and deep water transports.

Zonal momentum balance

What slows down the Antarctic Circumpolar Current ?

- It is accelerated by the zonal wind stress.
- Like the gyre circulation, neither lateral dissipation nor bottom friction are strong enough to equilibrate wind stress.
- The "bottom form drag". Indeed, the zonally and vertically-integrated zonal pressure force is :

$$< \int_{-h}^{\eta} \frac{\partial P}{\partial x} dz > = < \frac{\partial}{\partial x} (\int_{-h}^{\eta} P dz) > - < p(\eta) \frac{\partial \eta}{\partial x} > + < P(-h) \frac{\partial -h}{\partial x} >$$

$$\simeq - < P(-h) \frac{\partial h}{\partial x} >$$

For the zonal pressure force to slow down zonal motion, bottom pressure must be on average higher upstream than downstream of seamounts. In this case :

The pressure force exerted by seamount on the ocean is directed westward, hence compensating for the surface wind stress.

Zonal momentum balance

Figure 23 – a) Main fronts of the Antarctic Circumpolar Current (ACC), b) their sea level and temperature signatures, and c) zonal potential density section at $60°S$ (Sokolov et al 2009, Olbers et al 2007).

Zonal momentum balance

- Because velocities are surface-intensified, bottom pressure gradients are dominated by surface pressure, that is by sea level variations, although they are almost entirely equilibrated by baroclinic pressure gradients so that bottom velocities are small.

- Positive sea level anomaly, although also a negative density anomaly, upstream of the main topographic accidents, and reversely downstream of them.

Role of eddies
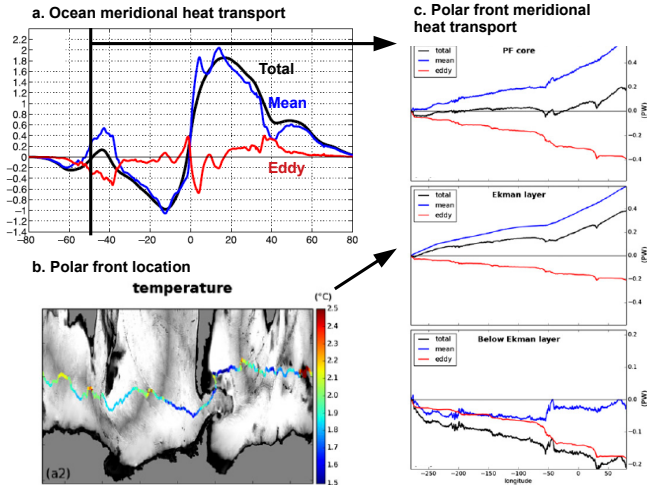
Figure 24 – a) Meridional heat flux decomposed between the mean and eddy transport in a high-resolution ocean model, and b-c) its decomposition as a function of depth and longitude along the Polar Front (Griffies et al 2015, Duffour et al 2016).

Role of eddies

- Similarly to the atmosphere, the Southern Ocean poleward heat transport is dominated by transient eddies. This is a unique characteristic in the world ocean which transports heat by the mean circulation at all other latitudes.

- In the Southern Ocean, the mean meridional circulation, the Deacon Cell, transports heat equatorward.

- On the contrary, eddy heat advection is poleward. The physical mechanism is identical to the atmosphere : baroclinic instability advects buoyancy (mostly heat) southward.

Role of eddies

"Standing meanders" can advect heat poleward :

▶ Let us decompose mean transports as

$$\overline{\mathbf{U_h}} = \overline{<\mathbf{U_h}>} + \overline{\mathbf{U_h}^*}$$

with $\overline{<\mathbf{U_h}>}$ the zonal mean average transports and $\overline{\mathbf{U_h}^*}$ their zonal anomaly. Similarly we have mean temperatures :

$$\overline{\theta} = \overline{<\theta>} + \overline{\theta^*}$$

▶ Neglecting vertical variations of $\mathbf{U_h}$ and θ, we deduce zonally averaged meridional heat transports by standing meanders : $<\overline{V^*\theta^*}>$.

▶ Meanders advect anomalously cold waters $\overline{\theta^*} < 0$ northward $\overline{V^*} > 0$, and reversely anomalously warm waters southward, so that the zonal mean transport by those meanders is poleward :

$$<\overline{V^*\theta^*}> < 0$$

Indeed, waters advected northward come from the south and are colder, and vice versa for waters advected southward.

Role of eddies

a. Detection of 2495 eddies on 28-Aug-1996

SH (cm)

b. Temporal standard deviation of sea level
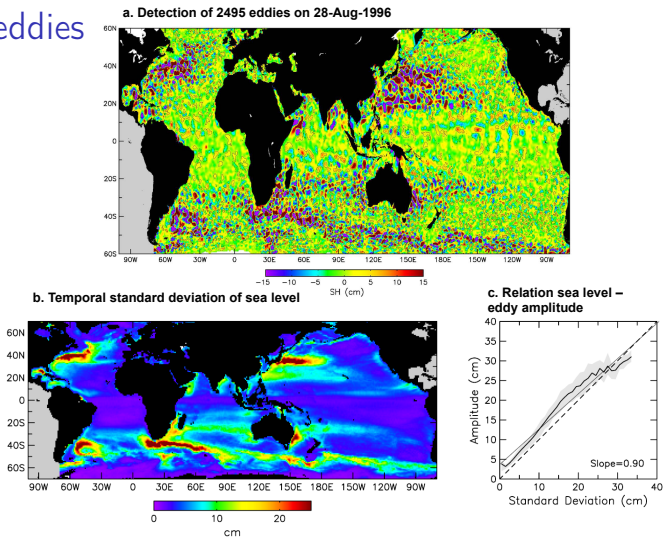
cm

c. Relation sea level – eddy amplitude

Slope=0.90

Figure 25 – a) Dynamic sea level on 28-Aug-1996 (shades) and eddy tracking (contours). b) Temporal standard deviation of sea level (high-pass flitered) and c) its spatial scatterplot with the average eddy sea level amplitude (Chelton et al 2011).

Role of eddies

Transient mesoscale eddies :

- ► Are ubiquitous in the world ocean.
- ► Are particularly intense in baroclinic mid-latitude oceanic regions such as western boundary currents and the Antarctic Circumpolar Current, which indicates their dominant formation through baroclinic instability.
- ► Their magnitude is most commonly diagnosed by computing the temporal standard deviation of dynamic sea level. This is an indication that instantaneous surface geostrophic velocities are dominated by those eddies, as we have seen in Chapter 1.

Role of eddies

Impact of eddies on the mean circulation :

- ► They play a large role at mid-latitude in ocean meridional heat transports.
- ► They induce a meridional overturning transport of buoyancy which is opposed to the Deacon cell, although it is not visible in terms of time mean volume transports. This effect can be quantified with the so-called "transformed Eulerian mean" formalism.
- ► They modify the zonal mean flow. This effect is qualified as the "eddy rectification" of the mean zonal flow. The thermal wind relation states that the magnitude of zonal transports is closely related to meridional gradients of buoyancy, which are reduced by eddies.

Role of eddies

The "Transformed Eulerian Mean" formalism :

▶ The principle is to decompose eddy density fluxes (mostly temperature fluxes) into an isopycnal component that writes as a transport streamfunction and a diapycnal component that write as a diffusivity.

▶ Let us consider the zonally-averaged mean density equation in the adiabatic ocean interior :

$$\frac{\partial \overline{\rho}}{\partial t} + (\overline{\mathbf{v}}.\boldsymbol{\nabla})\overline{\rho} + \boldsymbol{\nabla}.\overline{\mathbf{v}'\rho'} = 0$$

where we have decomposed the advection term into the mean and eddy transports, $\mathbf{v} = (v, w)$ and $\boldsymbol{\nabla} = (\frac{\partial}{\partial y}, \frac{\partial}{\partial z})$ denote the zonally-averaged velocities and gradient. Density is conserved, so that its time evolution at a given location is only due to its advection in the meridional plane by mean and eddy velocities.

Role of eddies

▶ Let us decompose the eddy transport into a component parallel to isopycnals and another one normal to them :

$$\overline{\mathbf{v'}\rho'} = \chi \underset{\neg}{\boldsymbol{\nabla}} \overline{\rho} + K \boldsymbol{\nabla} \overline{\rho}$$

with $\underset{\neg}{\boldsymbol{\nabla}} = (\frac{\partial}{\partial z}, -\frac{\partial}{\partial y})$ the rotated gradient operator ($90°$ clockwise to $\boldsymbol{\nabla}$), χ the along-isopycnal eddy transport and K the across-isopycnal eddy transport given by the formulas :

$$\chi = \frac{\overline{\mathbf{v'}\rho'}.\underset{\neg}{\boldsymbol{\nabla}}\overline{\rho}}{(\boldsymbol{\nabla}\overline{\rho})^2}$$

$$K = \frac{\overline{\mathbf{v'}\rho'}.\boldsymbol{\nabla}\overline{\rho}}{(\boldsymbol{\nabla}\overline{\rho})^2}$$

We have just projected the eddy transport term onto a new coordinate system following density surfaces.

Role of eddies

- The eddy transport divergence term of the density equation now writes as :

$$\boldsymbol{\nabla} . \overline{\mathbf{v'}\rho'} = \boldsymbol{\nabla} . (\chi \underset{\urcorner}{\boldsymbol{\nabla}} \overline{\rho}) + \boldsymbol{\nabla} . (K \boldsymbol{\nabla} \overline{\rho})$$

The second term has the form of the usual eddy diffusivity introduced for the closure of turbulence, as we have seen in Chapter 2. However, there is an additional term which does not take the form of a diffusivity.

- Let us develop it :

$$
\begin{aligned}
\boldsymbol{\nabla} . (\chi \underset{\urcorner}{\boldsymbol{\nabla}} \overline{\rho}) &= \frac{\partial}{\partial y}(\chi \frac{\partial \overline{\rho}}{\partial z}) + \frac{\partial}{\partial z}(\chi(-\frac{\partial \overline{\rho}}{\partial y})) \\
&= \frac{\partial \chi}{\partial y}\frac{\partial \overline{\rho}}{\partial z} - \frac{\partial \chi}{\partial z}\frac{\partial \overline{\rho}}{\partial y} \\
&= -\underset{\urcorner}{\boldsymbol{\nabla}}\chi . \boldsymbol{\nabla} \overline{\rho}
\end{aligned}
$$

Role of eddies

▶ This shows that χ is the streamfunction of the eddy-induced transport. Indeed, if we introduce $v^* = -\frac{\partial \chi}{\partial z}$ and $w^* = +\frac{\partial \chi}{\partial y}$, we find the following density equation :

$$\frac{\partial \overline{\rho}}{\partial t} + ((\overline{\mathbf{v}} + \mathbf{v}^*).\boldsymbol{\nabla})\overline{\rho} + \boldsymbol{\nabla}.(K\boldsymbol{\nabla}\overline{\rho}) = 0$$

So far we have made no approximation. Our decomposition illustrates that eddy transports take the form of eddy-induced velocities plus eddy diffusivities.

Role of eddies

▶ Now we must close turbulent terms χ and K. We make the so-called "eddy adiabatic" approximation $K = 0$, which assumes that eddies transport tracers along isopycnals. This allows to introduce only one closure equation (e.g. for $\overline{v'\rho'}$) and deduce the second ($\overline{w'\rho'}$) so that the eddy flux is isopycnal :

$$
\begin{aligned}
\overline{v'\rho'} &= -\kappa^* \frac{\partial \overline{\rho}}{\partial y} \\
\Longrightarrow \overline{w'\rho'} &= -(\frac{\partial \overline{\rho}}{\partial y} / \frac{\partial \overline{\rho}}{\partial z})\overline{v'\rho'} \\
&= -\kappa^* s \frac{\partial \overline{\rho}}{\partial y} \\
&= +\kappa^* s^2 \frac{\partial \overline{\rho}}{\partial z}
\end{aligned}
$$

with κ^* the isopycnal eddy diffusivity and $s = -(\frac{\partial \overline{\rho}}{\partial y} / \frac{\partial \overline{\rho}}{\partial z})$ the isopycnal slope.

Role of eddies

Finally, we obtain a very simplified expression for χ :

$$
\begin{aligned}
\chi &= \frac{\overline{\mathbf{v}'\rho'}.\boldsymbol{\nabla}\overline{\rho}}{(\boldsymbol{\nabla}\overline{\rho})^2} \\
&= \frac{\overline{v'\rho'}}{\frac{\partial \overline{\rho}}{\partial z}} \times \frac{1+s^2}{1+s^2} \\
&= -\kappa^*(\frac{\partial \overline{\rho}}{\partial y}/\frac{\partial \overline{\rho}}{\partial z}) \\
&= +\kappa^* s
\end{aligned}
$$

After a lot of pain, we have just formulated the streamfunction that defines the "eddy-induced velocities" introduced in Chapter 2. It is used as a parametrization of eddies in all low resolution ($\sim 1°$) ocean models because it largely improves the mean flow and water mass distribution.

Role of eddies
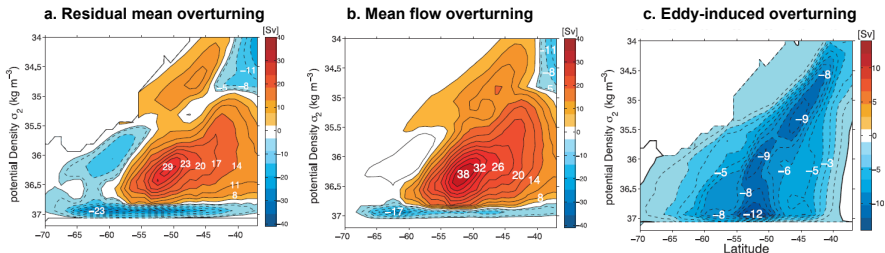
Figure 26 – Residual mean, mean and eddy-induced meridional overturning diagnosed in the Southern Ocean (Farneti et al 2010). The vertical coordinate is density and can be considered similar to depth.

Because of transient eddies, the wind-driven overturning is weakened, which transports heat southward and weakens the Antarctic Circumpolar Current.

Role of eddies

In the Southern Ocean :

- ▶ The eddy-induced overturning is opposite to the mean overturning.
- ▶ So that the overturning felt by tracers (e.g. temperature), called the residual circulation, is weakened.
- ▶ The eddy-induced overturning advects light (warm) waters poleward and dense (cold) waters equatorward, which also explains the intense poleward heat transport by eddies.

Interpretation in terms of the Lorenz energy cycle

The Southern Ocean circulation highlights the importance of both the mean and eddy dynamics. The oceanic Lorenz Energy Cycle describes exchanges between those reservoirs of mechanical energy :

- ▶ The available potential energy (P) is the gravitational potential energy with respect to a horizontally homogeneous reference state, that is the energy available for conversion into kinetic energy.
- ▶ We separate both P and kinetic energy (K) into a mean (Pm and Km) and eddy (Pe and Ke) component, respectively for the time mean and time-varying contributions.
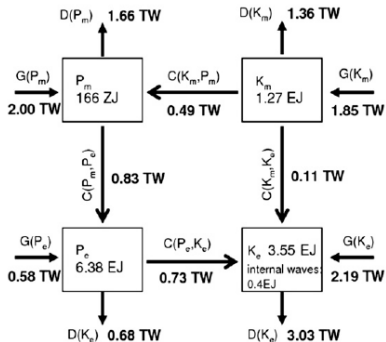
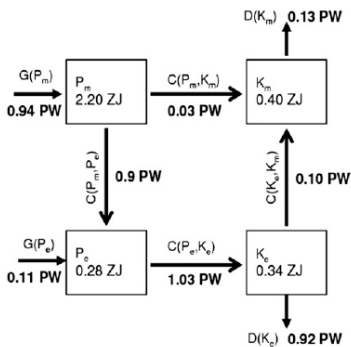# Interpretation in terms of the Lorenz energy cycle
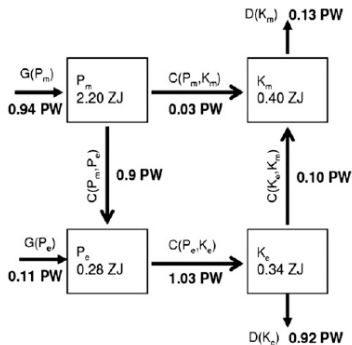
Figure 27 – a) Oceanic and b) atmospheric Lorenz Energy Cycles deduced from high-resolution simulations (von Storch et al 2012). Conversion terms are noted C, generation G and dissipation D.

Interpretation in terms of the Lorenz energy cycle

- ▶ Wind work ($G(Km)$) is the main source of Km : the mean circulation in the Southern Ocean is set up by the wind forcing.
- ▶ Positive $C(Km, Pm)$ flux : Ekman pumping builds meridional density gradients.
- ▶ Strong $G(Pm)$: Pm can also be directly created by buoyancy (mostly heat) fluxes at surface which contribute to meridional density gradients.
- ▶ Positive $C(Pm, Pe)$ and $C(Pe, Ke)$: once large density gradients are built, they can be converted by bariclinic instability into mesoscale eddies (Ke) which are the main kinetic energy reservoir of the ocean.
- ▶ Intense $D(Ke)$: kinetic energy is transferred to lower scales where it can ultimately dissipate.
- ▶ Transfers between Km and Ke are limited, meaning that neither component drives the other : their is neither intense barotropic instability ($C(Km, Ke)$) nor an intense inverse cascade ($C(Ke, Km)$).
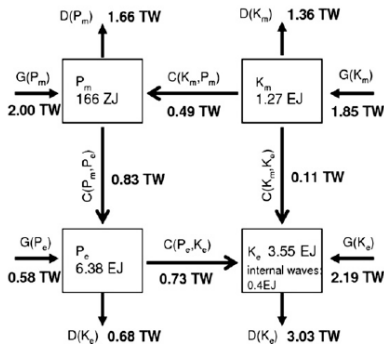
Interpretation in terms of the Lorenz energy cycle

Figure 28 – a) Oceanic and b) atmospheric Lorenz Energy Cycles deduced from high-resolution simulations (von Storch et al 2012). Conversion terms are noted C, generation G and dissipation D.

▶ Both fluids are strikingly different owing to the different nature of their forcings.

Interpretation in terms of the Lorenz energy cycle

Differences with the atmospheric Lorenz Energy cycle :

- ▶ In the atmosphere, the forcing is diabatic and not mechanical, so that it increases Pm.

- ▶ From that, atmospheric dynamics must extract this energy through baroclinic conversion to be set into motion.

- ▶ Ultimately, it is the transient eddies that drive the mean circulation through the so-called inverse cascade.

- ▶ In this sense, eddies have a greater role in atmosphere than ocean, and the atmosphere has a much less forced and more internal dynamical nature than the ocean.

Outline

Along the Equator, the ocean dynamics are fundamentally different because the Coriolis acceleration cancels out.

- ▶ Very fast (Kelvin, Rossby) waves that make of the ocean and atmosphere a coupled system, contrary to extratropical latitudes.
- ▶ Currents are very strong ($\sim 1m/s$) and unbalanced, with a wind-driven flow extending to higher depth ($\sim 100 - 200m$).
- ▶ Hence temporal variability (ENSO, monsoon, seasonal cycle) largely masks mean patterns.
- ▶ Similarities with Extratropics : geostrophic and Ekman balances are rapidly restored, and winds play a determinant role in the circulation.

Meridional circulation

Strong modulations of Ekman transports :

- ▶ From the maximum trade wind jet (around $\pm 10 - 15°$ of latitude), trade winds weaken to reach the so-called "doldrums" at the Inter-Tropical Convergence Zone.

- ▶ However, the Coriolis parameter also varies rapidly to cancel at the Equator.

- ▶ These variations generate the Tropical Gyres, the Tropical Cells and the Subtropical Cells.
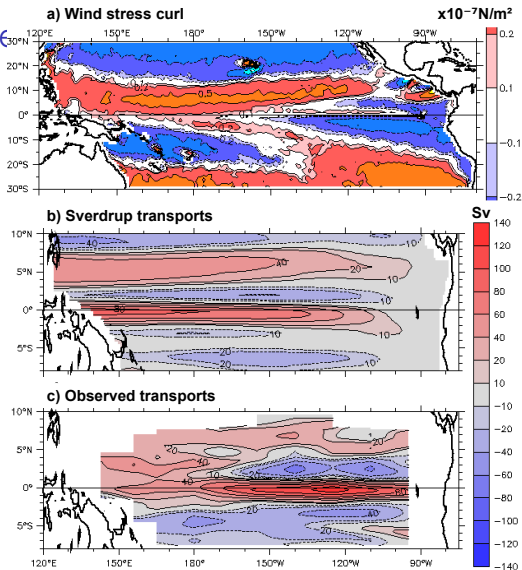
Tropical gyres

Figure 29 – a) Observed wind stress curl, b) vertically-integrated transports predicted from Sverdrup theory and c) observed vertically-integrated transports in the Tropical Pacific (Kessler et al 2003).

Tropical gyres

Interpretation :

- In the $\sim 5-15°$ latitude band, as predicted by Munk, tropical gyres are generated by the positive Ekman pumping due to the poleward intensification of trade winds.

- Symmetrically to subtropical gyres, the Ekman pumping induces a poleward interior Sverdrup transport which is returned by an equatorward western boundary current.

- The subtropical-tropical gyre separation defines the North Equatorial Current, whereas the southern edge of the tropical gyre defines the North Equatorial Countercurrent.

Tropical gyres

Limitations :

- ▶ Sverdrup theory predicts much weaker transports than what is observed.
- ▶ The North Equatorial Countercurrent is part of the Equatorial current system and largely deviates from Sverdrup balance.
- ▶ Much less steady features than their subtropical and subpolar counterparts :
  - ▶ The Ekman pumping almost cancels in the winter of each hemisphere, so that the corresponding tropical gyres can even disappear as is the case in the tropical North Atlantic.
  - ▶ Trade winds are interrupted by the seasonal to interannual variability associated with ENSO and the monsoon systems, and hence so are the tropical gyres.

Tropical cells

a) Meridional overturning (Sv) and potential density (kg/m³)

b) Location and volumic flux (mSv) of parcels reaching the Equatorial Undercurrent
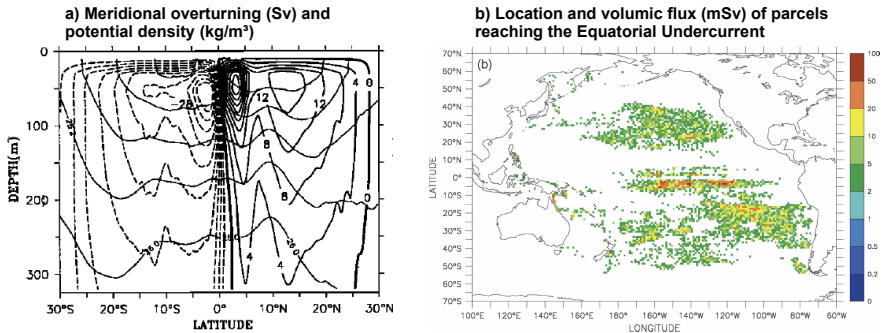
Figure 30 – a) Meridional overturning streamfunction and potential density section from a numerical model of the Tropical Indo-Pacific and b) location and volumic flux of simulated Lagrangian parcels reaching the Pacific Equatorial Undercurrent (Hazeleger et al 2001, Goodman et al 2005).

Tropical cells

In the Deep Tropics ($\pm 5°$ latitude band), the Ekman pumping is not dominated by the wind stress curl (the trade wind weakening) anymore but by the beta effect. In the Equatorial beta plane, the Coriolis parameter is : $f = \beta y \simeq \frac{2\Omega}{R_a} y$ with R_a the Earth's radius. Hence the Ekman pumping becomes :

Tropical cells

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$$
\begin{aligned}
w(-h_E) &= \frac{1}{\rho_0} Curl(\frac{\tau}{\beta y}) \\
&= \frac{1}{\rho_0}\left[\frac{\partial \tau_y/(\beta y)}{\partial x} - \frac{\partial \tau_x/(\beta y)}{\partial y}\right] \\
&= -\frac{1}{\rho_0}\left[\frac{1}{\beta y} Curl(\tau) - \frac{\tau_x}{\beta y^2}\right] \\
&= \frac{1}{\rho_0 \beta y}\left[\frac{\tau_x}{y} - Curl(\tau)\right]
\end{aligned}
$$

Near the Equator, the first term, that is the beta effect, becomes dominant in the Ekman pumping.

Tropical cells

Interpretation :

- ▶ Stronger currents are required for the Coriolis acceleration to balance both wind stress and pressure gradients.
- ▶ Consequence for Ekman transports :
    - ▶ The Ekman pumping becomes negative around $5°$ of latitude.
    - ▶ At the Equator, Coriolis acceleration cancels out and so do the Ekman transports, so that there is an intense Ekman pumping.
- ▶ Consequence for geostrophic transports :
    - ▶ A zonal pressure gradient must prevail to permit an interior return flow and satisfy continuity.
    - ▶ This zonal pressure gradient is itself built by zonal Equatorial currents driven by trade winds that push warm waters in the so-called western "Warm Pools".

We have just described wind-driven meridional cells : the Tropical Cells.

Tropical Instability Waves
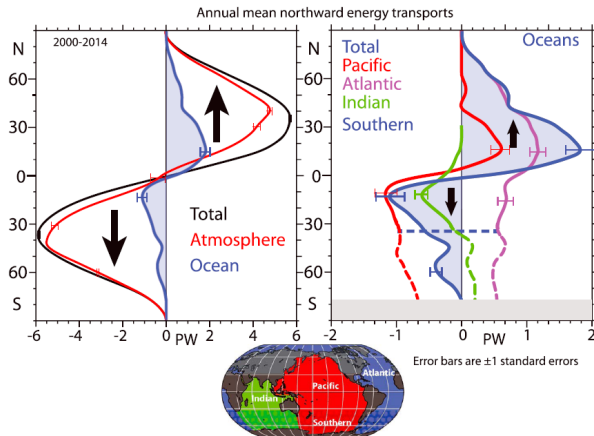
Figure 31 – 2000–2014 average meridional heat transport for the atmosphere and ocean, and per main oceanic basinc (Trenberth and Fasullo 2017).

- The Deep Tropics are the only region where the ocean dominates over the atmosphere in the meridional heat transport.
- Due to Tropical Cells
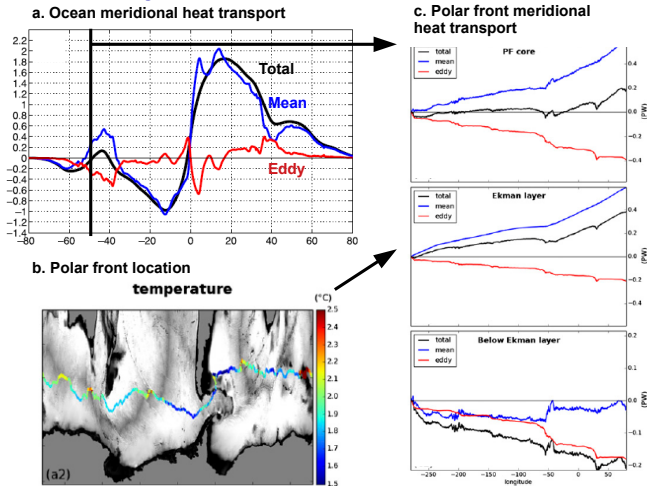
Tropical Instability Waves

Figure 32 – a) Mean and eddy meridional heat flux.

- Similarity with the Southern Ocean : eddy and mean opposed.
- Due to Tropical Instability Waves.

Tropical Instability Waves
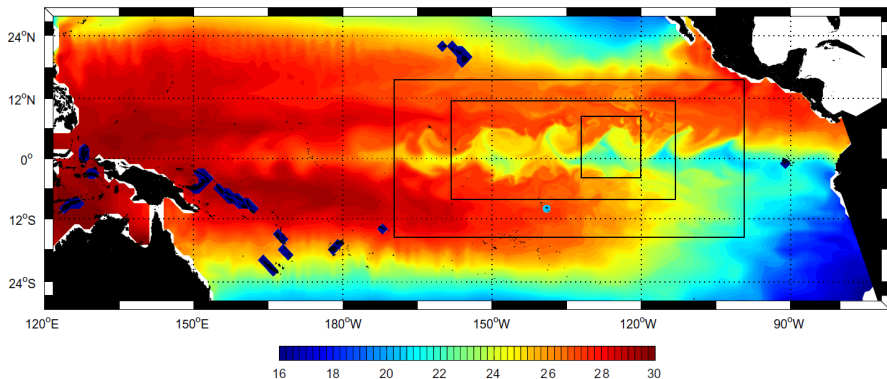
Figure 33 – Sea surface temperature in November 1998 from a nested high-resolution simulation of the Tropical Pacific (Marchesiello et al 2011).

Tropical Instability Waves

Tropical Instability Waves :

- ▶ They are larger than mesoscale eddies ($L \sim 1000 km$) because of the low Coriolis parameter.
- ▶ They are fed by the strong meridional shear of zonal Equatorial currents (barotropic instability).
- ▶ The residual (mean plus eddy-driven) overturning transport is weakened : poleward heat advection is reduced.
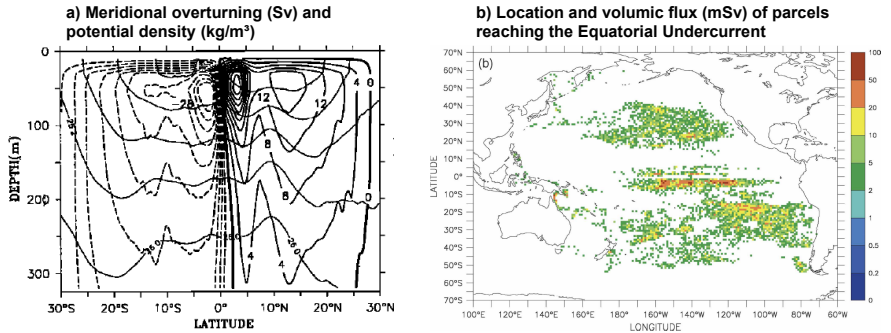
Subtropical cells
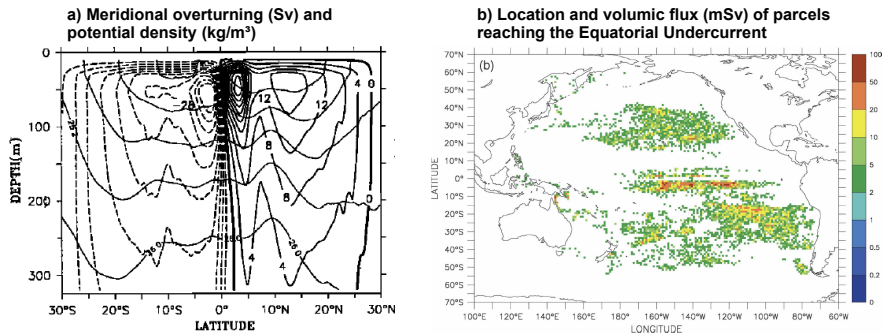
Figure 34 – a) Meridional overturning streamfunction and potential density section from a numerical model of the Tropical Indo-Pacific and b) location and volumic flux of simulated Lagrangian parcels reaching the Pacific Equatorial Undercurrent (Hazeleger et al 2001, Goodman et al 2005).

Subtropical cells

- In the wide subtropical band ($\pm 30°$ of latitude), there is poleward Ekman transport driven by Easterly winds.
- It corresponds to the upper branch of a planetary overturning cell that upwells in the Equator and sinks within the subtropical gyre.
- By continuity, a return flow must exist at depth, defining the Subtropical Cells.
- Because of the northward interior flow within the Tropical Gyres, this southward flow occurs southwestward within the Subtropical Gyre, and then through the western boundary current of the Tropical Gyre.

Subtropical cells

a) Meridional overturning (Sv) and potential density (kg/m³)

b) Location and volumic flux (mSv) of parcels reaching the Equatorial Undercurrent

Figure 35 – a) Meridional overturning streamfunction and potential density section from a numerical model of the Tropical Indo-Pacific and b) location and volumic flux of simulated Lagrangian parcels reaching the Pacific Equatorial Undercurrent (Hazeleger et al 2001, Goodman et al 2005).

Similarly to Subtropical Gyres, Subtropical Cells trap mode waters for years to decades in the adiabatic interior ocean!

Zonal circulation

- The most intense circulation features of the Deep Tropics are zonal.
- Their exceptional magnitude is related to the cancellation of the Coriolis acceleration at the Equator.

South Equatorial Currents

Equatorial Rossby number :

South Equatorial Currents

Equatorial Rossby number : $Ro = \frac{U}{fL} = \frac{U}{\beta L^2} \simeq \frac{R_a U}{2\Omega y^2}$

- ▶ Submesoscale dynamics :

South Equatorial Currents

Equatorial Rossby number : $Ro = \frac{U}{fL} = \frac{U}{\beta L^2} \simeq \frac{R_a U}{2\Omega y^2}$

▶ Submesoscale dynamics :

$$Ro \simeq 1 \iff y \simeq 250km$$

that is at $\phi \simeq 2°$ of latitude. Interpretation :

- ▸ $Ro = \zeta/f \simeq 1$: the effect of rotation is still felt by water masses, but advective effects become as important.
- ▸ Usually very small oceanic scales ($\sim 1-10km$), but also concerns large scales along the Equator !
- ▸ All terms of the horizontal momentum equations become important !

▶ Quasi-geostrophic dynamics :

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- ► $Ro = \zeta/f \simeq 1$: the effect of rotation is still felt by water masses, but advective effects become as important.
- ► Usually very small oceanic scales ($\sim 1 - 10 km$), but also concerns large scales along the Equator !
- ► All terms of the horizontal momentum equations become important !

► Quasi-geostrophic dynamics :

$$Ro \simeq 0.1 \iff y \simeq 800 km$$

that is at $\phi \simeq 7°$ of latitude, where dynamics becomes similar to Extratropics again.

► Both dynamical regimes characterize the South Equatorial Currents which gather westward currents extending from $3°N$ to $20°S$.
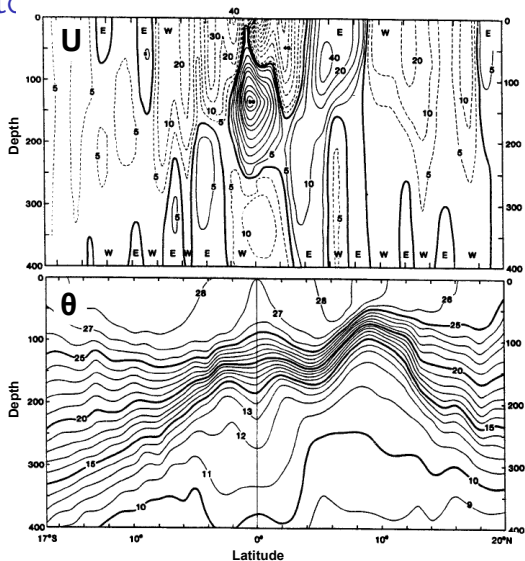
Figure 36 – Observed meridional section of zonal currents and potential temperature across the central Tropical Pacific ($155°W$, between Tahiti and Hawaii, Lu et al 1998).

South Equatorial Currents

Equatorial South Equatorial Current (eSEC) :

- ▶ Along the Equator, Coriolis acceleration can no longer balance the westward wind stress caused by Easterlies.
- ▶ Hence the flow is accelerated westards until water masses accumulate in the western Warm Pool and build an eastward pressure gradient that balances wind stress.

South Equatorial Currents
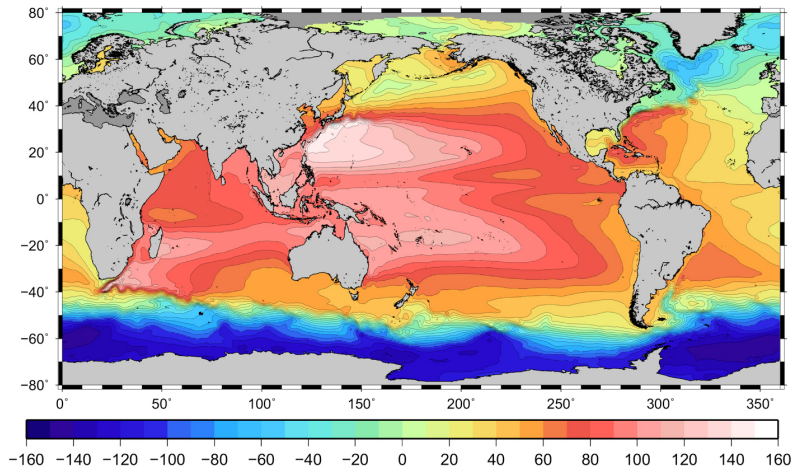
Figure 37 – Mean dynamic topography (cm) deduced from satellite (ESA/CNES/CLS).

▶ Pressure gradient visible at surface from the zonal slope of the dynamic sea level !
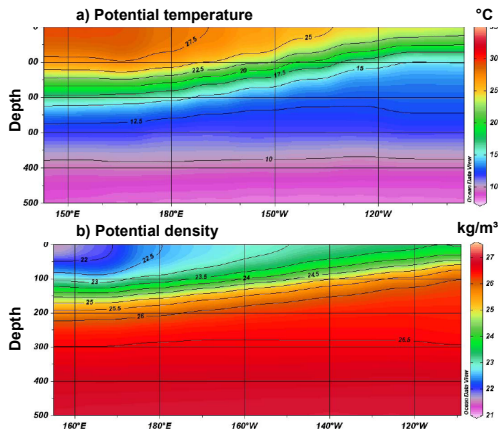
South Equatorial Currents

Figure 38 – Observed mean zonal section of a) potential temperature and b) potential density along the Equatorial Pacific (LT's lecture).

▶ The abyssal ocean being mostly at rest, an opposite thermocline slope exists in the interior ocean (Margules's relation)!

South Equatorial Currents

Margules's relation at the Equator :

- With a typical dynamic sea level difference of $\Delta\eta \simeq -60cm$ in the Equatorial Pacific, we get a thermocline depth difference of

South Equatorial Currents

Margules's relation at the Equator :

- With a typical dynamic sea level difference of $\Delta\eta \simeq -60cm$ in the Equatorial Pacific, we get a thermocline depth difference of $\Delta h \simeq -200 \times \Delta\eta \simeq +120m$.
- Typical in the tropical Pacific of a neutral ENSO anomaly, and enhanced during La Niña events.

South Equatorial Currents

Exercise : zonal sea level slope within the frictional surface layer.

We suppose a surface frictional layer of depth $H = 100m$ where zonal pressure gradients balance the surface wind stress. We assume a homogeneous layer of density $\rho_0 = 1025 kg/m^3$. Deduce the zonal sea level gradient balancing a trade wind of magnitude $u_{10m} = -5m/s$. To what zonally-integrated sea level difference does it correspond in the Equatorial Pacific of width $W = 8,000km$? We assume $g \sim 10m^2/s$, $\rho_a \sim 1kg/m^3$ and $C_d \sim 2 \times 10^{-3}$.

South Equatorial Currents

Solution : the zonal momentum equation writes as :

$$0 = -\frac{1}{\rho_0}\frac{\partial P}{\partial x} + \frac{\partial}{\partial z}(\kappa_{zu}\frac{\partial u}{\partial z})$$

$$\Longleftrightarrow 0 = -g\frac{\partial \eta}{\partial x} + \frac{\partial}{\partial z}(\kappa_{zu}\frac{\partial u}{\partial z})$$

Integrating over the frictional surface layer yields :

$$gH\frac{\partial \eta}{\partial x} = \left[\kappa_{zu}\frac{\partial u}{\partial z}\right]_{-H}^{\eta}$$

$$= \frac{\tau_x}{\rho_0} = -\frac{\rho_a C_d u_{10m}^2}{\rho_0}$$

$$\Longrightarrow \Delta\eta = -\frac{W\rho_a C_d u_{10m}^2}{\rho_0 gH} \simeq -40cm$$

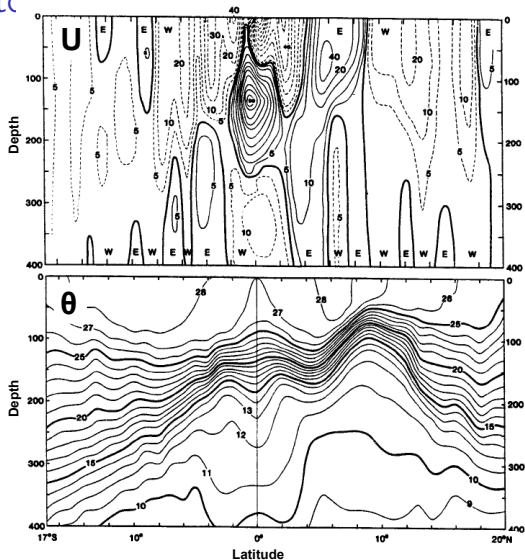*which is of the right order of magnitude.*

Figure 39 – Observed meridional section of zonal currents and potential temperature across the central Tropical Pacific ($155°W$, between Tahiti and Hawaii, Lu et al 1998).

South Equatorial Currents

On each side of the eSEC, in the Deep Tropics, lie the Northern and Central South Equatorial Currents (nSEC and cSEC).

- ▶ The wind stress still accelerates their westward flow, but the Coriolis acceleration becomes important so that the meridional pressure gradient also drives the westward flow.

- ▶ Because of the Equatorial upwelling, an intense equatorial sea level minimum causes a poleward pressure gradient and hence a westward geostrophic flow.

- ▶ Despite these dominant mechanisms, all terms of the zonal momentum equation matter!

South Equatorial Currents

Finally, outside of the Deep Tropics, the southern South Equatorial Current (sSEC) :

- ▶ Southern Hemisphere counterpart of the North Equatorial Current, that is the westward return flow of the subtropical gyre.
- ▶ Its dynamic is quasi-geostrophic and hence simpler and not equatorial anymore.

South Equatorial Currents

The very large magnitude of the South Equatorial Currents located in the Deep Tropics ($\sim 1m/s$) reveals the weakness of Coriolis acceleration :

- ▶ Because surface currents are more aligned to surface winds, the wind work is largely increased which permits higher levels of kinetic energy, despite modest winds along the Equator.

- ▶ The vertical extent of the wind-driven current is not limited anymore by Coriolis acceleration, so that it can reach much higher depth ($h \sim 200m$) than Ekman currents ($h_E \sim 50m$). Indeed, vertical turbulent momentum fluxes are not balanced anymore by Coriolis acceleration, so that they must penetrate deeper for other terms to balance them.
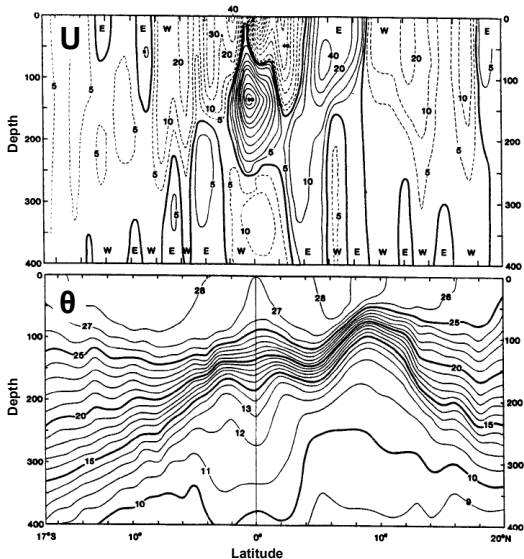
North Equatorial Countercurrent

Figure 40 – Observed meridional section of zonal currents and potential temperature across the central Tropical Pacific.

North Equatorial Countercurrent

- The North Equatorial Countercurrent's very large magnitude (up to $\sim 1m/s$) cannot be simply explained by gyre dynamics.

- It is above all the geostrophic response to the intense Ekman suction at the northern edge of the northern Tropical Cell.

- The meridional pressure gradient is opposite to that along the Equator driving the nSEC and cSEC, so that geostrophic currents are eastward.

- Similarly to Ekman currents within the Tropical Cells, due to the weakness of Coriolis, intense geostrophic currents are required for the geostrophic balance to be reached.
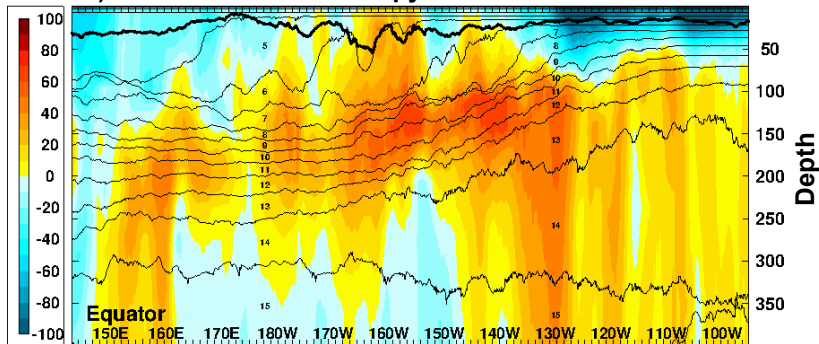
Equatorial Undercurrent

Figure 41 – c) Near real-time zonal current and isopycnal thickness in September 2018 from MYCOM analysis (US Navy).

Equatorial Undercurrent

The Equatorial Undercurrent is probably the most striking illustration of the unique nature of Equatorial dynamics :

- ► Depth-intensified current reaching a maximum of $\sim 1 m/s$ around $100 - 200m$ depth with no surface signature !
- ► Strongest interior current and largest vertical shears of the World ocean.
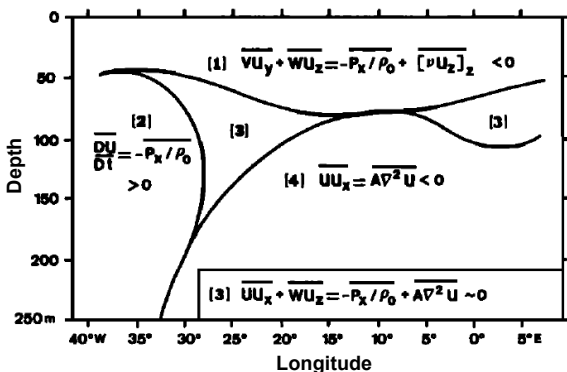- ► Opposite to local wind forcing.

Equatorial Undercurrent

Figure 42 – Zonal momentum balance along the Equatorial Atlantic from a numerical model (Wacongne 1989).

1. Surface eSEC under the effect of wind stress.
2. Western Equatorial Undercurrent accelerated by the pressure gradient
3. Eastern Equatorial Undercurrent slowed down by meridional export of momentum
4. Transitional areas involving all terms except vertical friction.

Equatorial Undercurrent

Western acceleration :

- ▶ The zonal pressure gradient generated by surface South Equatorial currents not equilibrated below the frictional layer. Hence intense zonal acceleration of the flow.
- ▶ Assuming the zonal pressure gradient is purely barotropic (driven by sea level), the zonal momentum equation is :

Equatorial Undercurrent

Western acceleration :

- ▶ The zonal pressure gradient generated by surface South Equatorial currents not equilibrated below the frictional layer. Hence intense zonal acceleration of the flow.
- ▶ Assuming the zonal pressure gradient is purely barotropic (driven by sea level), the zonal momentum equation is :

$$\frac{du}{dt} = -g\frac{\partial \eta}{\partial x} \simeq +10 \times \frac{0.60}{8 \times 10^6} \simeq 10^{-6} m/s^2$$

with $\frac{du}{dt} \simeq u\frac{\partial u}{\partial y}$ for a purely zonal steady state.

- ▶ Integrating from the western boundary where there is no normal flow,

Equatorial Undercurrent

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with $\frac{du}{dt} \simeq u\frac{\partial u}{\partial y}$ for a purely zonal steady state.

- ▶ Integrating from the western boundary where there is no normal flow,

$$
\begin{aligned}
u(t) &= 10^{-6}t; \quad x(t) = 0.5 \times 10^{-6} t^2 \\
\implies u = 1.5 m/s &\iff t \simeq 15\, days; \quad x \simeq 1,000 km
\end{aligned}
$$

This gives a lower bound for the western boundary width where the Equatorial undercurrent is accelerated.
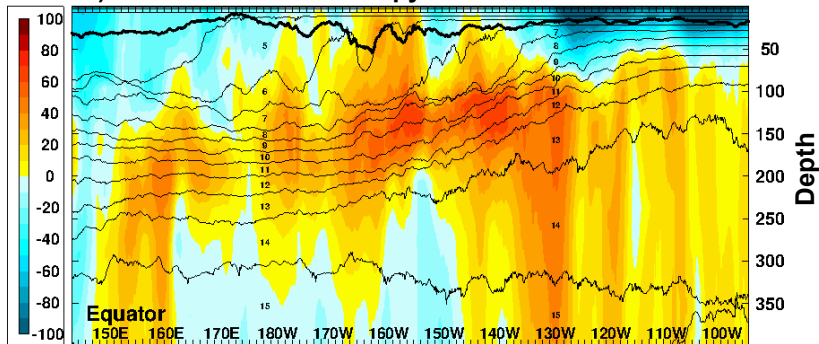
Equatorial Undercurrent

Figure 43 – c) Near real-time zonal current and isopycnal thickness in September 2018 from MYCOM analysis (US Navy).

Equatorial Undercurrent

Eastern deceleration :

- ▶ The zonal pressure gradients weaken.
- ▶ Meridional eddy momentum fluxes due to Tropical Instability Waves export zonal momentum.
- ▶ Due to the intense equatorial upwelling, the Equatorial Undercurrent is slanted : in the East, it can ultimately feel surface wind stress that slows it down.
- ▶ In this case, the zonal momentum equation writes as :

Equatorial Undercurrent

Eastern deceleration :

- ▶ The zonal pressure gradients weaken.
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- ▶ In this case, the zonal momentum equation writes as :

$$\frac{du}{dt} = -g\frac{\partial \eta}{\partial x} + \frac{\partial}{\partial z}(\kappa_{zu}\frac{\partial u}{\partial z}) + \frac{\partial}{\partial y}(\kappa_{hu}\frac{\partial u}{\partial y})$$

The second and third terms to the right hand side, due to wind stress and Tropical Instability Waves, are negative.

Equatorial Undercurrent

Exercise : what would happen to the tropical Pacific circulation if the Equatorial trades were suddenly reversed into Westerlies ?

Equatorial Undercurrent

Response : El Niño. Namely :

- ▶ *Reversal of the eSEC and of the Tropical Gyres, hence also of the nSEC, cSEC*
- ▶ *Cancellation of the Tropical Gyre circulation and of the North Equatorial Countercurrent.*
- ▶ *Consequence for temperature distributions : weakening to cancellation of the zonal sea level, temperature and hence pressure gradients, and as a consequence also weakening of the Equatorial Undercurrent.*
- ▶ *This response is not instantaneous and involves mostly Equatorial Kelvin and Rossby waves of typical propagation times ~ 2 months and ~ 6 months across the Tropical Pacific. Note that mid-latitude Rossby waves take typically decades to cross oceanic basins : this is why the Equatorial ocean variability is coupled to the atmosphere, contrary to mid-latitudes.*